Alıştırmalar ve Problemler-4.1

1. a. $\vec{A}=(-2,3)$, $\vec{B}=(3,-1)$, $\vec{C}=(-4,2)$,

$\overrightarrow{CD}=(1,2)$

$\vec{A}+2\vec{B}-\vec{C}=(-2,3)+2(3,-1)-(4,2)$

$\Rightarrow \hat{A}+2\vec{B}-\vec{C}=(8,-1)$

b. $\overrightarrow{AC}+\overrightarrow{BP}=2\overrightarrow{CD}$

$\Rightarrow \vec{P}=2\overrightarrow{CD}+\vec{B}-\vec{C}+\vec{A}$

$\Rightarrow \vec{P}=2(1,2)+(3,-1)-(-4,2)+(-2,3)$

$\Rightarrow \vec{P}=(7,4)$

c. $\overrightarrow{AB}+2\overrightarrow{BC}+3\overrightarrow{CD}=\overrightarrow{AR}$

$\Rightarrow \vec{B}-\vec{A}+2\vec{C}-2\vec{B}+3\overrightarrow{CD}=\vec{R}-\vec{A}$

$\Rightarrow \vec{R}=3\overrightarrow{CD}+2\vec{C}-\vec{B}$

$\Rightarrow \vec{R}=3(1,2)+2(-4,2)-(3,-1)$

$\Rightarrow \vec{R}=(-8,11)$

d. $\overrightarrow{AS}-2\overrightarrow{BC}=\overrightarrow{SC}$

$\Rightarrow \vec{S}-\vec{A}-2\vec{C}+2\vec{B}=\vec{C}-\vec{S}$

$\Rightarrow 2\vec{S}=\vec{A}-2\vec{B}+3\vec{C}$

$=(-2,3)-2(3,-1)+3(-4,2)$

$\Rightarrow 2\vec{S}=(-20,11)\Rightarrow \vec{S}=\left(-10,\frac{11}{2}\right)$

2. a. $\overrightarrow{AB}=(3,-2)$, $\overrightarrow{BC}=(-1,1)$, $\vec{C}=(2,-3)$,

$\overrightarrow{CD}=(k,2)$

$\vec{B}=\vec{C}+\overrightarrow{CB}$

$\Rightarrow \vec{B}=(2,-3)+(1,-1)\Rightarrow \vec{B}=(3,-4)$

$\vec{A}=\vec{B}+\overrightarrow{BA}$

$\Rightarrow \vec{A}=(3,-4)+(-3,2)\Rightarrow \vec{A}=(0,-2)$

b. $\overrightarrow{AB}=x\vec{A}+y\vec{B}$

$\Rightarrow \vec{B}-\vec{A}=x\vec{A}+y\vec{B}$

$\vec{A}$ ve $\vec{B}$ doğrusal bağımsız vektörler olduğundan, $x=-1$ ve $y=1$ olmalıdır.

c. $\vec{D}=\vec{C}+\overrightarrow{CD}$

$\Rightarrow \vec{D}=(2,-3)+(k,2)\Rightarrow \vec{D}=(k+2,-1)$

$\vec{C}$ ve $\vec{D}$ doğrusal bağımlı ise,

$\frac{k+2}{2}=\frac{-1}{-3}\Rightarrow k=-\frac{4}{3}$ bulunur

d. $x(0,-2)+y(3,-4)+z(2,-3)=0$

$\Rightarrow \left.\begin{array}{ll} 3/ & 3y+2z=0 \\ 2/ & -2x-4y-3z=0 \end{array}\right\}$

$\Rightarrow -4x+y=0\Rightarrow y=4x;$

$3y+2z=0$

$\Rightarrow 3\cdot 4x+2z=0\Rightarrow z=-6x$ bulunur.

$x=1$ için $y=4$ ve $z=-6$ olur.

$\vec{A}+4\vec{B}-6\vec{C}=0$ dır

e. $\vec{A}+\vec{B}+\vec{C}=x\overrightarrow{AB}+y\overrightarrow{BC}$

$\Rightarrow (0,-2)+(3,-4)+(2,-3)$

$=x(3,-2)+y(-1,1)$

$\Rightarrow \left.\begin{array}{l} 3x-y=5 \\ -2x+y=-9 \end{array}\right\}$

$\Rightarrow x=-4,\ y=-17$ bulunur.

f. A, B, D doğrusal ise,

örneğin; $\overrightarrow{AB}$ ve $\overrightarrow{BD}$ doğrusal bağımlı olurlar.

$\overrightarrow{AB}=(3,-2);$

$\overrightarrow{BD}=\vec{D}-\vec{B}\Rightarrow \overrightarrow{BD}=(k+2,-1)-(3,-4)$

$\Rightarrow \overrightarrow{BD}=(k-1,3)$

$\frac{3}{k-1}=\frac{-2}{3}\Rightarrow -2k+2=9$

$\Rightarrow k=-\frac{7}{2}$ bulunur.

3. a. $A(-3,4)$, $B(2,-6)$, $C(x,y)$

$\frac{\overrightarrow{CA}}{\overrightarrow{CB}}=\frac{-2}{3}\Rightarrow 3\overrightarrow{CA}=-2\overrightarrow{CB}$

$\Rightarrow 3(-3-x,4-y)=-2(2-x,-6-y)$

$\Rightarrow \left.\begin{array}{l} -9-3x=-4+2x\Rightarrow x=-1 \\ 12-3y=12+2y\Rightarrow y=0 \end{array}\right\}$

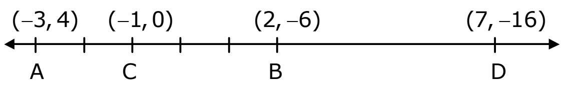
$\vec{C}=(-1,0)$ bulunur.

b. $\overrightarrow{DA} = 2\overrightarrow{DB} \Rightarrow (-3-x, 4-y) = 2(2-x, -6-y)$

$\Rightarrow \left.\begin{array}{l} -3-x = 4-2x \Rightarrow x = 7 \\ 4-y = -12-2y \Rightarrow y = -16 \end{array}\right\}$

$\vec{D} = (7, -16)$ bulunur.

c.



