Problem -1

$$\lim_{n\to\infty}\left(\frac{1}{n^3}+\frac{4}{n^3}+\frac{9}{n^3}+\ldots+\frac{n^2}{n^3}\right)$$

değerini bulunuz.

Çözüm

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n} f\left(\frac{k}{n}\right)=\int_0^1 f(x)\,dx$$

olduğunu biliyoruz.

$$\lim_{n\to\infty}\left(\frac{1}{n^3}+\frac{4}{n^3}+\frac{9}{n^3}+\ldots+\frac{n^2}{n^3}\right)$$

$$=\lim_{n\to\infty}\frac{1}{n}\left(\frac{1}{n^2}+\frac{4}{n^2}+\frac{9}{n^2}+\ldots+\frac{n^2}{n^2}\right)$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{k^2}{n^2}$$

$$=\int_0^1 x^2dx$$

$$=\left.\frac{x^3}{3}\right|_0^1$$

$$=\frac{1}{3}$$

Problem -2

$$\lim_{n\to\infty}\left(\frac{1^{10}+2^{10}+3^{10}+\ldots+n^{10}}{n^{11}}\right)$$

değerini bulunuz.

Çözüm

$$\lim_{n\to\infty}\left(\frac{1^{10}}{n^{11}}+\frac{2^{10}}{n^{11}}+\frac{3^{10}}{n^{11}}+\ldots+\frac{n^{10}}{n^{11}}\right)$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[\left(\frac{1}{n}\right)^{10}+\left(\frac{2}{n}\right)^{10}+\left(\frac{3}{n}\right)^{10}\ldots+\left(\frac{n}{n}\right)^{10}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{k^{10}}{n^{10}}$$

$$=\int_0^1 x^{10}dx$$

$$=\left.\frac{x^{11}}{11}\right|_0^1$$

$$=\frac{1}{11}$$

Problem -3

$$\lim_{n\to\infty}\left(\frac{\sqrt[n]{e}+\sqrt[n]{e^2}+\sqrt[n]{e^3}+\ldots+\sqrt[n]{e^n}}{n}\right)$$

değerini bulunuz.

Çözüm

$$\lim_{n\to\infty}\frac{1}{n}\left(e^{\frac{1}{n}}+e^{\frac{2}{n}}+e^{\frac{3}{n}}+\ldots+e^{\frac{n}{n}}\right)$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}e^{\frac{k}{n}}$$

$$=\int_0^1 e^x dx$$

$$=\left.e^x\right|_0^1$$

$$=e-1$$