Problem -1

$\lim_{x\to 0} \frac{x-\sin x}{x^2}$ değeri kaçtır?

Çözüm

$\sin x = 3\cdot\sin\left(\frac{x}{3}\right) - 4\cdot\sin^3\left(\frac{x}{3}\right)$

özdeşliğini kullanalım:

$$\ell = \lim_{x\to 0}\frac{x-\sin x}{x^2} = \lim_{x\to 0}\frac{x-3\cdot\sin\left(\frac{x}{3}\right)+4\cdot\sin^3\left(\frac{x}{3}\right)}{x^2}$$

$\frac{x}{3} = t$ diyelim:

$$\ell = \lim_{x\to 0}\frac{x-\sin x}{x^2} = \lim_{t\to 0}\frac{3t-3\cdot\sin t+4\cdot\sin^3 t}{9\cdot t^2}$$

$$\Rightarrow \ell = \lim_{t\to 0}\frac{3t-3\cdot\sin t}{9\cdot t^2} + \lim_{t\to 0}\frac{4\cdot\sin^3 t}{9\cdot t^2}$$

$$\Rightarrow \ell = \frac{3}{9}\lim_{t\to 0}\frac{t-\sin t}{t^2} + \lim_{t\to 0}\frac{\sin^3 t}{t^3}\cdot\lim_{t\to 0}\frac{4\cdot t}{9}$$

$$\Rightarrow \ell = \frac{1}{3}\cdot\ell + 1\cdot 0$$

$$\Rightarrow \ell = 0$$

$\Rightarrow \lim_{x\to 0}\frac{x-\sin x}{x^2} = 0$ olur.

Problem -2

$\lim_{x\to 0}\frac{x\cdot\cos x-\sin x}{x^2}$ değeri kaçtır?

Çözüm

$$\lim_{x\to 0}\frac{x\cdot\cos x-\sin x}{x^2} = \lim_{x\to 0}\frac{x\cdot\cos x - x + x - \sin x}{x^2}$$

$$= \lim_{x\to 0}\frac{x\cdot\cos x - x}{x^2} + \lim_{t\to 0}\frac{x-\sin x}{x^2}$$

$$= \lim_{x\to 0}\frac{\cos x - 1}{x} + 0$$

$$= \lim_{x\to 0}\frac{-2\cdot\sin^2\left(\frac{x}{2}\right)}{x} = \lim_{x\to 0}\frac{-\sin\left(\frac{x}{2}\right)\cdot\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}$$

$\Rightarrow \lim_{x\to 0}\frac{x\cdot\cos x-\sin x}{x^2} = 0$ olur.

Problem -3

$\lim_{x\to 0}\frac{x-\tan x}{x-\sin x}$ değeri kaçtır?

Çözüm

$$\lim_{x\to 0}\frac{x-\tan x}{x-\sin x} = \lim_{x\to 0}\frac{x-\frac{\sin x}{\cos x}}{x-\sin x}$$

$$= \lim_{x\to 0}\frac{x\cdot\cos x-\sin x}{\cos x\cdot(x-\sin x)}$$

$$= \lim_{x\to 0}\frac{x\cdot\cos x-\sin x}{x-\sin x}\cdot\underbrace{\lim_{x\to 0}\frac{1}{\cos x}}_{1}$$

$$= \lim_{x\to 0}\frac{x\cdot\cos x - x + x - \sin x}{x-\sin x}$$

$$= \lim_{x\to 0}\frac{x\cdot\cos x - x}{x-\sin x} + \lim_{x\to 0}\frac{x-\sin x}{x-\sin x}$$

$$= \lim_{x\to 0}\frac{x\cdot(\cos x - 1)}{x-\sin x} + 1$$

$$= \lim_{x\to 0}\frac{-2\cdot x\cdot\sin^2\left(\frac{x}{2}\right)}{x-\sin x} + 1$$

Kesrin payını ve paydasını x^3 ile bölelim:

$$= \lim_{x\to 0}\frac{-2\cdot\frac{x\cdot\sin^2\left(\frac{x}{2}\right)}{x^3}}{\frac{x-\sin x}{x^3}} + 1$$

$$= \frac{\lim_{x\to 0}\frac{-2\cdot\sin\left(\frac{x}{2}\right)\cdot\sin\left(\frac{x}{2}\right)}{\frac{x}{2}\cdot\frac{x}{2}\cdot 2\cdot 2}}{\lim_{x\to 0}\frac{x-\sin x}{x^3}} + 1$$

$\lim_{x\to 0}\frac{x-\sin x}{x^3} = \frac{1}{6}$ olduğunu ayrıca hesaplamıştık.

$$\lim_{x\to 0}\frac{x-\tan x}{x-\sin x} = \frac{\frac{-1}{2}}{\frac{1}{6}} + 1$$

$\lim_{x\to 0}\frac{x-\tan x}{x-\sin x} = -2$ bulunur.