4. a. A, B, C doğrusal ise, örneğin $\overrightarrow{AB}$ ile $\overrightarrow{AC}$ doğrusal bağımlıdır.

$\overrightarrow{AB} = (5, m-4)$ ve $\overrightarrow{AC} = (2, m-7)$ doğrusal bağımlı ise,

$\frac{5}{2} = \frac{m-4}{m-7} \Rightarrow m = 9$ olur.

b. 1. yol

$[AD]$'nın ortası B ise,

$\overrightarrow{AB} = \overrightarrow{BD}$ olur.

$\overrightarrow{AB} = \overrightarrow{BD}$

$\Rightarrow (5, m-4) = (n-2, 2)$

$\Rightarrow n = 7,\ m = 6$ olur.

2. yol

$M(x_1, y_1),\ N(x_2, y_2),\ P(x, y)$ ve $[MN]$'nın orta noktası P ise

$x = \frac{x_1 + x_2}{2}$ ve $y = \frac{y_1 + y_2}{2}$ olur.

$2 = \frac{-3+n}{2} \Rightarrow n = 7;\ m = \frac{4+m+2}{2} \Rightarrow m = 6$

5. $|DC| = 2|AD| \Rightarrow \overrightarrow{AC} = 3\overrightarrow{AD}$

$\Rightarrow (6, -6) = 3 \cdot (x-1, y-3)$

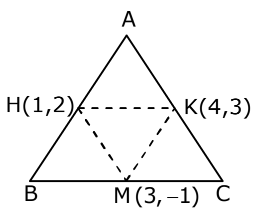
$\Rightarrow x = 3,\quad y = 1 \Rightarrow D(3, 1)$

$\Rightarrow \overrightarrow{BD} = (4, 2)$ olur.

6. $\overrightarrow{HA} = \overrightarrow{MK} \Rightarrow A(2, 6);$

$\overrightarrow{BM} = \overrightarrow{HK} \Rightarrow B(0, -2);$

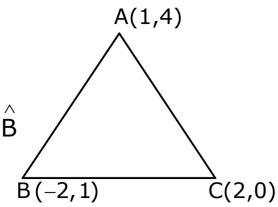
$\overrightarrow{MC} = \overrightarrow{HK} \Rightarrow C(6, 0)$



7. $\overrightarrow{AB} \cdot \overrightarrow{AC} = \|\overrightarrow{AB}\| \cdot \|\overrightarrow{AC}\| \cdot \cos\hat{A}$

$\Rightarrow (-3, -3) \cdot (1, -4) = 3\sqrt{2} \cdot \sqrt{17} \cdot \cos\hat{A}$

$\Rightarrow \cos\hat{A} = \frac{3\sqrt{34}}{34};$



$\overrightarrow{BA} \cdot \overrightarrow{BC} = \|\overrightarrow{BA}\| \cdot \|\overrightarrow{BC}\| \cdot \cos\hat{B}$

$\Rightarrow (3, 3) \cdot (4, -1)$

$= 3\sqrt{2} \cdot \sqrt{17} \cdot \cos\hat{B}$

$\Rightarrow \cos\hat{B} = \frac{3\sqrt{34}}{34};$

$\overrightarrow{CA} \cdot \overrightarrow{CB} = \|\overrightarrow{CA}\| \cdot \|\overrightarrow{CB}\| \cdot \cos\hat{C}$

$\Rightarrow (-1, 4) \cdot (-4, 1) = \sqrt{17} \cdot \sqrt{17} \cdot \cos\hat{C}$

$\Rightarrow \cos\hat{C} = \frac{8}{17}$

8. 2/ $\quad 2\vec{a} + \vec{b} = (1, 6)$

$\underline{3\vec{a} - 2\vec{b} = (-9, 16)}$

$7\vec{a} = (-7, 28) \Rightarrow \vec{a} = (-1, 4)$

$2 \cdot (-1, 4) + \vec{b} = (1, 6) \Rightarrow \vec{b} = (3, -2)$

9. $x\vec{a} + y\vec{b} = \vec{v}$

$\Rightarrow x(2, -1) + y(-1, 3) = (1, 7)$

$\Rightarrow \left.\begin{array}{l} 2x - y = 1 \\ -x + 3y = 7 \end{array}\right\} x = 2,\ y = 3$

$\overrightarrow{v_a} = x(2, -1) \Rightarrow \overrightarrow{v_a} = (4, -2)$

$\overrightarrow{v_b} = y(-1, 3) \Rightarrow \overrightarrow{v_b} = (-3, 9)$

$\overrightarrow{v_a} \cdot \overrightarrow{v_b} = (4, -2) \cdot (-3, 9)$

$\Rightarrow \overrightarrow{v_a} \cdot \overrightarrow{v_b} = -30$ bulunur.

