

Etkinlik Çözümleri**Etkinlik – 3.1**

$$\text{a. } \vec{A} = (x_1, y_1) \Rightarrow \vec{A} = x_1 \vec{e}_1 + y_1 \vec{e}_2$$

$$\vec{B} = (x_2, y_2) \Rightarrow \vec{B} = x_2 \vec{e}_1 + y_2 \vec{e}_2$$

$$\Rightarrow \vec{A} + \vec{B} = x_1 \vec{e}_1 + y_1 \vec{e}_2 + x_2 \vec{e}_1 + y_2 \vec{e}_2$$

$$\Rightarrow \vec{A} + \vec{B} = (x_1 + x_2) \vec{e}_1 + (y_1 + y_2) \vec{e}_2$$

$$\Rightarrow \vec{A} + \vec{B} = (x_1 + x_2, y_1 + y_2)$$

$$\Rightarrow (x_1 + y_1) + (x_2 + x_2) = (x_1 + x_2, y_1 + y_2)$$

$$\text{b. } \vec{M} = (x, y) \Rightarrow \vec{M} = x \vec{e}_1 + y \vec{e}_2$$

$$\Rightarrow k\vec{M} = k(x \vec{e}_1 + y \vec{e}_2)$$

$$\Rightarrow k\vec{M} = k(x \vec{e}_1 + y \vec{e}_2)$$

$$\Rightarrow k\vec{M} = kx \vec{e}_1 + ky \vec{e}_2$$

$$\Rightarrow k\vec{M} = (kx, ky)$$

$$\Rightarrow k(x, y) = (kx, ky)$$

$$\text{c. } \vec{A} = (-3, 1), \vec{B} = (2, -4), \vec{C} = (1, -5)$$

$$\Rightarrow \vec{A} + 2\vec{B} - \vec{C} = (-3, 1) + 2(2, -4) - (1, -5)$$

$$\Rightarrow \vec{A} + 2\vec{B} - \vec{C} = (-3, 1) + (4, -8) + (-1, 5)$$

$$\Rightarrow \vec{A} + 2\vec{B} - \vec{C} = (-3 + 4 - 1, 1 - 8 + 5)$$

$$\Rightarrow \vec{A} + 2\vec{B} - \vec{C} = (0, -2)$$

Etkinlik – 3.2

$$\text{a. } A(x_1, y_1), B(x_2, y_2)$$

olmak üzere,

$$\vec{OA} + \vec{AB} = \vec{OB}$$

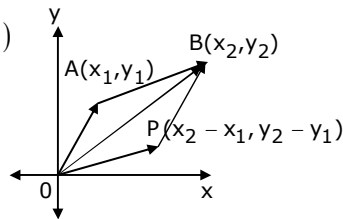
$$\Rightarrow \vec{A} + \vec{AB} = \vec{B}$$

$$\Rightarrow \vec{AB} = \vec{B} - \vec{A}$$

$$\Rightarrow \vec{AB} = (x_2, y_2) - (x_1, y_1)$$

$$\Rightarrow \vec{AB} = (x_2 - x_1, y_2 - y_1) \text{ olur.}$$

$(x_2 - x_1, y_2 - y_1)$ ikilisi analitik düzlemde P gibi bir noktayı, aynı zamanda orijinden P'ye yönelmiş \vec{OP} (ya da \vec{P}) gibi bir vektörü gösterir.



$$\text{b. } A(-2, 1) \text{ ve } B(2, -2)$$

$$\Rightarrow \vec{AB} = (2 - (-2), -2 - 1)$$

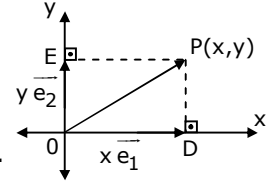
$$\Rightarrow \vec{AB} = (4, -3) \text{ olur.}$$

Etkinlik – 3.3

$$\text{a. } \vec{P} = (x, y)$$

$$\Rightarrow \vec{P} = x \vec{e}_1 + y \vec{e}_2$$

$$x \vec{e}_1 = \vec{OD} \text{ diyelim.}$$



$$\vec{P} = \vec{OD} + \vec{DP} \text{ ve } \vec{P} = \vec{OD} + y \vec{e}_2$$

$$\Rightarrow \vec{DP} = y \vec{e}_2 \text{ olur.}$$

$\vec{e}_1 \perp \vec{e}_2$ olduğundan $\vec{OD} \perp \vec{DP}$ olur.

POD dik üçgeninde Pisagor bağıntısından

$$|\vec{OP}|^2 = |\vec{OD}|^2 + |\vec{DP}|^2$$

$$\Rightarrow |\vec{P}|^2 = x^2 + y^2 \quad (|\vec{OD}| = x, |\vec{DP}| = y)$$

$$\Rightarrow |\vec{P}| = \sqrt{x^2 + y^2} \text{ bulunur.}$$

$$\text{b. } \vec{P} = (x, y)$$

$$\Rightarrow |\vec{P}| = \sqrt{x^2 + y^2} \text{ olduğunu gösterdik.}$$

$$A(x_1, y_1) \text{ ve } B(x_2, y_2)$$

$\Rightarrow \vec{AB} = (x_2 - x_1, y_2 - y_1)$ olduğunu da Etkinlik-3.2'de göstermiştik.

Öyleyse;

$$|\vec{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ olur.}$$

$$\text{c. } A(-3, 4) \text{ ve } B(1, -4)$$

$$\Rightarrow \vec{AB} = (1 + 3, -4 - 4)$$

$$\Rightarrow \vec{AB} = (4, -8)$$

$$\Rightarrow |\vec{AB}| = \sqrt{4^2 + (-8)^2}$$

$$\Rightarrow |\vec{AB}| = 4\sqrt{5} \text{ olur.}$$

Etkinlik – 3.4

a. $\vec{u} = (x_1, y_1)$ ve $\vec{v} = (x_2, y_2)$ doğrusal bağımlı ise;

$$\vec{u} = \lambda \vec{v}$$

$$\Rightarrow (x_1, y_1) = \lambda(x_2, y_2)$$

$$\Rightarrow x_1 = \lambda x_2 \text{ ve } y_1 = \lambda y_2$$

$$\Rightarrow \lambda = \frac{x_1}{x_2} \text{ ve } \lambda = \frac{y_1}{y_2}$$

$$\Rightarrow \frac{x_1}{x_2} = \frac{y_1}{y_2} \text{ bulunur.}$$

b. $\vec{u} = (m+1, -2)$ ve $\vec{v} = (1-m, 1)$ doğrusal bağımlı ise;

$$\frac{m+1}{1-m} = \frac{-2}{1} \Rightarrow m+1 = -2+2m$$

$$\Rightarrow m = 3 \text{ olur.}$$

Etkinlik – 3.5

a. 1. yol

$$\vec{A} = (-3, 1), \vec{AB} = (-2, 3), \vec{BC} = (1, 2)$$

$$\Rightarrow \vec{A} - 2\vec{B} + \vec{C} = \vec{A} - \vec{B} + \vec{C} - \vec{B}$$

