İntegral - Birkaç İndirgeme Formülü

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Kısmi integrasyon yöntemini hatırlayarak başlayalım:

$$d(u\cdot v)=u\cdot dv+v\cdot du \Rightarrow \int d(u\cdot v)=\int u\cdot dv+\int v\cdot du \Rightarrow u\cdot v=\int u\cdot dv+\int v\cdot du$$

$$\Rightarrow \int u\cdot dv = u\cdot v-\int v\cdot du$$

I. $\int \sin^n x\cdot dx = ?$

$u=\sin^{n-1}x$ ve $dv=\sin x\cdot dx$ diyelim.

$du=(n-1)\cdot\sin^{n-2}x\cdot\cos x\cdot dx$ ve $v=-\cos x$ olur.

$$\int \sin^n x\cdot dx=-\cos x\cdot\sin^{n-1}x+\int \cos^2 x\cdot(n-1)\cdot\sin^{n-2}x\cdot dx$$

$$\Rightarrow \int \sin^n x\cdot dx=-\cos x\cdot\sin^{n-1}x+(n-1)\int(1-\sin^2 x)\cdot\sin^{n-2}x\cdot dx$$

$$\Rightarrow \int \sin^n x\cdot dx=-\cos x\cdot\sin^{n-1}x+(n-1)\int(\sin^{n-2}x-\sin^n x)\cdot dx$$

$$\Rightarrow \int \sin^n x\cdot dx=-\cos x\cdot\sin^{n-1}x+(n-1)\cdot\int\sin^{n-2}x\cdot dx-(n-1)\cdot\int\sin^n x\cdot dx$$

$$\Rightarrow n\cdot\int \sin^n x\cdot dx=-\cos x\cdot\sin^{n-1}x+(n-1)\cdot\int\sin^{n-2}x\cdot dx$$

$$\Rightarrow \int \sin^n x\cdot dx=-\frac{1}{n}\cdot\sin^{n-1}x\cdot\cos x+\frac{n-1}{n}\cdot\int\sin^{n-2}x\cdot dx$$ elde edilir.

Örnekler

1. $\int \sin^2 x\cdot dx=-\frac{1}{2}\cdot\sin x\cdot\cos x+\frac{1}{2}\cdot x+C$

2. $\int \sin^3 x\cdot dx=-\frac{1}{3}\cdot\sin^2 x\cdot\cos x+\frac{2}{3}\cdot\int\sin x\cdot dx=-\frac{1}{3}\cdot\sin^2 x\cdot\cos x-\frac{2}{3}\cdot\cos x+C$

3. $\int \sin^4 x\cdot dx=-\frac{1}{4}\cdot\sin^3 x\cdot\cos x+\frac{3}{4}\cdot\int\sin^2 x\cdot dx$

$$\Rightarrow \int \sin^4 x\cdot dx=-\frac{1}{4}\cdot\sin^3 x\cdot\cos x-\frac{3}{8}\cdot\sin x\cdot\cos x+\frac{3}{8}+C$$

4. $\int \sin^5 x\cdot dx=-\frac{1}{5}\cdot\sin^4 x\cdot\cos x+\frac{4}{5}\cdot\int\sin^3 x\cdot dx$

$$\Rightarrow \int \sin^5 x\cdot dx=-\frac{1}{5}\cdot\sin^4 x\cdot\cos x-\frac{4}{15}\cdot\sin^2 x\cdot\cos x-\frac{8}{15}\cos x+C$$

II. $\int \cos^n x\cdot dx = ?$

$u=\cos^{n-1}x$ ve $dv=\cos x\cdot dx$ denirse;

$$\Rightarrow \int \cos^n x\cdot dx=\frac{1}{n}\cdot\cos^{n-1}x\cdot\sin x+\frac{n-1}{n}\cdot\int\cos^{n-2}x\cdot dx$$ elde edilir.