10. a. $\|\vec{A}\| = \sqrt{(2\sqrt{2}\vec{a} - 4\vec{b}) \cdot (2\sqrt{2}\vec{a} - 4\vec{b})}$

$\Rightarrow \|\vec{A}\| = \sqrt{8 + 16 - 16\sqrt{2}\vec{a} \cdot \vec{b}}$

$\Rightarrow \|\vec{A}\| = \sqrt{24 - 16\sqrt{2} \cdot 1 \cdot 1 \cdot \frac{\sqrt{2}}{2}}$

$\Rightarrow \|\vec{A}\| = 2\sqrt{2}$

b. $\|\vec{B}\| = \sqrt{(2\vec{a}+\sqrt{2}\vec{b})(2\vec{a}+\sqrt{2}\vec{b})}$

$\Rightarrow \|\vec{B}\| = \sqrt{4+2+4\sqrt{2}\cdot 1\cdot 1\cdot \frac{\sqrt{2}}{2}}$

$\Rightarrow \|\vec{B}\| = \sqrt{10}$

c. $\vec{A}\cdot\vec{B} = \|\vec{A}\|\cdot\|\vec{B}\|\cdot\cos\alpha$

$(2\sqrt{2}\vec{a}-4\vec{b})\cdot(2\vec{a}+\sqrt{2}\vec{b}) = 2\sqrt{2}\cdot\sqrt{20}\cdot\cos\alpha$

$\Rightarrow 4\sqrt{2}-4\sqrt{2}-4\vec{a}\cdot\vec{b} = 4\sqrt{5}\cdot\cos\alpha$

$\Rightarrow -4\cdot 1\cdot 1\cdot\frac{\sqrt{2}}{2} = 4\sqrt{5}\cdot\cos\alpha$

$\Rightarrow \cos\alpha = \frac{-\sqrt{10}}{10}$

11. $\vec{a} = x\cdot\vec{u} \Rightarrow \vec{a} = (x,-2x)$

$\vec{b} = y\cdot\vec{v} \Rightarrow \vec{b} = (-2y,3y)$

$\vec{a}-2\vec{b} = (-11,+6)$

$\Rightarrow (x,-2x)-2(-2y,3y) = (-11,+6)$

$\Rightarrow \left.\begin{array}{l} x+4y=-11 \\ -2x-6y=+6 \end{array}\right\} \Rightarrow x=21,\ y=-8$

$\vec{a} = (21,-42)$ ve $\vec{b} = (16,-24)$ olur.

12. a. $\vec{a} = (3,-1),\ \vec{b} = (2,1),\ \vec{c} = (-1,-3)$

$a+kb = (3+2k,-1+k)$

$\frac{3+2k}{-1} = \frac{-1+k}{-3} \Rightarrow k=-2$

b. Koordinatlarına bakıldığında $\vec{a}$ ve $\vec{b}$ vektörlerinin doğrusal bağımsız oldukları görülür. $2\vec{a}+k(\vec{a}+\vec{b})$ ile $t(\vec{a}-\vec{b})$ vektörleri doğrusal bağımlı olduğuna gore, bu iki vektördeki $\vec{a}$'nün katsayıları ile $\vec{b}$'nün kat sayıları orantılı olmalıdır.

$\frac{2+k}{t} = \frac{k}{-t} \Rightarrow -2-k=k$

$\Rightarrow k=-1$ olur.

13. a. $(\vec{a}-2\vec{b})\cdot(\vec{a}-2\vec{b}) = \|\vec{a}\|^2+4\|\vec{b}\|^2-4\vec{a}\cdot\vec{b}$

$\Rightarrow (2\sqrt{3},-2)\cdot(2\sqrt{3},-2)$

$= 16+4-4\cdot 4\cdot 2\cdot\cos\alpha$

$\Rightarrow 16 = 20-32\cos\alpha \Rightarrow \cos\alpha = \frac{1}{8}$

bulunur.

b. a'da $\vec{a}\cdot\vec{b} = 1$ bulunur.

$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = \|\vec{a}+\vec{b}\|\cdot\|\vec{a}-\vec{b}\|\cdot\cos\beta$

$\Rightarrow \|\vec{a}\|^2-\|\vec{b}\|^2 = \sqrt{\|\vec{a}\|^2+\|\vec{b}\|^2+2\vec{a}\cdot\vec{b}}$

$\sqrt{\|\vec{a}\|^2+\|\vec{b}\|^2-2\vec{a}\cdot\vec{b}}\cdot\cos\beta$

$\Rightarrow 16-4 = \sqrt{16+4+2}\cdot\sqrt{16+4-2}\cdot\cos\beta$

$\Rightarrow \cos\beta = \frac{2\sqrt{11}}{11}$ bulunur.

14. a. $(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = \|\vec{a}\|^2+\|\vec{b}\|^2+2\vec{a}\cdot\vec{b}$

$\Rightarrow 64 = 36+16+2\vec{a}\cdot\vec{b}$

$\Rightarrow \vec{a}\cdot\vec{b} = 6$

$\Rightarrow \|\vec{a}\|\cdot\|\vec{b}\|\cdot\cos\alpha = 6 \Rightarrow \cos\alpha = \frac{1}{4}$ olur.

b. $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = \|\vec{a}+\vec{b}\|\cdot\|\vec{a}-\vec{b}\|\cdot\cos\beta$

$\Rightarrow \|\vec{a}\|^2-\|\vec{b}\|^2 = \|\vec{a}+\vec{b}\|\cdot\sqrt{\|\vec{a}\|^2+\|\vec{b}\|^2-2\underbrace{\vec{a}\cdot\vec{b}}_{6}}\cdot\cos\beta$

$\Rightarrow 36-16 = 8\cdot\sqrt{36+16-2\cdot 6}\cdot\cos\beta$

$\Rightarrow \cos\beta = \frac{\sqrt{10}}{4}$ olur.

c. $(\vec{a}-2\vec{b})\cdot\vec{a} = \|\vec{a}-2\vec{b}\|\cdot\|\vec{a}\|\cdot\cos\theta$

$\Rightarrow \|\vec{a}\|^2-2\underbrace{\vec{a}\cdot\vec{b}}_{6} =$

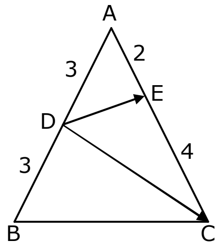
$\sqrt{\|\vec{a}\|^2+4\|\vec{b}\|^2-4\vec{a}\cdot\vec{b}}\cdot\|\vec{a}\|\cdot\cos\theta$

$\Rightarrow 36-12 = \sqrt{36+64-4\cdot 6}\cdot 6\cdot\cos\theta$

$\Rightarrow 24 = \sqrt{76}\cdot 6\cdot\cos\theta$

$\Rightarrow \cos\theta = \frac{2\sqrt{19}}{19}$ bulunur.