$$\Rightarrow \vec{A} - 2\vec{B} + \vec{C} = \vec{BA} + \vec{BC}$$

$$\Rightarrow \vec{A} - 2\vec{B} + \vec{C} = (2, -3) + (1, 2)$$

$$\Rightarrow \vec{A} - 2\vec{B} + \vec{C} = \vec{A}(3, -1) \text{ bulunur.}$$

2. yol

$$\vec{AB} = \vec{B} - \vec{A} = (-2, 3)$$

$$+ \vec{A} = (-3, 1)$$

$$\vec{B} = (-5, 4);$$

$$\vec{BC} = \vec{C} - \vec{B} = (1, 2)$$

$$+ \vec{B} = (-5, 4)$$

$$\vec{C} = (-4, 6) \text{ olur.}$$

$$\vec{A} - 2\vec{B} + \vec{C} = (-3, 1) - 2(-5, 4) + (-4, 6)$$

$$\Rightarrow \vec{A} - 2\vec{B} + \vec{C} = (3, -1) \text{ bulunur.}$$

b. $\vec{AC} + \vec{BP} = \vec{BA}$

$$\Rightarrow \vec{C} - \vec{A} + \vec{P} - \vec{B} = \vec{A} - \vec{B}$$

$$\Rightarrow \vec{P} = 2\vec{A} - \vec{C}$$

\vec{C} vektörü a'daki gibi

$$\vec{A} = (-3, 1), \vec{C} = (-4, 6) \text{ ve } \vec{P} = 2\vec{A} - \vec{C} \text{ bulunur.}$$

$$\Rightarrow \vec{P} = 2(-3, 1) - (-4, 6) \Rightarrow \vec{P} = (-2, -4)$$

\vec{C} vektörü şöyle de bulunabilirdi.

$$\vec{AB} + \vec{BC} = \vec{AC} \text{ ve } \vec{A} + \vec{AC} = \vec{C} \text{ olduğundan,}$$

$$\vec{C} = \vec{A} + \vec{AB} + \vec{BC}$$

$$\Rightarrow \vec{C} = (-3, 1) + (-2, 3) + (1, 2)$$

$$\Rightarrow \vec{C} = (-4, 6) \text{ olur.}$$

c. $\vec{AR} + 2\vec{RC} = \vec{BR}$

$$\Rightarrow \vec{R} - \vec{A} + 2\vec{C} - 2\vec{R} = \vec{R} - \vec{B}$$

$$\Rightarrow 2\vec{R} = 2\vec{C} + \vec{B} - \vec{A}$$

$$\Rightarrow \vec{R} = \vec{C} + \frac{1}{2}\vec{AB}$$

$$\Rightarrow \vec{R} = (-4, 6) + \frac{1}{2}(-2, 3)$$

$$\Rightarrow \vec{R} = \left(-5, \frac{15}{2}\right) \text{ bulunur.}$$

Etkinlik – 3.6

a. $\vec{AB} = (-1, 2)$ ve $\vec{B} = (3, -1)$

$$\Rightarrow \vec{B} - \vec{A} = (-1, 2) \text{ ve } \vec{B} = (3, -1)$$

$$\Rightarrow \vec{A} = (4, -3) \text{ olur.}$$

$\vec{A} = (4, -3)$ ve $\vec{CD} = (k, 2-k)$ doğrusal bağımlı ise,

$$\frac{4}{k} = \frac{-3}{2-k} \Rightarrow k = 8 \text{ dir.}$$

$$\vec{C} = (-2, 4) \text{ ve } \vec{CD} = \vec{D} - \vec{C} = (8, -6) \text{ olduğundan,}$$

$$\vec{D} = (6, -2) \text{ olur.}$$

$$\vec{AD} = \vec{D} - \vec{A}$$

$$\Rightarrow \vec{AD} = (6, -2) - (4, -3)$$

$$\Rightarrow \vec{AD} = (2, 1) \text{ bulunur.}$$

$$\text{b. } x\overline{AB} + y\overline{BC} = \overline{A};$$

$$\Rightarrow x \cdot (-1, 2) + y[(-2, 4) - (3, -1)] = (4, -3)$$

$$\Rightarrow x \cdot (-1, 2) + y(-5, 5) = (4, -3)$$

$$\Rightarrow \begin{cases} -x - 5y = 4 \\ 2x + 5y = -3 \end{cases}$$

$$\Rightarrow x = 1 \text{ ve } y = -1$$

$$\text{c. } x\overline{A} + y\overline{B} + z\overline{C} = 0$$

$$\Rightarrow x(4, -3) + y(3, -1) + z(-2, 4) = 0$$

$$\Rightarrow \begin{cases} 4x + 3y - 2z = 0 \\ -3x - y + 4z = 0 \end{cases} \text{ olur.}$$

x ve y'yi z türünden bulalım:

$$4x + 3y - 2z = 0 \quad (1)$$

$$3/ \quad -3x - y + 4z = 0 \quad (2)$$

$$-5x + 10z = 0 \Rightarrow x = 2z$$

x = 2z değerine (2)'de yerine koyalım:

$$-3(2z) - y + 4z = 0$$

$$\Rightarrow y = -2z \text{ bulunur.}$$

$$z = 1 \text{ alınırsa, } x = 2 \text{ ve } y = -2 \text{ olur.}$$

d. A, B, D noktaları doğrusal ise, örneğin;

\overline{AB} ile \overline{BD} doğrusal bağımlıdır.

$$\overline{AB} = (-1, 2) \text{ ve } \overline{BD} = \overline{BC} + \overline{CD}$$

$$\Rightarrow \overline{BD} = \overline{C} - \overline{B} + \overline{CD}$$

$$\Rightarrow \overline{BD} = (-2, 4) - (3, -1) + (k, 2 - k)$$

$$\Rightarrow \overline{BD} = (k - 5, 7 - k) \text{ olur.}$$

$$\frac{-1}{k - 5} = \frac{2}{7 - k} \Rightarrow k = 3 \text{ bulunur.}$$

Etkinlik - 3.7

A(-2, -3), B(1, -1), C(3, 2) olsun.

[AB] köşegen olabilir.

Bu durumda 4. köşe K(x, y) ise,

$$\overline{AK} = \overline{CB}$$

$$\Rightarrow \overline{CB} + \overline{A}$$

$$\Rightarrow (x, y) = (2, 3) + (-2, -3)$$

$$\Rightarrow (x, y) = (0, 0) \Rightarrow K(0, 0) \text{ olur.}$$

[AC] köşegen ve 4. köşe P(x, y) ise

$$\overline{AP} = \overline{BC} \Rightarrow \overline{P} = \overline{BC} + \overline{A}$$

$$\Rightarrow (x, y) = (-2, -3) + (-2, -3) \Rightarrow (x, y) = (-4, -6)$$

P(-4, -6) olur.

