Riemann in ADM decomposition

i. Gauss relation

$$\perp R_{\mu\nu\rho\sigma} \equiv h^{\alpha}{}_{\mu} h^{\beta}{}_{\nu} h^{\gamma}{}_{\rho} h^{\delta}{}_{\sigma} R_{\alpha\beta\gamma\delta} = {}^{(3)}R_{\mu\nu\rho\sigma} + K_{\mu\rho}K_{\nu\sigma} - K_{\mu\sigma}K_{\nu\rho} ;$$

ii. Codazzi relation

$$\perp R_{\mu\nu\rho\boldsymbol{n}} \equiv h^{\alpha}{}_{\mu} h^{\beta}{}_{\nu} h^{\gamma}{}_{\rho} n^{\delta} R_{\alpha\beta\gamma\delta} = 2D_{[\mu}K_{\nu]\rho} ;$$

iii. Ricci relation

$$\perp R_{\mu\boldsymbol{n}\nu\boldsymbol{n}} \equiv h^{\alpha}{}_{\mu} n^{\beta} h^{\gamma}{}_{\nu} n^{\delta} R_{\alpha\beta\gamma\delta} = K_{\mu\rho}K_{\nu}{}^{\rho} - \mathcal{L}_n K_{\mu\nu} + D_{(\mu}a_{\nu)} + a_{\mu}a_{\nu} .$$

Ricci in ADM decomposition

$$\begin{aligned}
\perp R_{\mu\nu} &= {}^{(3)}R_{\mu\nu} + K_{\mu\nu}K - 2K_{\mu\rho}K^{\rho}{}_{\nu} + \mathcal{L}_n K_{\mu\nu} - D_{(\mu}a_{\nu)} - a_{\mu}a_{\nu} , \\
\perp R_{\mu\boldsymbol{n}} &= D_{\nu}K^{\nu}{}_{\mu} - D_{\mu}K , \\
R_{\boldsymbol{n}\boldsymbol{n}} &= K_{\mu\nu}K^{\mu\nu} - h^{\mu\nu}\mathcal{L}_n K_{\mu\nu} + D_{\mu}a^{\mu} + a_{\mu}a^{\mu} ,
\end{aligned}$$