15. $\left(\overrightarrow{DE}+\overrightarrow{DA}\right)\cdot\overrightarrow{DC}$



$=\left(\overrightarrow{DA}+\overrightarrow{AE}+\overrightarrow{DA}\right)\cdot\left(\overrightarrow{DA}+\overrightarrow{AC}\right)$

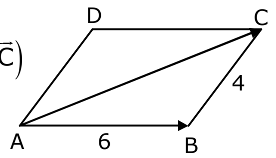
$=\left(2\overrightarrow{DA}+\overrightarrow{AE}\right)\cdot\left(\overrightarrow{DA}+\overrightarrow{AC}\right)$

$=2\left\|\overrightarrow{DA}\right\|^2+2\overrightarrow{DA}\cdot\overrightarrow{AC}+\overrightarrow{AE}\cdot\overrightarrow{DA}+\overrightarrow{AE}\cdot\overrightarrow{AC}$

$=2\cdot9+2\cdot3\cdot6\cdot\dfrac{-1}{2}+3\cdot2\cdot\dfrac{-1}{2}+2\cdot6$

$=9$ bulunur.

16. $\overrightarrow{AC}\cdot\left(\overrightarrow{AB}+\overrightarrow{CB}\right)$

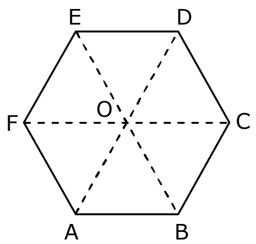


$\Rightarrow\left(\overrightarrow{AB}+\overrightarrow{BC}\right)\left(\overrightarrow{AB}-\overrightarrow{BC}\right)$

$=\left\|\overrightarrow{AB}\right\|^2-\left\|\overrightarrow{BC}\right\|^2$

$=36-16=20$

17. a. $\overrightarrow{AC}\cdot\overrightarrow{AD}$



$=\left(\overrightarrow{AB}+\overrightarrow{BC}\right)\cdot2\overrightarrow{BC}$

$=2\overrightarrow{AB}\cdot\overrightarrow{BC}+2\left\|\overrightarrow{BC}\right\|^2$

$=2\cdot6\cdot6\cdot\dfrac{1}{2}+2\cdot6^2$

$=108$

b. $\left(\overrightarrow{AB}+\overrightarrow{CD}\right)\cdot\overrightarrow{EF}$

$=\left(\overrightarrow{AB}+\overrightarrow{BO}\right)\cdot\overrightarrow{CB}$

$=\overrightarrow{AO}\cdot\overrightarrow{CB}=\overrightarrow{BC}\cdot\overrightarrow{CB}=-\left\|\overrightarrow{BC}\right\|^2=-36$

c. $\left(\overrightarrow{AB}+\overrightarrow{AF}\right)\cdot\left(\overrightarrow{AC}+\overrightarrow{AE}\right)$

$=\overrightarrow{AO}\cdot\left(\overrightarrow{AO}+\overrightarrow{OC}+\overrightarrow{AO}+\overrightarrow{OE}\right)$

$=\overrightarrow{AO}\cdot\left(2\overrightarrow{AO}+\overrightarrow{OD}\right)=\overrightarrow{AO}\cdot3\overrightarrow{AO}=3\cdot6^2=108$

d. $\overrightarrow{AC}\cdot\overrightarrow{BD}$

$=\left(\overrightarrow{AB}+\overrightarrow{BC}\right)\cdot\left(\overrightarrow{BC}+\overrightarrow{CD}\right)$

$=\overrightarrow{AB}\cdot\overrightarrow{BC}+\overrightarrow{AB}\cdot\overrightarrow{CD}+\left\|\overrightarrow{BC}\right\|^2+\overrightarrow{BC}\cdot\overrightarrow{CD}$

$=6\cdot6\cdot\dfrac{1}{2}+6\cdot6\cdot\dfrac{-1}{2}+6^2+6\cdot6\cdot\dfrac{1}{2}=54$

e. $\overrightarrow{AB}\cdot\overrightarrow{BD}=\overrightarrow{AB}\cdot\left(\overrightarrow{BA}+\overrightarrow{AD}\right)$

$=\overrightarrow{AB}\cdot\overrightarrow{BA}+\overrightarrow{AB}\cdot\overrightarrow{AD}=-36+6\cdot12\cdot\dfrac{1}{2}=0$

f. $\overrightarrow{DE}\cdot\overrightarrow{BF}=\overrightarrow{BA}\cdot\left(\overrightarrow{BA}+\overrightarrow{AF}\right)$

$=\overrightarrow{BA}\cdot\overrightarrow{BA}+\overrightarrow{BA}\cdot\overrightarrow{AF}=36+6\cdot6\cdot\dfrac{1}{2}=54$

18. a. $\overrightarrow{AB'}\cdot\overrightarrow{BC'}=\left(\overrightarrow{AB}+\overrightarrow{BB'}\right)\cdot\left(\overrightarrow{BB'}+\overrightarrow{B'C'}\right)$

$=\overrightarrow{AB}\cdot\overrightarrow{BB'}+\overrightarrow{AB}\cdot\overrightarrow{B'C'}+\overrightarrow{BB'}\cdot\overrightarrow{BB'}+\overrightarrow{BB'}\cdot\overrightarrow{B'C'}$

$=0+0+1+0=1$

b. $\overrightarrow{AB'}\cdot\overrightarrow{AC}=\left(\overrightarrow{AB}+\overrightarrow{BB'}\right)\cdot\left(\overrightarrow{AB}+\overrightarrow{BC}\right)$

$=\overrightarrow{AB}\cdot\overrightarrow{AB}+\overrightarrow{AB}\cdot\overrightarrow{BC}+\overrightarrow{BB'}\cdot\overrightarrow{AB}+\overrightarrow{BB'}\cdot\overrightarrow{BC}$