[BC] köşegen ve 4. köşe T(x, y) ise,

$$\overline{BT} = \overline{AC} \Rightarrow T = \overline{AC} + \overline{B}$$

$$\Rightarrow (x, y) = (5, 5) + (1, -1) \Rightarrow (x, y) = (6, 4) T(6, 4)$$

Olur.

Etkinlik - 3.8

a. $\overline{A} = (-2, 1)$ ve $\overline{AB} = (3, 6) \Rightarrow \overline{B} = (1, 7)$ dir.

$$\frac{\overline{AE}}{\overline{EB}} = \frac{1}{2} \Rightarrow \overline{EB} = 2\overline{AE}$$

$$\Rightarrow \overline{B} - \overline{E} = 2\overline{E} - 2\overline{A}$$

$$\Rightarrow 3\overline{E} = 2\overline{A} + \overline{B}$$

$$\Rightarrow 3\overline{E} = 2(-2, 1) + (1, 7)$$

$$\Rightarrow 3\overline{E} = (-3, 9)$$

$$\Rightarrow \overline{E} = (-1, 3) \text{ bulunur.}$$

$$\text{b. } \frac{\overline{FA}}{\overline{FB}} = \frac{3}{2} \Rightarrow 2\overline{FA} = 3\overline{FB}$$

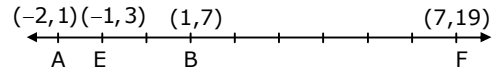
$$\Rightarrow 2\overline{A} - 2\overline{F} = 3\overline{B} - 3\overline{F}$$

$$\Rightarrow \overline{F} = 3\overline{B} - 2\overline{A}$$

$$\Rightarrow \overline{F} = 3(1, 7) - 2(-2, 1)$$

$$\Rightarrow \overline{F} = (7, 19) \text{ bulunur.}$$

c.



Etkinlik - 3.9

$$\text{a. } \vec{a} \cdot \vec{b} = (x_1, y_1) \cdot (x_2, y_2) = x_1x_2 + y_1y_2 \quad (1)$$

$$\vec{b} \cdot \vec{a} = (x_2, y_2) \cdot (x_1, y_1) = x_2x_1 + y_2y_1 \quad (2)$$

(1) ve (2)'den $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ olduğu görülür.

b. $(k\vec{a} + \vec{b}) \cdot \vec{c}$

$$\begin{aligned} &= [k(x_1y_1) + (x_2y_2)] \cdot (x_3y_3) \\ &= (kx_1 + x_2, ky_1 + y_2) \cdot (x_3, y_3) \\ &= kx_1x_3 + x_2x_3 + ky_1y_3 + y_2y_3 \\ &= k(x_1x_3 + y_1y_3) + x_2x_3 + y_2y_3 \\ &= k\vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c} \text{ olur.} \end{aligned}$$

c. $\vec{a} \cdot (\vec{b} + k\vec{c})$

$$\begin{aligned} &= (x_1, y_1) \cdot [(x_2, y_2) + k(x_3, y_3)] \\ &= (x_1, y_1) \cdot (x_2 + kx_3, y_2 + ky_3) \\ &= x_1x_2 + kx_1x_3 + y_1y_2 + ky_1y_3 \\ &= x_1x_2 + y_1y_2 + k(x_1x_3 + y_1y_3) \\ &= \vec{a} \cdot \vec{b} + k \cdot \vec{a} \cdot \vec{c} \text{ olur.} \end{aligned}$$

d. $\vec{a} \cdot \vec{a} = (x_1y_1) \cdot (x_1y_1)$

$$\Rightarrow \vec{a} \cdot \vec{a} = x_1^2 + y_1^2$$

$$x_1, y_1 \in \mathbb{R} \text{ ise } x_1^2 \geq 0 \text{ ve } y_1^2 \geq 0$$

$$\Rightarrow \vec{a} \cdot \vec{a} \geq 0 \text{ olur.}$$

$$x_1 = y_1 = 0 \Leftrightarrow \vec{a} = \vec{0} \text{ geçerli önermesi}$$

$$\vec{a} \cdot \vec{a} \Leftrightarrow \vec{a} = \vec{0} \text{ önermesini gerektirir.}$$

$$\vec{a} \neq \vec{0} \Leftrightarrow \vec{a} \cdot \vec{a} > 0 \text{ önermesi de geçerlidir.}$$

(Daima doğrudur.)

Etkinlik – 3.10

$$\vec{a} = (x, y) \Rightarrow |\vec{a}| = \sqrt{x^2 + y^2} \quad (1)$$

$$\vec{a} \cdot \vec{a} = (x, y) \cdot (x, y) \Rightarrow \vec{a} \cdot \vec{a} = x^2 + y^2 \quad (2)$$

(1) ve (2) den, $|\vec{a}| = \sqrt{\vec{a} \cdot \vec{a}}$ bulunur.

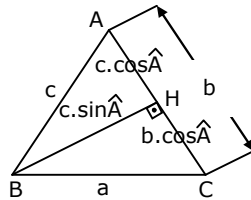
Etkinlik – 3.11

a. ABH dik üçgeninde,

$$\sin \hat{A} = \frac{|BH|}{c} \Rightarrow |BH| = c \cdot \sin \hat{A};$$

$$\cos \hat{A} = \frac{|AH|}{c} \Rightarrow |AH| = c \cdot \cos \hat{A};$$

$$|HC| = |AC| - |AH| \Rightarrow |HC| = b - c \cos A \text{ olur.}$$



b. HBC dik üçgeninde,

$$|BC|^2 = |BH|^2 + |HC|^2$$

$$\Rightarrow a^2 = (c \sin \hat{A})^2 + (b - c \cos A)^2$$

$$\Rightarrow a^2 = c^2 \sin^2 \hat{A} + b^2 - 2b \cos \hat{A} + c^2 \cos^2 A$$

$$\Rightarrow a^2 = b^2 + c^2 \left(\underbrace{\sin^2 A + \cos^2 A}_1 \right) - 2bc \cos A$$

$$\Rightarrow a^2 = b^2 + c^2 - 2bc \cos A$$

elde edilir.

c. $a = 6, b = 7, c = 5$

$$a^2 = b^2 + c^2 - 2bc \cos \hat{A}$$

$$\Rightarrow 36 = 49 + 25 - 2 \cdot 7 \cdot 5 \cdot \cos A \Rightarrow \cos \hat{A} = \frac{19}{35};$$

$$b^2 = a^2 + c^2 - 2ac \cos \hat{B}$$

$$\Rightarrow 49 = 36 + 25 - 2 \cdot 6 \cdot 5 \cdot \cos B \Rightarrow \cos \hat{B} = \frac{1}{5};$$

$$c^2 = a^2 + b^2 - 2ab \cos \hat{C}$$

$$\Rightarrow 25 = 36 + 49 - 2 \cdot 6 \cdot 7 \cdot \cos \hat{C} \Rightarrow \cos \hat{C} = \frac{1}{7}$$

bulunur.

d. $a = 8, b = 6, m(\hat{C}) = 60^\circ$

$$c^2 = a^2 + b^2 - 2ab \cos 60^\circ$$

$$\Rightarrow c^2 = 8^2 + 6^2 - 2 \cdot 8 \cdot 6 \cdot \frac{1}{2} \Rightarrow c = 2\sqrt{13}$$

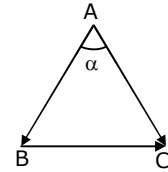
olur.