Örnekler

1. $\int \cos^3 x\cdot dx=\frac{1}{3}\cdot\cos^2 x\cdot\sin x+\frac{2}{3}\cdot\int\cos x\cdot dx=\frac{1}{3}\cdot\cos^2 x\cdot\sin x+\frac{2}{3}\cdot\sin x+C$

2. $\int \cos^6 x\cdot dx=\frac{1}{6}\cdot\cos^5 x\cdot\sin x+\frac{5}{6}\cdot\int\cos^4 x\cdot dx$

$$\Rightarrow \int \cos^6 x\cdot dx=\frac{1}{6}\cdot\cos^5 x\cdot\sin x+\frac{5}{6}\cdot(\frac{1}{4}\cdot\cos^3 x\cdot\sin x+\frac{3}{4}\int\cos^2 x\cdot dx)$$

$$\Rightarrow \int \cos^6 x\cdot dx=\frac{1}{6}\cdot\cos^5 x\cdot\sin x+\frac{5}{24}\cdot\cos^3 x\cdot\sin x+\frac{5}{16}\cdot\cos x\cdot\sin x+\frac{5}{16}\cdot x+C$$

III. $\int \sec^n x \cdot dx = ?$

$u = \sec^{n-2} x$ ve $dv = \sec^2 x \cdot dx$ diyelim.

$du = (n-2) \cdot \sec^{n-3} x \cdot \sec x \cdot \tan x \cdot dx$ ve $v = \tan x$ olur.

$$\int \sec^n x \cdot dx = \sec^{n-2} x \cdot \tan x - \int (n-2) \cdot \sec^{n-2} x \cdot \tan^2 x \cdot dx$$

$$\Rightarrow \int \sec^n x \cdot dx = \sec^{n-2} x \cdot \tan x - (n-2) \cdot \int \sec^{n-2} x \cdot (\sec^2 x - 1) \cdot dx$$

$$\Rightarrow \int \sec^n x \cdot dx = \sec^{n-2} x \cdot \tan x - (n-2) \cdot \int \sec^n x \cdot dx + (n-2) \cdot \int \sec^{n-2} x \cdot dx$$

$$\Rightarrow (n-1) \cdot \int \sec^n x \cdot dx = \sec^{n-2} x \cdot \tan x + (n-2) \cdot \int \sec^{n-2} x \cdot dx$$

$$\Rightarrow \int \sec^n x \cdot dx = \frac{1}{n-1} \cdot \sec^{n-2} x \cdot \tan x + \frac{n-2}{n-1} \cdot \int \sec^{n-2} x \cdot dx \text{ elde edilir.}$$

Örnekler

1. $\int \sec^3 x \cdot dx = \frac{1}{2} \cdot \sec x \cdot \tan x + \frac{1}{2} \int \sec x \cdot dx$

$$\Rightarrow \int \sec^3 x \cdot dx = \frac{1}{2} \cdot \sec x \cdot \tan x + \frac{1}{2} \cdot \ln \left| \sec x + \tan x \right| + C$$

2. $\int \sec^5 x \cdot dx = \frac{1}{4} \cdot \sec^3 x \cdot \tan x + \frac{3}{4} \cdot \int \sec^3 x \cdot dx$

$$\Rightarrow \int \sec^5 x \cdot dx = \frac{1}{4} \cdot \sec^3 x \cdot \tan x + \frac{3}{8} \cdot \sec x \cdot \tan x + \frac{3}{8} \cdot \ln \left| \sec x + \tan x \right| + C$$

3. $\int \sec^6 x \cdot dx = \frac{1}{5} \cdot \sec^4 x \cdot \tan x + \frac{4}{5} \cdot \int \sec^4 x \cdot dx$

$$\Rightarrow \int \sec^6 x \cdot dx = \frac{1}{5} \cdot \sec^4 x \cdot \tan x + \frac{4}{15} \cdot \sec^2 x \cdot \tan x + \frac{8}{15} \cdot \tan x + C$$