$=1+0+0+0=1$

c. $\overrightarrow{AB}\cdot\overrightarrow{AC'}=\overrightarrow{AB}\cdot\left(\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CC'}\right)$

$=\overrightarrow{AB}\cdot\overrightarrow{AB}+\overrightarrow{AB}\cdot\overrightarrow{BC}+\overrightarrow{AB}\cdot\overrightarrow{CC'}$

$=1+0+0=1$

d. $\overrightarrow{AC'}\cdot\overrightarrow{BD'}=\left(\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CC'}\right)\cdot\left(\overrightarrow{BC}+\overrightarrow{CC'}+\overrightarrow{C'D'}\right)$

$=\overrightarrow{AB}\cdot\overrightarrow{C'D'}+\overrightarrow{BC}\cdot\overrightarrow{BC}+\overrightarrow{CC'}\cdot\overrightarrow{CC'}$

$=-1+1+1=1$

e. $\left(\overrightarrow{AB}+\overrightarrow{AC}\right)\cdot\left(\overrightarrow{AB'}+\overrightarrow{AC'}\right)$

$=\left(\overrightarrow{AB}+\overrightarrow{AB}+\overrightarrow{BC}\right)\cdot\left(\overrightarrow{AB}+\overrightarrow{BB'}+\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CC'}\right)$

$=\left(2\overrightarrow{AB}+\overrightarrow{BC}\right)\left(2\overrightarrow{AB}+\overrightarrow{BC}+2\overrightarrow{BB'}\right)$

$=4\overrightarrow{AB}\cdot\overrightarrow{AB}+\overrightarrow{BC}\cdot\overrightarrow{BC}=4+1=5$

f. $\left(\overrightarrow{AB'}+\overrightarrow{A'C}\right)\cdot\left(\overrightarrow{A'B}+\overrightarrow{AC'}\right)$

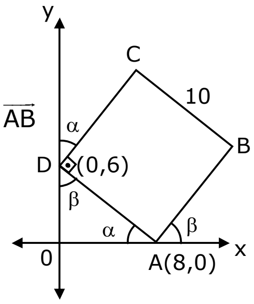
$\left(\overrightarrow{AA'}+\overrightarrow{A'B'}+\overrightarrow{A'A}+\overrightarrow{AB}+\overrightarrow{BC}\right)\cdot\left(\overrightarrow{A'A}+\overrightarrow{AB}+\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CC'}\right)$

$=\left(2\overrightarrow{AB}+\overrightarrow{BC}\right)\left(2\overrightarrow{AB}+\overrightarrow{BC}\right)$

$=2\overrightarrow{AB}\cdot\overrightarrow{AB}+\overrightarrow{BC}\cdot\overrightarrow{BC}=4+1=5$

19. a. $\vec{A}\cdot\overrightarrow{BC}=\vec{A}\cdot\overrightarrow{AD}=(8,0)\cdot(-8,6)=-64$

b. $\vec{B}\cdot\overrightarrow{CD}=\left(\vec{A}+\overrightarrow{AB}\right)\cdot\overrightarrow{BA}$



$=\vec{A}\cdot\overrightarrow{BA}+\overrightarrow{AB}\cdot\overrightarrow{BA}$

$=-\vec{A}\cdot\overrightarrow{AB}-\overrightarrow{AB}\cdot\overrightarrow{AB}$

$=-8\cdot 10\cdot\dfrac{6}{10}-10\cdot 10$

$=-148$

c. $\vec{A}\cdot\overrightarrow{BD}=\vec{A}\cdot\left(\overrightarrow{BA}+\overrightarrow{AD}\right)$

$=-\vec{A}\cdot\overrightarrow{AB}-\vec{A}\cdot\overrightarrow{DA}=-8\cdot 10\cdot\dfrac{6}{10}-8\cdot 10\cdot\dfrac{8}{10}$

$=-48-64=-112$

d. $\vec{B}\cdot\overrightarrow{AC}=\left(\vec{A}+\overrightarrow{AB}\right)\left(\overrightarrow{AB}+\overrightarrow{BC}\right)$

$=\left(\vec{A}+\overrightarrow{AB}\right)\left(\overrightarrow{AB}+\overrightarrow{AD}\right)=\vec{A}\cdot\overrightarrow{AB}+\vec{A}\cdot\overrightarrow{AD}$

$+\overrightarrow{AB}\cdot\overrightarrow{AB}+\overrightarrow{AB}\cdot\overrightarrow{AD}$

$=8\cdot 10\cdot\dfrac{6}{10}+8\cdot 10\cdot\dfrac{-8}{10}+10\cdot 10+0=84$

e. $\left(\vec{A}+\vec{D}\right)\left(\vec{B}+\vec{C}\right)=\left(\vec{A}+\vec{D}\right)\left(\vec{A}+\overrightarrow{AB}+\vec{D}+\overrightarrow{DC}\right)$

$=\left(\vec{A}+\vec{D}\right)\left(\vec{A}+\vec{D}+2\overrightarrow{AB}\right)$

$=\vec{A}\cdot\vec{A}+\vec{A}\cdot\vec{D}+2\vec{A}\cdot\overrightarrow{AB}+\vec{D}\cdot\vec{A}$

$+\vec{D}\cdot\vec{D}+2\vec{D}\cdot\overrightarrow{DC}$

$=64+0+2\cdot 8\cdot 10\cdot\dfrac{6}{10}+0+36$

$+2\cdot 6\cdot 10\cdot\dfrac{8}{10}$

$=64+96+36+96=292$

f. $\left(\vec{A}+\vec{B}\right)\left(\vec{C}+\vec{D}\right)=\left(\vec{A}+\vec{A}+\overrightarrow{AB}\right)\left(\vec{D}+\overrightarrow{DC}+\vec{D}\right)$