Etkinlik – 3.12

$$\vec{BC} = \vec{AC} - \vec{AB}$$

$$\begin{aligned} \Rightarrow \vec{BC} \cdot \vec{BC} &= (\vec{AC} - \vec{AB}) \cdot (\vec{AC} - \vec{AB}) \\ \Rightarrow \vec{BC} \cdot \vec{BC} &= \vec{AC} \cdot \vec{AC} - \vec{AC} \cdot \vec{AB} \\ &\quad - \vec{AB} \cdot \vec{AC} + \vec{AB} \cdot \vec{AB} \end{aligned}$$

$$\Rightarrow \|\vec{BC}\|^2 = \|\vec{AB}\|^2 + \|\vec{AC}\|^2 - 2\vec{AB} \cdot \vec{AC} \quad (1) \text{ olur.}$$



Kosinüs Teoremine göre

$$|BC|^2 = |AB|^2 + |AC|^2 - 2|AB| \cdot |AC| \cdot \cos \alpha \quad (2) \text{ yazılır.}$$

$|\overline{BC}|^2$ ile $|BC|^2$ ifadeleri özdeştir.

(1) ve (2) den

$$\overline{AB} \cdot \overline{AC} = \|\overline{AB}\| \cdot \|\overline{AC}\| \cdot \cos \alpha \text{ elde edilir.}$$

Etkinlik – 3.13

a. $\vec{a} = (-1, 2)$ ve $\vec{b} = (3, 4)$

$$\Rightarrow \vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos \alpha$$

$$\Rightarrow (-1, 2) \cdot (3, 4) = \sqrt{5} \cdot \sqrt{25} \cdot \cos \alpha$$

$$\Rightarrow 5 = 5\sqrt{5} \cos \alpha$$

$$\Rightarrow \cos \alpha = \frac{\sqrt{5}}{5} \text{ olur.}$$

b. $\vec{a} = (-4, 2)$, $\vec{b} = (k, -6)$ ve $\vec{a} \perp \vec{b}$

$$\Rightarrow \vec{a} \cdot \vec{b} = 0 \Rightarrow -4k - 12 = 0 \Rightarrow k = -3 \text{ olur.}$$

Etkinlik – 3.14

a. $\vec{a} = (2, -3)$, $\vec{b} = (-1, 1)$, $\vec{c} = (1, 2)$ ve

$\vec{a} + k\vec{b}$ ile \vec{c} birbirine dik ise,

$$(\vec{a} + k\vec{b}) \cdot \vec{c} = 0$$

$$\Rightarrow (2 - k, -3 + k) \cdot (1, 2) = 0$$

$$\Rightarrow 2 - k - 6 + 2k = 0$$

$$\Rightarrow k = 4 \text{ olur.}$$

b. $\vec{a} + t\vec{b}$ ile \vec{c} doğrusal bağımlı ise,

$$\frac{2-t}{1} = \frac{-3+t}{2} \Rightarrow t = \frac{7}{3} \text{ olur.}$$

Etkinlik – 3.15

B(x,0) olsun.

$$\overline{AB} \perp \overline{BC}$$

$$\Rightarrow \overline{AB} \cdot \overline{BC} = 0$$

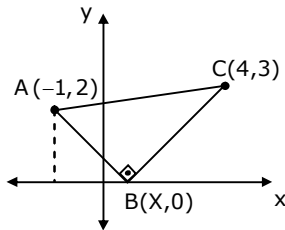
$$\Rightarrow (x+1, -2) \cdot (4-x, 3) = 0$$

$$\Rightarrow 4x - x^2 + 4 - x - 6 = 0$$

$$\Rightarrow x^2 - 3x + 2 = 0$$

$$\Rightarrow (x-1)(x-2) = 0$$

$$\Rightarrow x_1 = 1, \quad x_2 = 2$$



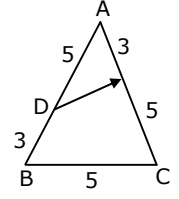
Etkinlik – 3.16

$$\overline{DE} \cdot \overline{BC} = (\overline{DA} + \overline{AE}) \cdot \overline{BC}$$

$$\Rightarrow \overline{DE} \cdot \overline{BC} = \overline{DA} \cdot \overline{BC} + \overline{AE} \cdot \overline{BC}$$

$$\Rightarrow \overline{DE} \cdot \overline{BC} = 5 \cdot 8 \cdot \frac{1}{2} + 3 \cdot 8 \cdot \frac{1}{2}$$

$$\Rightarrow \overline{DE} \cdot \overline{BC} = 32 \text{ olur.}$$



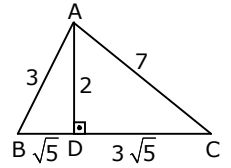
Etkinlik – 3.17

$$\overline{AB} \cdot \overline{AC} = (\overline{AD} + \overline{DB}) \cdot (\overline{AD} + \overline{DC})$$

$$\Rightarrow \overline{AB} \cdot \overline{AC} = \|\overline{AD}\|^2 + \|\overline{DB}\| \cdot \|\overline{DC}\|$$

$$\overline{AB} \cdot \overline{AC} = 2^2 + \sqrt{5} \cdot 3\sqrt{5} \cdot \underbrace{\cos 180^\circ}_{-1}$$

$$\overline{AB} \cdot \overline{AC} = -11 \text{ olur.}$$



Etkinlik – 3.18

Vektörleri \overline{BA} ve \overline{BC}

türünden ifade etmeliyiz.