Problem -4

$\lim\limits_{x\to 2} \dfrac{1+\sin\frac{3\pi}{x}}{(x-2)^2}$ değeri kaçtır?

Çözüm

$\sin\dfrac{3\pi}{x} = \cos\left(\dfrac{\pi}{2}-\dfrac{3\pi}{x}\right) = 2\cos^2\left(\dfrac{\pi}{4}-\dfrac{3\pi}{2x}\right)-1$

eşitliğini kullanalım:

$$\lim_{x\to 2} \frac{1+\sin\frac{3\pi}{x}}{(x-2)^2} = \lim_{x\to 2} \frac{2\cos^2\left(\frac{\pi}{4}-\frac{3\pi}{2x}\right)}{(x-2)^2}$$

$$\Rightarrow \lim_{x\to 2} \frac{1+\sin\frac{3\pi}{x}}{(x-2)^2} = \lim_{x\to 2} \frac{2\sin^2\left(\frac{\pi}{2}-\frac{\pi}{4}+\frac{3\pi}{2x}\right)}{(x-2)^2}$$

$$\Rightarrow \lim_{x\to 2} \frac{1+\sin\frac{3\pi}{x}}{(x-2)^2} = \lim_{x\to 2} \frac{2\sin^2\left(\frac{\pi}{4}+\frac{3\pi}{2x}\right)}{(x-2)^2}$$

$$\Rightarrow \lim_{x\to 2} \frac{1+\sin\frac{3\pi}{x}}{(x-2)^2} = 2\cdot\left[\lim_{x\to 2} \frac{\sin\left(\frac{\pi}{4}+\frac{3\pi}{2x}\right)}{(x-2)}\right]^2$$

$\sin\left(\dfrac{\pi}{4}+\dfrac{3\pi}{2x}\right) = \sin\left(\dfrac{3\pi}{4}-\dfrac{3\pi}{2x}\right)$ eşitliğini kullanalım:

$$\lim_{x\to 2} \frac{1+\sin\frac{3\pi}{x}}{(x-2)^2} = 2\cdot\left[\lim_{x\to 2} \frac{\sin\left(\frac{\pi}{4}+\frac{3\pi}{2x}\right)}{(x-2)}\right]^2$$

$$\Rightarrow \lim_{x\to 2} \frac{1+\sin\frac{3\pi}{x}}{(x-2)^2} = 2\cdot\left[\lim_{x\to 2} \frac{\sin\left(\frac{3\pi}{4}-\frac{3\pi}{2x}\right)}{(x-2)}\right]^2$$

$\dfrac{3\pi}{4}-\dfrac{3\pi}{2x} = t$ dersek, $x-2 = \dfrac{8t}{3\pi-4t}-\dfrac{3\pi}{2x}$ olur.

$$\lim_{x\to 2} \frac{1+\sin\frac{3\pi}{x}}{(x-2)^2} = 2\cdot\left[\lim_{x\to 2} \frac{\sin\left(\frac{3\pi}{4}-\frac{3\pi}{2x}\right)}{(x-2)}\right]^2$$

$$\Rightarrow \lim_{x\to 2} \frac{1+\sin\frac{3\pi}{x}}{(x-2)^2} = 2\cdot\left[\lim_{t\to 0} \frac{\sin t}{8t/(3\pi-4t)}\right]^2$$

$$\Rightarrow \lim_{x\to 2} \frac{1+\sin\frac{3\pi}{x}}{(x-2)^2} = 2\cdot\left[\lim_{t\to 0} \frac{\sin t}{8t}\cdot(3\pi-4t)\right]^2$$

$$\Rightarrow \lim_{x\to 2} \frac{1+\sin\frac{3\pi}{x}}{(x-2)^2} = \frac{9\pi^2}{32}$$

elde edilir.