$=\left(2\vec{A}+\overrightarrow{AB}\right)\left(2\vec{D}+\overrightarrow{DC}\right)$

$=4\vec{A}\cdot\vec{D}+2\vec{A}\cdot\overrightarrow{DC}+2\overrightarrow{AB}\cdot\vec{D}+\overrightarrow{AB}\cdot\overrightarrow{DC}$

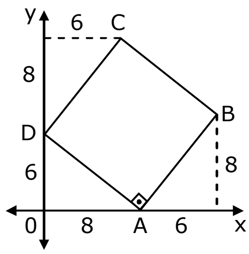
$=0+2\vec{A}\cdot\overrightarrow{AB}+2\overrightarrow{DC}\cdot\vec{D}+10\cdot 10$

$=2\cdot 8\cdot 10\cdot\dfrac{6}{10}+2\cdot 10\cdot 6\cdot\dfrac{8}{10}+100$

$=292$

Uyarı!

Sorunun tamamı B ve C köşelerinin koordinatları bulunarak da çözülebilirdi.



$\overset{\Delta}{AOD}\cong\overset{\Delta}{BFA}\cong\overset{\Delta}{DEC}$

$\Rightarrow B(14,8),\ C(6,14)$ olur.

f'deki çarpımı bir de koordinatlarla bulalım:

$\left(\vec{A}+\vec{B}\right)\cdot\left(\vec{C}+\vec{D}\right)$

$=\left[(8,0)+(14,8)\right]\cdot\left[(6,14)+(0,6)\right]$

$=(22,8)\cdot(6,20)$

$=132+160$

$=292$

20. $\vec{a}=(-6,3),\ \vec{b}=(4,2),\ \vec{c}=(6,-2)$

a. $\overrightarrow{b_c}=\dfrac{\vec{b}\cdot\vec{c}}{\vec{c}\cdot\vec{c}}\cdot\vec{c}$

$\Rightarrow\overrightarrow{b_c}=\dfrac{(4,2)\cdot(6,-2)}{(6,-2)\cdot(6,-2)}\cdot(6,-2)$

$\Rightarrow\overrightarrow{b_c}=(3,-1)$

b. $\overrightarrow{c_b}=\dfrac{\vec{b}\cdot\vec{c}}{\vec{b}\cdot\vec{b}}\cdot\vec{b}$

$\Rightarrow\overrightarrow{c_b}=\dfrac{(4,2)\cdot(6,-2)}{(4,2)\cdot(4,2)}\cdot(4,2)\Rightarrow\overrightarrow{c_b}=(4,2)$

c. $\overrightarrow{a_b}=\dfrac{\vec{a}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\cdot\vec{b}$

$\Rightarrow\overrightarrow{a_b}=\dfrac{(-6,3)\cdot(4,2)}{(4,2)\cdot(4,2)}\cdot(4,2)$

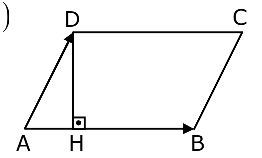
$\Rightarrow\overrightarrow{a_b}=\left(\dfrac{-18}{5},\dfrac{-9}{5}\right)$

d. $\overrightarrow{b_a}=\dfrac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\cdot\vec{a}$

$\Rightarrow\overrightarrow{b_a}=\dfrac{(-6,3)\cdot(4,2)}{(-6,3)\cdot(-6,3)}\cdot(-6,3)$

$\Rightarrow\overrightarrow{b_a}=\left(\dfrac{12}{5},\dfrac{-6}{5}\right)$

21. $\overrightarrow{AB}=(10,5)$, $\overrightarrow{AD}=(2,6)$



a. $\overrightarrow{AH}=\dfrac{\overrightarrow{AD}\cdot\overrightarrow{AB}}{\overrightarrow{AB}\cdot\overrightarrow{AB}}\cdot\overrightarrow{AB}$

$\Rightarrow \overrightarrow{AH}=\dfrac{(2,6)\cdot(10,5)}{(10,5)(10,5)}\cdot(10,5)\Rightarrow\overrightarrow{AH}=(4,2)$

Şöyle de yapılabilirdi:

$\overrightarrow{AH}=\left(\overrightarrow{AD}\cdot\overrightarrow{u_{AB}}\right)\cdot\overrightarrow{u_{AB}}$

$\Rightarrow \overrightarrow{AH}=\left[(2,6)\cdot\dfrac{(10,5)}{\sqrt{125}}\right]\left(\dfrac{2}{\sqrt5},\dfrac{1}{\sqrt5}\right)$

$=\dfrac{10}{\sqrt5}\left(\dfrac{2}{\sqrt5},\dfrac{1}{\sqrt5}\right)$

$\Rightarrow \overrightarrow{AH}=(4,2)$

b. $\overrightarrow{DH}=\overrightarrow{DA}+\overrightarrow{AH}\Rightarrow\overrightarrow{DH}=(-2,-6)+(4,2)$

$\Rightarrow\overrightarrow{DH}=(2,-4)$

c. $A(ABCD)=\left\|\overrightarrow{AB}\right\|\cdot\left\|\overrightarrow{DH}\right\|$

$\Rightarrow A(ABCD)=\sqrt{125}\cdot\sqrt{20}$

$\Rightarrow A(ABCD)=50$ birim^2

d. $A(ABCD)=\sqrt{125\cdot40-\left[(10,5)\cdot(2,6)\right]^2}$

$\Rightarrow A(ABCD)=\sqrt{5000-2500}=50$ birim^2

e. $\overrightarrow{AB}\cdot\overrightarrow{AD}=\left\|\overrightarrow{AB}\right\|\cdot\left\|\overrightarrow{AD}\right\|\cdot\cos\alpha$