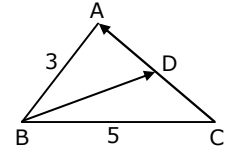
$$\overline{BD} \cdot \overline{DA} = \left(\overline{BC} + \frac{1}{2} \overline{CA} \right) \cdot \frac{1}{2} \overline{CA}$$

$$\Rightarrow \overline{BD} \cdot \overline{DA} = \left[\overline{BC} + \frac{1}{2} (-\overline{BC} + \overline{BA}) \right] \cdot \left[\frac{1}{2} (-\overline{BC} + \overline{BA}) \right]$$

$$\Rightarrow \overline{BD} \cdot \overline{DA} = \frac{1}{2} (\overline{BA} + \overline{BC}) \cdot \frac{1}{2} (\overline{BA} - \overline{BC})$$

$$\Rightarrow \overline{BD} \cdot \overline{DA} = \frac{1}{4} (\|\overline{BA}\|^2 - \|\overline{BC}\|^2)$$

$$\Rightarrow \overline{BD} \cdot \overline{DA} = -4 \text{ bulunur.}$$



Etkinlik – 3.19

$$(\vec{a} + 2\vec{b}) \cdot (\vec{a} + 2\vec{b}) = \|\vec{a}\|^2 + 4\|\vec{b}\|^2 + 4\vec{a} \cdot \vec{b}$$

$$\Rightarrow (-3, -4) \cdot (-3, -4) = 5^2 + 4 \cdot 3^2 + 4\vec{a} \cdot \vec{b}$$

$$\Rightarrow 25 = 25 + 36 + 4\vec{a} \cdot \vec{b}$$

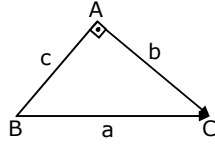
$$\Rightarrow \vec{a} \cdot \vec{b} = -9 \text{ olur.}$$

Etkinlik – 3.20

$$\begin{aligned} \vec{u} \cdot \vec{v} &= \|\vec{u}\| \cdot \|\vec{v}\| \cdot \cos \alpha \\ \Rightarrow (2\vec{a} + \vec{b}) \cdot (\vec{a} - 2\vec{b}) &= \|2\vec{a} + \vec{b}\| \cdot \|\vec{a} - 2\vec{b}\| \cdot \cos \alpha \\ \Rightarrow 2\|\vec{a}\|^2 - 2\|\vec{b}\|^2 - 3\vec{a} \cdot \vec{b} \\ &= \sqrt{(2\vec{a} + \vec{b}) \cdot (2\vec{a} + \vec{b})} \cdot \sqrt{(\vec{a} - 2\vec{b}) \cdot (\vec{a} - 2\vec{b})} \cdot \cos \alpha \\ \Rightarrow 2\|\vec{a}\|^2 - 2\|\vec{b}\|^2 - 3\|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos 60^\circ \\ &= \sqrt{4\|\vec{a}\|^2 + \|\vec{b}\|^2 + 4\|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos 60^\circ} \\ &\quad \sqrt{\|\vec{a}\|^2 + 4\|\vec{b}\|^2 - 4\|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos 60^\circ} \\ \Rightarrow 2 - 2 - \frac{3}{2} &= \sqrt{4 + 1 + 4 \cdot \frac{1}{2}} \cdot \sqrt{1 + 4 - 4 \cdot \frac{1}{2}} \cdot \cos \alpha \\ \Rightarrow -\frac{3}{2} &= \sqrt{7} \cdot \sqrt{3} \cdot \cos \alpha \\ \Rightarrow \cos \alpha &= -\frac{\sqrt{21}}{2} \text{ bulunur.} \end{aligned}$$

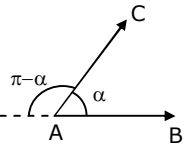
Etkinlik – 3.21

$$\begin{aligned} \vec{BC} \cdot \vec{BC} &= (\vec{AC} - \vec{AB}) \cdot (\vec{AC} - \vec{AB}) \\ \Rightarrow \|\vec{BC}\|^2 &= \|\vec{AC}\|^2 + \|\vec{AB}\|^2 - 2\vec{AC} \cdot \vec{AB} \\ \Rightarrow a^2 &= b^2 + c^2 \text{ olur.} \end{aligned}$$

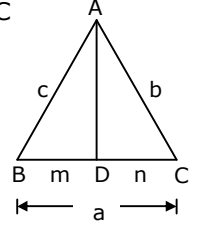
**Etkinlik – 3.22**

$$\begin{aligned} \vec{AB} \cdot \vec{AC} &= \|\vec{AB}\| \cdot \|\vec{AC}\| \cdot \cos \alpha \quad (1) \\ \vec{BA} \cdot \vec{AC} &= \|\vec{BA}\| \cdot \|\vec{AC}\| \cdot \cos(\pi - \alpha) \\ \Rightarrow -\vec{BA} \cdot \vec{AC} &= \|\vec{AB}\| \cdot \|\vec{AC}\| \cdot \cos(\pi - \alpha) \\ \Rightarrow \vec{AB} \cdot \vec{AC} &= -\|\vec{AB}\| \cdot \|\vec{AC}\| \cdot \cos(\pi - \alpha) \quad (2) \end{aligned}$$

(1) ve (2) den $\cos(\pi - \alpha) = -\cos \alpha$ bulunur.

**Etkinlik – 3.23**

$$\begin{aligned} \text{a. } \vec{BC} \cdot \vec{BC} &= (\vec{AC} - \vec{AB}) \cdot (\vec{AC} - \vec{AB}) \\ \Rightarrow \vec{BC} \cdot \vec{BC} &= \vec{AC} \cdot \vec{AC} + \vec{AB} \cdot \vec{AB} - 2\vec{AB} \cdot \vec{AC} \\ \Rightarrow \|\vec{BC}\|^2 &= \|\vec{AC}\|^2 + \|\vec{AB}\|^2 - 2\vec{AB} \cdot \vec{AC} \\ \Rightarrow a^2 &= b^2 + c^2 - 2\vec{AB} \cdot \vec{AC} \\ \Rightarrow \vec{AB} \cdot \vec{AC} &= \frac{1}{2}(b^2 + c^2 - a^2) \text{ olur.} \end{aligned}$$

**b. 1. yol**

$$\begin{aligned} \vec{AB} \cdot \vec{AC} &= \frac{1}{2}(b^2 + c^2 - a^2) \\ \vec{BA} \cdot \vec{BC} &= \frac{1}{2}(a^2 + c^2 - b^2) \\ \vec{CA} \cdot \vec{CB} &= \frac{1}{2}(a^2 + b^2 - c^2) \\ \vec{AB} \cdot \vec{AC} + \vec{BA} \cdot \vec{BC} + \vec{CA} \cdot \vec{CB} &= \frac{1}{2}(a^2 + b^2 + c^2) \end{aligned}$$

