3D Airplane Attitude Controls

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Given:

$$\mathbf{R_d}(t) = \mathbf{R_{desired}}(t) = \text{desired rotation matrix}$$

$$\hat{\mathbf{R}}(t) = \text{estimated rotation matrix}$$

$$\mathbf{\Omega} = \text{measured rotation rate in body frame}$$

$$\mathrm{V} = \text{measured air speed}$$

Motion of the plane is described by desired evolution of the attitude.

Control:

$$\dot{\mathbf{R}}_\mathbf{d} = \frac{d\mathbf{R_d}}{dt}$$

$$\mathbf{\Omega_d} = \frac{1}{2}\text{vex}\left(\mathbf{R_d^T} \cdot \dot{\mathbf{R}}_\mathbf{d} - \dot{\mathbf{R}}_\mathbf{d}^\mathbf{T} \cdot \mathbf{R_d}\right)$$

$$\overline{\mathbf{R}} = \mathbf{R_d^T} \cdot \hat{\mathbf{R}}$$

$$\boldsymbol{\varepsilon} = \mathbf{\Omega} - \mathbf{\Omega_d} + \frac{\mathbf{1}}{\mathbf{2}} K_P \cdot \text{vex}\left(\overline{\mathbf{R}} - \overline{\mathbf{R}}^\mathbf{T}\right)$$

$$\mathbf{servos} = \frac{\mathbf{\Omega_d} - K_D \cdot \boldsymbol{\varepsilon} - \frac{\mathbf{1}}{\mathbf{2}} K_P \cdot \text{vex}\left(\overline{\mathbf{R}} - \overline{\mathbf{R}}^\mathbf{T}\right)}{V}$$

Navigation, altitude, and speed control are separate, but they are easily built on top of attitude control