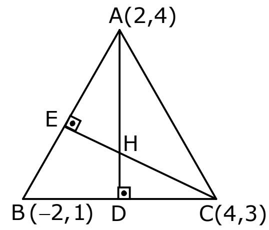
$\Rightarrow(10,5)\cdot(2,6)=\sqrt{125}\cdot\sqrt{40}\cdot\cos\alpha$

$\Rightarrow\cos\alpha=\dfrac{\sqrt2}{2}\Rightarrow\sin\alpha=\dfrac{\sqrt2}{2}$

$A(ABCD)=\left\|\overrightarrow{AB}\right\|\cdot\left\|\overrightarrow{AD}\right\|\cdot\sin\alpha$

$\Rightarrow A(ABCD)=\sqrt{125}\cdot\sqrt{40}\cdot\dfrac{\sqrt2}{2}=50$ birim^2

22. a. 1. yol



$\overrightarrow{AB}=(-4,-3)$,

$\overrightarrow{BC}=(6,2)$,

$\overrightarrow{BC}$'ne dik bir vectör $\lambda\overrightarrow{AD}$ ise

$\lambda\overrightarrow{AD}=(2,-6)$ alınabilir.

$\overrightarrow{u_{AD}}=\left(\dfrac{1}{\sqrt{10}},\dfrac{-3}{\sqrt{10}}\right)$

$\overrightarrow{AD}=\left(\overrightarrow{AB}\cdot\overrightarrow{u_{AD}}\right)\overrightarrow{u_{AD}}$

$\Rightarrow\overrightarrow{AD}=\left[(-4,-3)\cdot\left(\dfrac{1}{\sqrt{10}},\dfrac{-3}{\sqrt{10}}\right)\right]\left(\dfrac{1}{\sqrt{10}},\dfrac{-3}{\sqrt{10}}\right)$

$\Rightarrow\overrightarrow{AD}=\dfrac{5}{\sqrt{10}}\left(\dfrac{1}{\sqrt{10}},\dfrac{-3}{\sqrt{10}}\right)\Rightarrow\overrightarrow{AD}=\left(\dfrac12,\dfrac{-3}{2}\right)$

2. yol

$\overrightarrow{BC}$'ne dik bir vectör $\lambda\overrightarrow{AD}$ ise

$\lambda\overrightarrow{AD}=(2,-6)$ alınabilir.

$\overrightarrow{AD}$, $\overrightarrow{AB}$'nün $\lambda\overrightarrow{AD}$ üzerine dik izdüşümüdür.

$\overrightarrow{AD}=\dfrac{(-4,-3)\cdot(2,-6)}{(2,-6)\cdot(2,-6)}(2,-6)$

$\Rightarrow\overrightarrow{AD}=\left(\dfrac12,\dfrac{-3}{2}\right)$

b. $\overrightarrow{AB}$'ne dik bir vectör $\lambda\overrightarrow{CE}$ ise

$\lambda\overrightarrow{CE}=(-3,4)\Rightarrow\overrightarrow{u_{CE}}=\left(\dfrac{-3}{5},\dfrac45\right)$

$\overrightarrow{CE}=\left(\overrightarrow{CB}\cdot\overrightarrow{u_{CE}}\right)\cdot\overrightarrow{u_{CE}}$

$\Rightarrow\overrightarrow{CE}=\left[(-6,-2)\cdot\left(\dfrac{-3}{5},\dfrac45\right)\right]\left(\dfrac{-3}{5},\dfrac45\right)$

$\Rightarrow\overrightarrow{CE}=\left(\dfrac{-6}{5},\dfrac85\right)$

2. yol

$\overrightarrow{AB}$'ne dik bir vectör $\lambda\overrightarrow{CE} = (-3,4)$ alınabilir.

$\overrightarrow{CE}$, $\overrightarrow{CB}$'nin $\lambda\overrightarrow{CE}$ üzerine dik izdüşümüdür.

$$\overrightarrow{CE} = \frac{(-6,-2)\cdot(-3,4)}{(-3,4)\cdot(-3,4)}\cdot(-3,4)$$

$$\Rightarrow \overrightarrow{CE} = \left(\frac{-6}{5},\frac{8}{5}\right)$$

c. $H(x,y)$ olsun.

$$\overrightarrow{AH} \perp \overrightarrow{BC} \Rightarrow \overrightarrow{AH}\cdot\overrightarrow{BC} = 0;$$
$$\overrightarrow{CH} \perp \overrightarrow{AB} \Rightarrow \overrightarrow{CH}\cdot\overrightarrow{AB} = 0$$

$$\Rightarrow \left.\begin{array}{l}(x-2,y-4)\cdot(6,2)=0\\(x-4,y-3)\cdot(-4,-3)=0\end{array}\right\}$$

$$\Rightarrow \left.\begin{array}{l}3x+y-10=0\\-4-3y+25=0\end{array}\right\} \Rightarrow x=1,\ y=7$$

$H(1,7)$ bulunur.

23. a. $\overrightarrow{BC} = (4,2)$, $\overrightarrow{BA} = (2,6)$

$$A(A\overset{\Delta}{B}C) = \frac{1}{2}\sqrt{\left\|\overrightarrow{BC}\right\|^2\cdot\left\|\overrightarrow{BA}\right\|^2 - \left(\overrightarrow{BC}\cdot\overrightarrow{BA}\right)^2}$$

$$\Rightarrow A(A\overset{\Delta}{B}C) = \frac{1}{2}\sqrt{20\cdot 40 - \left[(4,2)\cdot(2,6)\right]^2}$$

$$\Rightarrow A(A\overset{\Delta}{B}C) = \frac{1}{2}\sqrt{800-400}$$

$$\Rightarrow A(A\overset{\Delta}{B}C) = 10 \text{ birim}^2$$

b. $\overrightarrow{BC}\cdot\overrightarrow{BA} = \left\|\overrightarrow{BC}\right\|\cdot\left\|\overrightarrow{BA}\right\|\cdot\cos\alpha$