2. yol

$$\begin{aligned} \vec{AB} \cdot \vec{AC} + \vec{BA} \cdot \vec{BC} + \vec{CA} \cdot \vec{CB} \\ &= \vec{AB} \cdot \vec{AC} + \vec{BC}(\vec{BA} - \vec{CA}) \\ &= \vec{AB} \cdot \vec{AC} + \vec{BC}(\vec{BA} + \vec{AC}) \\ &= \vec{AB} \cdot \vec{AC} + \vec{BC} \cdot \vec{BC} \\ &= \frac{1}{2}(b^2 + c^2 - a^2) + a^2 \\ &= \frac{1}{2}(a^2 + b^2 + c^2) \text{ olur.} \end{aligned}$$

c. $\vec{AD} = \vec{AB} + \vec{BD}$

$$\begin{aligned} \Rightarrow \vec{AD} &= \vec{AB} + \frac{m}{a}\vec{BC} \\ \Rightarrow \vec{AD} \cdot \vec{AD} &= \left(\vec{AB} + \frac{m}{a}\vec{BC}\right) \cdot \left(\vec{AB} + \frac{m}{a}\vec{BC}\right) \\ \Rightarrow \|\vec{AD}\|^2 &= \|\vec{AB}\|^2 + \frac{2m}{a}\vec{AB} \cdot \vec{BC} + \frac{m^2}{a^2}\|\vec{BC}\|^2 \\ \Rightarrow \|\vec{AD}\|^2 &= c^2 + \frac{2m}{a} \cdot (-\vec{BA} \cdot \vec{BC}) + m^2 \\ \Rightarrow \|\vec{AD}\|^2 &= c^2 + m^2 - \frac{2m}{a} \cdot \frac{1}{2}(a^2 + c^2 - b^2) \\ \Rightarrow \|\vec{AD}\|^2 &= \frac{ac^2 + m^2a - ma^2 - mc^2 + mb^2}{a} \\ \Rightarrow \|\vec{AD}\|^2 &= \frac{(a-m)c^2 + mb^2 + ma(m-a)}{a} \end{aligned}$$

$$\Rightarrow \|\overline{AD}\|^2 = \frac{mb^2 + nc^2}{a} + m \cdot (m - a)$$

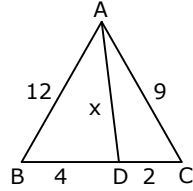
$$\Rightarrow \|\overline{AD}\|^2 = \frac{mb^2 + nc^2}{a} - mn$$

$$\Rightarrow \|\overline{AD}\|^2 = \frac{mb^2 + nc^2}{a} - m \cdot n \text{ bulunur.}$$

$$\text{d. } x^2 = \frac{4 \cdot 9^2 + 2 \cdot 12^2}{6} - 4 \cdot 2$$

$$\Rightarrow x^2 = 94$$

$$\Rightarrow x = \sqrt{94} \text{ birim olur.}$$

**Etkinlik – 3.24**

$$AH \perp BC$$

$$\Rightarrow \overline{AH} \cdot \overline{BC} = 0$$

$$\Rightarrow (\overline{AC} + \overline{CH}) \cdot \overline{BC} = 0$$

$$\Rightarrow \overline{AC} \cdot \overline{BC} + \overline{CH} \cdot \overline{BC} = 0 \quad (1)$$

$$BH \perp AC$$

$$\Rightarrow \overline{BH} \cdot \overline{AC} = 0$$

$$\Rightarrow (\overline{BC} + \overline{CH}) \cdot \overline{AC} = 0$$

$$\Rightarrow \overline{BC} \cdot \overline{AC} + \overline{CH} \cdot \overline{AC} = 0 \quad (2)$$

(1)'den (2) çıkarılırsa;

$$\overline{CH} \cdot \overline{BC} - \overline{CH} \cdot \overline{AC} = 0$$

$$\Rightarrow \overline{CH}(\overline{BC} + \overline{CA}) = 0$$

$$\Rightarrow \overline{CH} \cdot \overline{BA} = 0$$

$$\Rightarrow \overline{CH} \perp \overline{BA} \Rightarrow CH \perp AB \text{ bulunur.}$$

Etkinlik – 3.25

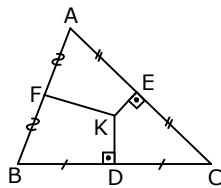
$$\left. \begin{array}{l} \overline{KD} \cdot \overline{BC} = 0 \\ \overline{KE} \cdot \overline{AC} = 0 \end{array} \right\} \Rightarrow \left(\overline{KF} + \frac{1}{2} \overline{AC} \right) \cdot \overline{BC} = 0$$

$$\left(\overline{KF} + \frac{1}{2} \overline{BC} \right) \cdot \overline{AC} = 0$$

$$\Rightarrow \overline{KF} \cdot \overline{BC} - \overline{KF} \cdot \overline{AC} = 0$$

$$\Rightarrow \overline{KF}(\overline{BC} + \overline{CA}) = 0$$

$$\Rightarrow \overline{KF} \cdot \overline{BA} = 0 \Rightarrow \overline{KF} \perp \overline{BA}$$

**Etkinlik – 3.26**

$$\text{a. } \overline{u}_a = \frac{\overline{a}}{\|\overline{a}\|} \Rightarrow \overline{u}_a = \frac{(2, 6)}{\sqrt{2^2 + 6^2}} \Rightarrow \overline{u}_a = \left(\frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10} \right);$$

$$\overline{u}_b = \frac{(4, 2)}{\sqrt{4^2 + 2^2}} \Rightarrow \overline{u}_b = \left(\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right);$$

$$\overline{u}_c = \frac{(-4, 2)}{\sqrt{(-4)^2 + 2^2}} \Rightarrow \overline{u}_c = \left(\frac{-2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right)$$

$$\text{b. } \overline{a}_b = (\overline{a} \cdot \overline{u}_b) \overline{u}_b$$

$$\Rightarrow \overline{a}_b = \left[(2, 6) \cdot \left(\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right) \right] \cdot \left(\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right)$$

$$\Rightarrow \overline{a}_b = 2\sqrt{5} \left(\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right)$$

$$\Rightarrow \overline{a}_b = (4, 2)$$

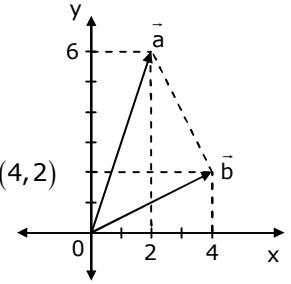
$\overline{a}_b = \overline{b}$ olduğuna dikkat ediniz.

Veya;

$$\overline{a}_b = \frac{\overline{a} \cdot \overline{b}}{\overline{b} \cdot \overline{b}} \cdot \overline{b}$$

$$\Rightarrow \overline{a}_b = \frac{(2, 6) \cdot (4, 2)}{(4, 2) \cdot (4, 2)} \cdot (4, 2)$$

$$\Rightarrow \overline{a}_b = (4, 2)$$



$$\text{c. } \overline{b}_a = (\overline{b} \cdot \overline{u}_a) \overline{u}_a$$

$$\Rightarrow \overline{b}_a = \left[(4, 2) \cdot \left(\frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10} \right) \right] \cdot \left(\frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10} \right)$$

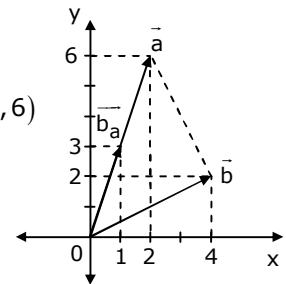
$$\Rightarrow \overline{b}_a = \sqrt{10} \cdot \left(\frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10} \right)$$

$$\Rightarrow \overline{b}_a = (1, 3)$$

Veya

$$\overline{b}_a = \frac{(2, 6) \cdot (4, 2)}{(2, 6) \cdot (2, 6)} \cdot (2, 6)$$

$$\Rightarrow \overline{b}_a = (1, 3)$$



$$d. \vec{c}_a = (\vec{c} \cdot \vec{u}_a) \cdot \vec{u}_a$$

$$\Rightarrow \vec{c}_a = \left[(-4, 2) \cdot \left(\frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10} \right) \right] \cdot \left(\frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10} \right)$$

$$\Rightarrow \vec{c}_a = \frac{\sqrt{10}}{5} \cdot \left(\frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10} \right)$$