IV. $\int \operatorname{cosec}^n x \cdot dx = ?$

$u = \operatorname{cosec}^{n-2} x$ ve $dv = \operatorname{cosec}^2 x \cdot dx$ denirse;

$$\int \operatorname{cosec}^n x \cdot dx = -\frac{1}{n-1} \cdot \operatorname{cosec}^{n-2} x \cdot \cot x - \frac{n-2}{n-1} \cdot \int \operatorname{cosec}^{n-2} x \cdot dx \quad \text{elde edilir.}$$

Örnekler

1. $\int \operatorname{cosec}^3 x \cdot dx = -\frac{1}{2} \cdot \operatorname{cosec} x \cdot \cot x - \frac{1}{2} \int \operatorname{cosec} x \cdot dx$

$$\Rightarrow \int \operatorname{cosec}^3 x \cdot dx = -\frac{1}{2} \cdot \operatorname{cosec} x \cdot \cot x + \frac{1}{2} \cdot \ln \left| \operatorname{cosec} x + \cot x \right| + C$$

2. $\int \operatorname{cosec}^4 x \cdot dx = -\frac{1}{3} \cdot \operatorname{cosec}^2 x \cdot \cot x - \frac{2}{3} \cdot \int \operatorname{cosec}^2 x \cdot dx$

$$\Rightarrow \int \operatorname{cosec}^4 x \cdot dx = -\frac{1}{3} \cdot \operatorname{cosec}^2 x \cdot \cot x + \frac{2}{3} \cdot \cot x + C$$

V. $\int \tan^n x \cdot dx = ?$

$$\int \tan^n x \cdot dx = \int \tan^{n-2} x \cdot (\sec^2 x - 1) \cdot dx = \int \tan^{n-2} x \cdot \sec^2 x \cdot dx - \int \tan^{n-2} x \cdot dx$$

$$\Rightarrow \int \tan^n x \cdot dx = \frac{1}{n-1} \cdot \tan^{n-1} x - \int \tan^{n-2} x \cdot dx \text{ olur.}$$

Örnekler

1. $\int \tan^2 x \cdot dx = \tan x - x + C$

2. $\int \tan^3 x \cdot dx = \frac{1}{2} \cdot \tan^2 x - \int \tan x \cdot dx = \frac{1}{2} \cdot \tan^2 x + \ln|\cos x| + C$

3. $\int \tan^4 x \cdot dx = \frac{1}{3} \cdot \tan^3 x - \int \tan^2 x \cdot dx = \frac{1}{3} \cdot \tan^3 x - \tan x + x + C$

VI. $\int \cot^n x \cdot dx = ?$

$$\int \cot^n x \cdot dx = \int \cot^{n-2} x \cdot (\operatorname{co}\sec^2 x - 1) \cdot dx$$

$$\Rightarrow \int \cot^n x \cdot dx = \int \cot^{n-2} x \cdot \operatorname{cosec}^2 x \cdot dx - \int \cot^{n-2} x \cdot dx$$

$$\Rightarrow \int \cot^n x \cdot dx = -\frac{1}{n-1} \cdot \cot^{n-1} x - \int \cot^{n-2} x \cdot dx \text{ olur.}$$

Örnekler

1. $\int \cot^4 x \cdot dx = -\frac{1}{3} \cdot \cot^3 x - \int \cot^2 x \cdot dx$

$$\Rightarrow \int \cot^4 x \cdot dx = -\frac{1}{3} \cdot \cot^3 x + \cot x + x + C$$

2. $\int \cot^5 x \cdot dx = -\frac{1}{4} \cdot \cot^4 x - \int \cot^3 x \cdot dx$

$$\Rightarrow \int \cot^5 x \cdot dx = -\frac{1}{4} \cdot \cot^4 x + \frac{1}{2}\cot^2 x + \int \cot x \cdot dx$$

$$\Rightarrow \int \cot^5 x \cdot dx = -\frac{1}{4} \cdot \cot^4 x + \frac{1}{2}\cot^2 x + \ln|\sin x| + C$$