Örnek Problem -1

$\sum_{k=1}^{n}(-1)^k\cdot k^2$ toplamını bulunuz.

Çözüm

n çift sayı ise;

$n=2p$ olsun.

$$\sum_{k=1}^{n}(-1)^k\cdot k^2=-1^2+2^2-\ldots-(2p-1)^2+(2p)^2$$
$$=\sum_{k=1}^{p}(2k)^2-\sum_{k=1}^{p}(2k-1)^2$$
$$=\sum_{k=1}^{p}\left[(2k)^2-(2k-1)^2\right]$$
$$=\sum_{k=1}^{p}(4k-1)$$
$$=4\cdot\frac{p\cdot(p+1)}{2}-p$$
$$=2p^2+p$$
$$=\frac{n^2+n}{2}$$ bulunur.

n tek sayı ise;

$n=2p-1$ olsun.

$$\sum_{k=1}^{n}(-1)^k\cdot k^2=-1^2+2^2-\ldots+(2p-2)^2+(2p-1)^2$$
$$=\sum_{k=1}^{p}(2k-1)^2-\sum_{k=1}^{p}(2k-2)^2$$
$$=\sum_{k=1}^{p}\left[(2k-1)^2-(2k-2)^2\right]$$
$$=\sum_{k=1}^{p}(3-4k)$$
$$=3p-4\cdot\frac{p\cdot(p+1)}{2}$$
$$=p-2p^2$$
$$=\frac{-n^2-n}{2}$$ bulunur.

Uygulama - 1

$\sum_{k=1}^{9}(-1)^k\cdot(2k-1)^2$ toplamını bulunuz.

Çözüm

n = 2p-1 ise;

$$\sum_{k=1}^{n}(-1)^k\cdot(2k-1)^2$$
$$=-1^2+3^2-\ldots+(4p-5)^2+(4p-3)^2$$
$$=\sum_{k=2}^{p}(4k-5)^2-\sum_{k=1}^{p}(4k-3)^2$$
$$=-1+\sum_{k=1}^{p}\left[(4k-5)^2-(4k-3)^2\right]$$
$$\Rightarrow\sum_{k=1}^{9}(-1)^k\cdot(2k-1)^2$$
$$=-1+\sum_{k=1}^{5}(16-16k)$$
$$=-1+5\cdot16-4\cdot\frac{5\cdot6}{2}$$
$$=-161$$ bulunur.

Uygulama - 2

$\sum_{k=1}^{14}(-1)^k\cdot(2k+1)^2$ toplamını bulunuz.

Uygulama - 3

$\sum_{k=1}^{13}(-1)^k\cdot\left(k^2-2k\right)$ toplamını bulunuz.

Uygulama - 4

$\sum_{k=1}^{12}(-1)^k\cdot\left(2k^2+k\right)$ toplamını bulunuz.

Örnek Problem -2

$$\sum_{k=1}^{n}\frac{1}{(2k-1)\cdot\sqrt{2k+1}+(2k+1)\cdot\sqrt{2k-1}}$$

toplamını bulunuz.

Çözüm

$$\sum_{k=1}^{n}\frac{1}{(2k-1)\cdot\sqrt{2k+1}+(2k+1)\cdot\sqrt{2k-1}}$$

$$=\sum_{k=1}^{n}\frac{1}{\sqrt{2k-1}\cdot\sqrt{2k+1}\left(\sqrt{2k-1}+\sqrt{2k+1}\right)}$$

$$=\sum_{k=1}^{n}\frac{\sqrt{2k+1}-\sqrt{2k-1}}{\sqrt{2k-1}\cdot\sqrt{2k+1}(2k+1-2k+1)}$$

$$=\frac{1}{2}\cdot\sum_{k=1}^{n}\left(\frac{1}{\sqrt{2k-1}}-\frac{1}{\sqrt{2k+1}}\right)$$

$$=\frac{1}{2}\cdot\left(\frac{1}{\sqrt{1}}-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{7}}+\dots+\frac{1}{\sqrt{2n-1}}-\frac{1}{\sqrt{2n+1}}\right)$$

$$=\frac{1}{2}\cdot\left(1-\frac{1}{\sqrt{2n+1}}\right)$$ bulunur.

Uygulama - 5

$$\frac{1}{9\sqrt{11}+11\sqrt{9}}+\frac{1}{11\sqrt{13}+13\sqrt{11}}+\dots+\frac{1}{223\sqrt{225}+225\sqrt{223}}$$

toplamını bulunuz.

Çözüm

$$\sum_{k=1}^{n}\frac{1}{(2k-1)\cdot\sqrt{2k+1}+(2k+1)\cdot\sqrt{2k-1}}$$

$$=\sum_{k=1}^{n}\frac{1}{\sqrt{2k-1}\cdot\sqrt{2k+1}\left(\sqrt{2k-1}+\sqrt{2k+1}\right)}$$

$$=\sum_{k=1}^{n}\frac{\sqrt{2k+1}-\sqrt{2k-1}}{\sqrt{2k-1}\cdot\sqrt{2k+1}(2k+1-2k+1)}$$

$$=\frac{1}{2}\cdot\sum_{k=1}^{n}\left(\frac{1}{\sqrt{2k-1}}-\frac{1}{\sqrt{2k+1}}\right)$$ olduğuna göre;

$$\frac{1}{9\sqrt{11}+11\sqrt{9}}+\frac{1}{11\sqrt{13}+13\sqrt{11}}+\dots+\frac{1}{223\sqrt{225}+225\sqrt{223}}$$

$$=\frac{1}{2}\cdot\sum_{k=5}^{112}\left(\frac{1}{\sqrt{2k-1}}-\frac{1}{\sqrt{2k+1}}\right)$$

$$=\frac{1}{2}\cdot\left(\frac{1}{\sqrt{9}}-\frac{1}{\sqrt{11}}+\frac{1}{\sqrt{11}}-\frac{1}{\sqrt{13}}+\frac{1}{\sqrt{13}}-\frac{1}{\sqrt{15}}+\dots+\frac{1}{\sqrt{223}}-\frac{1}{\sqrt{225}}\right)$$

$$=\frac{1}{2}\cdot\left(\frac{1}{3}-\frac{1}{15}\right)$$

$$=\frac{2}{15}$$ bulunur.