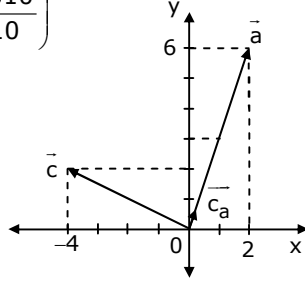
$$\Rightarrow \vec{c}_a = \left(\frac{1}{5}, \frac{3}{5} \right)$$

Veya

$$\vec{c}_a = \frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{a}} \cdot \vec{a}$$

$$\Rightarrow \vec{c}_a = \frac{(2, 6) \cdot (-4, 2)}{(2, 6) \cdot (2, 6)} \cdot (2, 6)$$

$$\Rightarrow \vec{c}_a = \left(\frac{1}{5}, \frac{3}{5} \right)$$



$$e. \vec{b}_c = (\vec{b} \cdot \vec{u}_c) \cdot \vec{u}_c$$

$$\Rightarrow \vec{b}_c = \left[(4, 2) \cdot \left(\frac{-2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right) \right] \cdot \left(\frac{-2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right)$$

$$\Rightarrow \vec{b}_c = \frac{-6\sqrt{5}}{5} \cdot \left(\frac{-2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right)$$

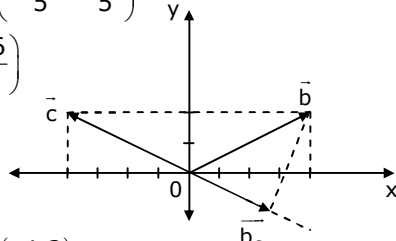
$$\Rightarrow \vec{b}_c = \left(\frac{12}{5}, \frac{-6}{5} \right)$$

Veya

$$\vec{b}_c = \frac{\vec{b} \cdot \vec{c}}{\vec{c} \cdot \vec{c}} \cdot \vec{c}$$

$$\Rightarrow \vec{b}_c = \frac{(4, 2) \cdot (-4, 2)}{(-4, 2) \cdot (-4, 2)} \cdot (-4, 2)$$

$$\Rightarrow \vec{b}_c = \left(\frac{12}{5}, \frac{-6}{5} \right)$$

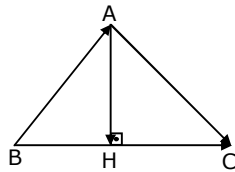


Etkinlik – 3.27

$$a. \vec{B}\vec{H} = \frac{\vec{B}\vec{A} \cdot \vec{B}\vec{C}}{\vec{B}\vec{C} \cdot \vec{B}\vec{C}} \cdot \vec{B}\vec{C}$$

$$\Rightarrow \vec{B}\vec{H} = \frac{(3, 4) \cdot (6, 3)}{(6, 3) \cdot (6, 3)} \cdot (6, 3)$$

$$\Rightarrow \vec{B}\vec{H} = \frac{2}{3}(6, 3) \Rightarrow \vec{B}\vec{H} = (4, 2) \text{ bulunur.}$$



Şöyle de yapılabilir:

$$\vec{B}\vec{H} = (\vec{B}\vec{A} \cdot \vec{u}_{\vec{B}\vec{C}}) \cdot \vec{u}_{\vec{B}\vec{C}}$$

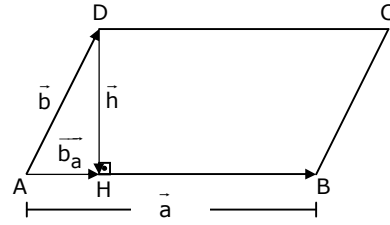
$$\Rightarrow \vec{B}\vec{H} = \left[(3, 4) \cdot \frac{(6, 3)}{\sqrt{45}} \right] \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right)$$

$$\Rightarrow \vec{B}\vec{H} = \frac{30}{3\sqrt{5}} \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right) \Rightarrow \vec{B}\vec{H} = (4, 2)$$

$$b. \vec{A}\vec{H} = \vec{A}\vec{B} + \vec{B}\vec{H}$$

$$\Rightarrow \vec{A}\vec{H} = (-3, -4) + (4, 2) \Rightarrow \vec{A}\vec{H} = (1, -2) \text{ olur.}$$

Etkinlik – 3.28



$$a. \vec{A}\vec{H} = \vec{b}_a = \frac{\vec{a} \cdot \vec{b}}{\vec{a} \cdot \vec{a}} \cdot \vec{a}$$

$$\Rightarrow \vec{b}_a = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|^2} \cdot \vec{a}$$

$$\vec{D}\vec{H} = \vec{D}\vec{A} + \vec{A}\vec{H}$$

$$\Rightarrow \vec{h} = -\vec{b} + \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|^2} \cdot \vec{a} \text{ olur.}$$

$$b. \|\vec{h}\| = \sqrt{\vec{h} \cdot \vec{h}}$$

$$\Rightarrow \|\vec{h}\| = \sqrt{\left(-\vec{b} + \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|^2} \cdot \vec{a} \right) \cdot \left(-\vec{b} + \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|^2} \cdot \vec{a} \right)}$$

$$\Rightarrow \|\vec{h}\| = \sqrt{\vec{b} \cdot \vec{b} - 2 \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|^2} \cdot \vec{a} \cdot \vec{b} + \frac{(\vec{a} \cdot \vec{b})^2}{\|\vec{a}\|^4} \cdot \vec{a} \cdot \vec{a}}$$

$$\Rightarrow \|\vec{h}\| = \sqrt{\|\vec{b}\|^2 - 2 \frac{(\vec{a} \cdot \vec{b})^2}{\|\vec{a}\|^2} + \frac{(\vec{a} \cdot \vec{b})^2}{\|\vec{a}\|^4} \cdot \|\vec{a}\|^2}$$

$$\Rightarrow \|\vec{h}\| = \sqrt{\|\vec{b}\|^2 - \frac{(\vec{a} \cdot \vec{b})^2}{\|\vec{a}\|^2}} \text{ olur.}$$

c. $A(ABCD) = |AB| \cdot |DH|$

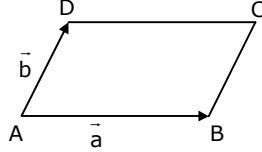
$$\Rightarrow A(ABCD) = \|\vec{a}\| \cdot \|\vec{h}\|$$

