Problem -1

$$\lim_{n\to\infty}\left(\frac{1}{n^3}+\frac{4}{n^3}+\frac{9}{n^3}+\ldots+\frac{n^2}{n^3}\right)$$

değerini bulunuz.

Çözüm

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n} f\left(\frac{k}{n}\right)=\int_0^1 f(x)\,dx$$

olduğunu biliyoruz.

$$\lim_{n\to\infty}\left(\frac{1}{n^3}+\frac{4}{n^3}+\frac{9}{n^3}+\ldots+\frac{n^2}{n^3}\right)$$

$$=\lim_{n\to\infty}\frac{1}{n}\left(\frac{1}{n^2}+\frac{4}{n^2}+\frac{9}{n^2}+\ldots+\frac{n^2}{n^2}\right)$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{k^2}{n^2}$$

$$=\int_0^1 x^2\,dx$$

$$=\left.\frac{x^3}{3}\right|_0^1$$

$$=\frac{1}{3}$$

Problem -2

$$\lim_{n\to\infty}\left(\frac{1^{10}+2^{10}+3^{10}+\ldots+n^{10}}{n^{11}}\right)$$

değerini bulunuz.

Çözüm

$$\lim_{n\to\infty}\left(\frac{1^{10}}{n^{11}}+\frac{2^{10}}{n^{11}}+\frac{3^{10}}{n^{11}}+\ldots+\frac{n^{10}}{n^{11}}\right)$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[\left(\frac{1}{n}\right)^{10}+\left(\frac{2}{n}\right)^{10}+\left(\frac{3}{n}\right)^{10}\ldots+\left(\frac{n}{n}\right)^{10}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{k^{10}}{n^{10}}$$

$$=\int_0^1 x^{10}\,dx$$

$$=\left.\frac{x^{11}}{11}\right|_0^1$$

$$=\frac{1}{11}$$

Problem -3

$$\lim_{n\to\infty}\left(\frac{\sqrt[n]{e}+\sqrt[n]{e^2}+\sqrt[n]{e^3}+\ldots+\sqrt[n]{e^n}}{n}\right)$$

değerini bulunuz.

Çözüm

$$\lim_{n\to\infty}\frac{1}{n}\left(e^{\frac{1}{n}}+e^{\frac{2}{n}}+e^{\frac{3}{n}}+\ldots+e^{\frac{n}{n}}\right)$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}e^{\frac{k}{n}}$$

$$=\int_0^1 e^x\,dx$$

$$=\left.e^x\right|_0^1$$

$$=e-1$$

Problem - 4

$\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left\{\left[\left(1+\frac{3\cdot i}{n}\right)^2+2\right]\cdot\frac{3}{n}\right\}$ ifadesi aşağıdakilerden hangisine eşittir?

A) $\int\limits_1^3\left(x^2+1\right)dx$ B) $\int\limits_1^3\left(x^2+2\right)dx$

C) $\int\limits_1^2\left(x^2+2\right)dx$ D) $\int\limits_0^4\left(x^2+2\right)dx$

E) $\int\limits_1^4\left(x^2+2\right)dx$

Çözüm

$\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{k=1}^{n}f\left(\frac{k}{n}\right)=\int\limits_0^1 f(x)dx$

olduğunu biliyoruz.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left\{\left[\left(1+\frac{3\cdot i}{n}\right)^2+2\right]\cdot\frac{3}{n}\right\}$$

$$=\lim_{n\to\infty}\frac{3}{n}\cdot\sum_{i=1}^{n}\left[\left(1+\frac{3\cdot i}{n}\right)^2+2\right]$$

$$=3\cdot\lim_{n\to\infty}\frac{1}{n}\cdot\sum_{i=1}^{n}\left[\left(1+3\cdot\frac{i}{n}\right)^2+2\right]$$

$$=3\cdot\int_0^1\left[(1+3\cdot x)^2+2\right]\cdot dx$$

$1+3x=t$ denirse, $3\cdot dx=dt$ ve

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left\{\left[\left(1+\frac{3\cdot i}{n}\right)^2+2\right]\cdot\frac{3}{n}\right\}$$

$$=\int_1^4\left(t^2+2\right)\cdot dt=\int_1^4\left(x^2+2\right)\cdot dx$$

bulunur.

Alıştırma

$\lim\limits_{n\to\infty}\frac{2}{n}\cdot\sum\limits_{j=1}^{n}\left(1+\frac{2\cdot j}{n}\right)^4$ ifadesi aşağıdakilerden hangisine eşittir?

A) $\int\limits_0^3 x^4\cdot dx$ B) $\int\limits_1^3 x^4\cdot dx$ C) $\int\limits_2^4 x^4\cdot dx$

D) $\int\limits_1^2 x^3\cdot dx$ E) $\int\limits_1^3 x^3\cdot dx$

Problem - 5

$\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{k=0}^{n}\left(\frac{n+k^2}{n^2}\right)$ değeri kaçtır?

Çözüm

$\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{k=1}^{n}f\left(\frac{k}{n}\right)=\int\limits_0^1 f(x)dx$

olduğunu biliyoruz.

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n}\left(\frac{n+k^2}{n^2}\right)$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n}\left(\frac{1}{n}+\frac{k^2}{n^2}\right)$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[\sum_{k=0}^{n}\left(\frac{1}{n}\right)+\sum_{k=0}^{n}\left(\frac{k^2}{n^2}\right)\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n}\left(\frac{1}{n}\right)+\lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n}\left(\frac{k^2}{n^2}\right)$$

$$=\lim_{n\to\infty}\frac{1}{n}\cdot\frac{n+1}{n}+\int_0^1 x^2dx$$

$$=0+\left.\frac{x^3}{3}\right|_0^1$$

$=\frac{1}{3}$ bulunur.