Not

x, 2'ye yaklaşırken, $\dfrac{\pi}{4}+\dfrac{3\pi}{2x}$ ifadesi sıfıra yaklaşmaz. Bu yüzden,

$\sin\left(\dfrac{\pi}{4}+\dfrac{3\pi}{2x}\right) = \sin\left(\dfrac{3\pi}{4}-\dfrac{3\pi}{2x}\right)$ eşitliğini kullandık.

Problem -5

$\lim\limits_{x\to 0^+} \dfrac{1-\sqrt{\cos x}}{\left(1-\cos\sqrt{x}\right)\cdot\sin x}$ değeri kaçtır?

Çözüm

$$\lim_{x\to 0^+} \frac{1-\sqrt{\cos x}}{\left(1-\cos\sqrt{x}\right)\cdot\sin x}$$

$$= \lim_{x\to 0^+} \frac{1-\sqrt{\cos x}}{\left(1-\cos\sqrt{x}\right)\cdot\sin x}\cdot\frac{\left(1+\sqrt{\cos x}\right)\cdot\left(1+\cos\sqrt{x}\right)}{\left(1+\sqrt{\cos x}\right)\cdot\left(1+\cos\sqrt{x}\right)}$$

$$= \lim_{x\to 0^+} \frac{1-\cos x}{\left(1-\cos^2\sqrt{x}\right)\cdot\sin x}\cdot\frac{\left(1+\cos\sqrt{x}\right)}{\left(1+\sqrt{\cos x}\right)}$$

$$= \lim_{x\to 0^+} \frac{2\sin^2\frac{x}{2}}{\sin^2\sqrt{x}\cdot 2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}\cdot\frac{\left(1+\cos\sqrt{x}\right)}{\left(1+\sqrt{\cos x}\right)}$$

$$= \lim_{x\to 0^+} \frac{\sin\frac{x}{2}}{\sin\sqrt{x}\cdot\sin\sqrt{x}\cdot\cos\frac{x}{2}}\cdot\frac{\frac{1}{2}\cdot\sqrt{x}\cdot\sqrt{x}\cdot\left(1+\cos\sqrt{x}\right)}{\frac{1}{2}\cdot\sqrt{x}\cdot\sqrt{x}\cdot\left(1+\sqrt{\cos x}\right)}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{2}\cdot\sin\frac{x}{2}}{\frac{x}{2}\cdot\cos\frac{x}{2}}\cdot\frac{\sqrt{x}\cdot\sqrt{x}\cdot\left(1+\cos\sqrt{x}\right)}{\sin\sqrt{x}\cdot\sin\sqrt{x}\cdot\left(1+\sqrt{\cos x}\right)}$$

$$= \frac{1}{2}$$

Problem -6

$\lim\limits_{x \to \frac{\pi}{2}} \left(2x \cdot \tan x - \dfrac{\pi}{\cos x} \right)$ değeri kaçtır?

Çözüm

$$\lim_{x \to \frac{\pi}{2}} \left(2x \cdot \tan x - \frac{\pi}{\cos x} \right) = \lim_{x \to \frac{\pi}{2}} \left(\frac{2x \cdot \sin x - \pi}{\cos x} \right)$$

$x = \dfrac{\pi}{2} + t$ dönüşümü yapalım:

$$\lim_{x \to \frac{\pi}{2}} \left(2x \cdot \tan x - \frac{\pi}{\cos x} \right)$$

$$= \lim_{t \to 0} \frac{(\pi + 2t) \cdot \sin\left(\frac{\pi}{2} + t \right) - \pi}{\cos\left(\frac{\pi}{2} + t \right)}$$

$$= \lim_{t \to 0} \frac{\pi \cdot \cos t + 2t \cdot \cos t - \pi}{-\sin t}$$

$$= \lim_{t \to 0} \frac{\pi \cdot (1 - \cos t)}{\sin t} - \lim_{t \to 0} \frac{2t \cdot \cos t}{\sin t}$$

$$= \lim_{t \to 0} \frac{\pi \cdot 2\sin^2\left(\frac{t}{2} \right)}{2\sin\frac{t}{2} \cdot \cos\frac{t}{2}} - \lim_{t \to 0} \frac{t}{\sin t} \cdot \lim_{t \to 0} 2\cos t$$

$$= \lim_{t \to 0} \frac{\pi \cdot \sin\frac{t}{2}}{\cos\frac{t}{2}} - \lim_{t \to 0} \frac{t}{\sin t} \cdot \lim_{t \to 0} 2\cos t$$

$= 0 - 1 \cdot 2$

$= -2$

bulunur.