$$\Rightarrow A(ABCD) = \|\vec{a}\| \cdot \sqrt{\|\vec{b}\|^2 - \frac{(\vec{a} \cdot \vec{b})^2}{\|\vec{a}\|^2}}$$

$$\Rightarrow A(ABCD) = \sqrt{\|\vec{a}\|^2 \cdot \|\vec{b}\|^2 - (\vec{a} \cdot \vec{b})^2} \text{ bulunur.}$$

d. $\vec{a} = (4, 2)$ ve

$$\vec{b} = (1, 3) \text{ ise}$$



$$A(ABCD) = \sqrt{(4^2 + 2^2) \cdot (1^2 + 3^2) - [(4, 2) \cdot (1, 3)]^2}$$

$$\Rightarrow A(ABCD) = \sqrt{20 \cdot 10 - 10^2}$$

$$\Rightarrow A(ABCD) = 10 \text{ birim}^2$$

e. $\vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos \alpha$ ve

$$A(ABCD) = \sqrt{\|\vec{a}\|^2 \cdot \|\vec{b}\|^2 - (\vec{a} \cdot \vec{b})^2}$$

$$\Rightarrow A(ABCD) = \sqrt{\|\vec{a}\|^2 \cdot \|\vec{b}\|^2 - (\|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos \alpha)^2}$$

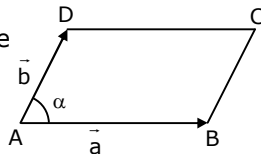
$$\Rightarrow A(ABCD) = \sqrt{\|\vec{a}\|^2 \cdot \|\vec{b}\|^2 (1 - \cos^2 \alpha)}$$

$$\Rightarrow A(ABCD) = \sqrt{\|\vec{a}\|^2 \cdot \|\vec{b}\|^2 \cdot \sin^2 \alpha}$$

$$\Rightarrow A(ABCD) = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \sin \alpha \text{ olur.}$$

f. $\vec{a} = (4, -2)$ ve $\vec{b} = (6, 2)$ ise

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos \alpha$$



$$\Rightarrow (4, -2) \cdot (6, 2) = \sqrt{4^2 + (-2)^2} \cdot \sqrt{6^2 + 2^2} \cdot \cos \alpha$$

$$\Rightarrow 20 = \sqrt{20} \cdot \sqrt{40} \cdot \cos \alpha \Rightarrow \cos \alpha = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \sin \alpha \sqrt{1 - \frac{2}{4}} \Rightarrow \sin \alpha = \frac{\sqrt{2}}{2} ;$$

$$A(ABCD) = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \sin \alpha$$

$$\Rightarrow A(ABCD) = \sqrt{20} \cdot \sqrt{40} \cdot \frac{\sqrt{2}}{2}$$

$$\Rightarrow A(ABCD) = 20 \text{ br}^2 \text{ bulunur.}$$

g. 1. yol

$$\vec{a} = (3, -4) \text{ ve}$$

$$\vec{b} = (8, 6) \text{ ise}$$

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos \alpha$$

$$(3, -4) \cdot (8, 6) = 5 \cdot 10 \cdot \cos \alpha$$

$$\cos \alpha = 0 \Rightarrow \sin \alpha = 1 \text{ olur.}$$

$$A(\triangle ABC) = \frac{1}{2} \|\vec{a}\| \cdot \|\vec{b}\| \cdot \sin \alpha$$

$$\Rightarrow A(\triangle ABC) = \frac{1}{2} \cdot 5 \cdot 10 \cdot 1$$

$$A(\triangle ABC) = 25 \text{ birim}^2 \text{ bulunur.}$$

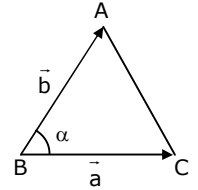
2. yol

$$A(\triangle ABC) = \frac{1}{2} \sqrt{\|\vec{a}\|^2 \cdot \|\vec{b}\|^2 - (\vec{a} \cdot \vec{b})^2}$$

$$\Rightarrow A(\triangle ABC) = \frac{1}{2} \cdot \sqrt{5^2 \cdot 10^2 - [(3, -4) \cdot (8, 6)]^2}$$

$$\Rightarrow A(\triangle ABC) = \frac{1}{2} \sqrt{25 \cdot 100}$$

$$\Rightarrow A(\triangle ABC) = 25 \text{ birim}^2$$



h. 1. yol

$$\vec{BC} = \vec{a} = (4, 2),$$

$$\vec{BA} = \vec{b} = (2, 6) \text{ ise}$$

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos \alpha$$

$$(4, 2) \cdot (2, 6) = \sqrt{20} \cdot \sqrt{40} \cdot \cos \alpha \Rightarrow \cos \alpha = \frac{\sqrt{2}}{2}$$

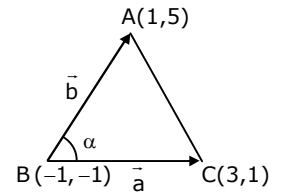
$$\sin \alpha = \sqrt{1 - \frac{1}{2}} \Rightarrow \sin \alpha = \frac{\sqrt{2}}{2}$$

$$A(\triangle ABC) = \frac{1}{2} \|\vec{a}\| \cdot \|\vec{b}\| \cdot \sin \alpha$$

$$\Rightarrow A(\triangle ABC) = \frac{1}{2} \cdot \sqrt{20} \cdot \sqrt{40} \cdot \frac{\sqrt{2}}{2}$$

$$\Rightarrow A(\triangle ABC) = 10 \text{ birim}^2 \text{ bulunur.}$$

2. yol



$$A(\triangle ABC) = \frac{1}{2} \sqrt{\|\vec{a}\|^2 \cdot \|\vec{b}\|^2 - (\vec{a} \cdot \vec{b})^2}$$

$$\Rightarrow A(\triangle ABC) = \frac{1}{2} \cdot \sqrt{20 \cdot 40 - [(4,2) \cdot (2,6)]^2}$$

$$\Rightarrow A(\triangle ABC) = 10 \text{ birim}^2 \text{ bulunur.}$$

3. yol

Bir noktanın koordinatlarının anlamını değerlendirerek, üçgenin alanını bir üçüncü yoldan bulabiliriz:

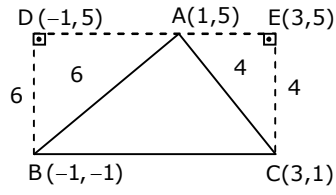
DE yatay

BD ve CE düşey
doğruları çizelim.

D(-1,5), E(3,5),

|DA| = 2, |BD| = 6,

|AE| = 4, |CE| = 4 olur.



$$A(\triangle ABC) = A(BCED) - A(\triangle ABD) - A(\triangle ACE)$$

$$\Rightarrow A(\triangle ABC) = \frac{(6+4)4}{2} - \frac{6 \cdot 2}{2} - \frac{2 \cdot 4}{2}$$

$$\Rightarrow A(\triangle ABC) = 10 \text{ birim}^2 \text{ bulunur.}$$