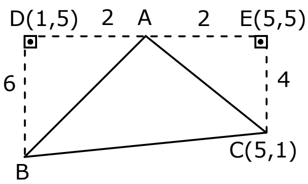
$$(4,2)\cdot(2,6) = \sqrt{20}\cdot\sqrt{40}\cdot\cos\alpha$$

$$\Rightarrow \cos\alpha = \frac{\sqrt{2}}{2},\quad \sin\alpha = \frac{\sqrt{2}}{2}$$

$$A(A\overset{\Delta}{B}C) = \frac{1}{2}\left\|\overrightarrow{BC}\right\|\cdot\left\|\overrightarrow{BA}\right\|\cdot\sin\alpha\,\frac{1}{2}\sqrt{20}\cdot\sqrt{40}\cdot\frac{1}{\sqrt{2}}$$

$$A(A\overset{\Delta}{B}C) = 10 \text{ birim}^2$$

c. DE yatay, BD ve CE düşey ise, $D(1,5)$ ve $E(5,5)$ olur.



$$A(A\overset{\Delta}{B}C) = A(BCED) - A(B\overset{\Delta}{A}D) - A(A\overset{\Delta}{E}C)$$

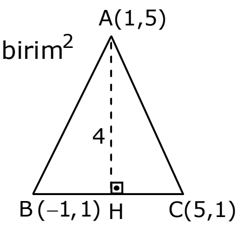
$$\Rightarrow A(A\overset{\Delta}{B}C) = \frac{(6+4)\cdot 4}{2} - \frac{2\cdot 6}{2} - \frac{2\cdot 4}{2}$$

$$\Rightarrow A(A\overset{\Delta}{B}C) = 10 \text{ birim}^2$$

24. a. $CH \perp AB$,

$|AB| = 6$, $|CH| = 4$

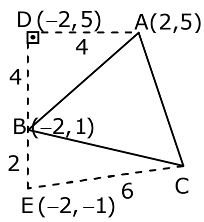
$$\Rightarrow A(A\overset{\Delta}{B}C) = \frac{6\cdot 4}{2} = 12 \text{ birim}^2$$



b. $|AH| = 4$, $|BC| = 6$

$$\Rightarrow A(A\overset{\Delta}{B}C) = \frac{6\cdot 4}{2} = 12 \text{ birim}^2$$

c. DE düşey, DA ve CE yatay ise $D(-2,5)$ ve $E(-2,-1)$ olur.



$$A(ABC) = A(ECAD) - A(ADB) - A(BEC)$$

$$\Rightarrow A(ABC) = \frac{(4+6)\cdot 6}{2} - \frac{4\cdot 4}{2} - \frac{6\cdot 2}{2}$$

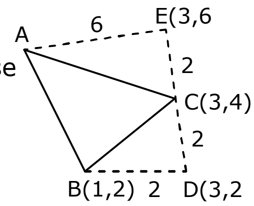
$$\Rightarrow A(ABC) = 16 \text{ birim}^2$$

d. DE düşey,

AE ve BD yatay ise

$D(3,2)$ ve $E(3,6)$

olur.

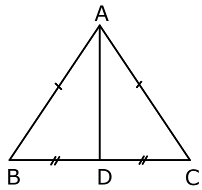


$A(ABC) = A(BDEA) - A(BDC) - A(AEC)$

$\Rightarrow A(ABC) = \dfrac{(6+2)\cdot 4}{2} - \dfrac{2\cdot 2}{2} - \dfrac{6\cdot 2}{2}$

$\Rightarrow A(ABC) = 8$ birim^2

25. $\overrightarrow{AD} = \dfrac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{AC}\right)$;



$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$\Rightarrow \overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$;

$\overrightarrow{AD}\cdot\overrightarrow{BC} = \dfrac{1}{2}\left(\overrightarrow{AC} + \overrightarrow{AB}\right)\cdot\left(\overrightarrow{AC} - \overrightarrow{AB}\right)$

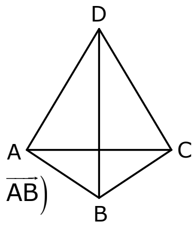
$\Rightarrow \overrightarrow{AD}\cdot\overrightarrow{BC} = \dfrac{1}{2}\left(\|AC\|^2 - \|AB\|^2\right)$

ve $\|AB\| = \|AC\|$

$\Rightarrow \overrightarrow{AD}\cdot\overrightarrow{BC} = 0$

$\Rightarrow AD \perp BC$

26. $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$;



$\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD}$

$\Rightarrow \overrightarrow{BD} = \overrightarrow{BC} - \overrightarrow{AB}$;

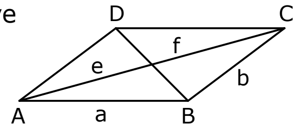
$\overrightarrow{AC}\cdot\overrightarrow{BD} = \left(\overrightarrow{BC} + \overrightarrow{AB}\right)\left(\overrightarrow{BC} - \overrightarrow{AB}\right)$

$\Rightarrow \overrightarrow{AC}\cdot\overrightarrow{BD} = \|\overrightarrow{BC}\|^2 - \|\overrightarrow{AB}\|^2$, $\|\overrightarrow{AB}\| = \|\overrightarrow{BC}\|$

$\Rightarrow \overrightarrow{AC}\cdot\overrightarrow{BD} = 0 \Rightarrow AC \perp BD$

27. $\overrightarrow{AC} = \overrightarrow{BC} + \overrightarrow{AB}$ ve

$\overrightarrow{BD} = \overrightarrow{BC} - \overrightarrow{AB}$



$\Rightarrow \overrightarrow{AC}\cdot\overrightarrow{AC} = \left(\overrightarrow{BC} + \overrightarrow{AB}\right)\cdot\left(\overrightarrow{BC} + \overrightarrow{AB}\right)$

$\overrightarrow{BD}\cdot\overrightarrow{BD} = \left(\overrightarrow{BC} - \overrightarrow{AB}\right)\cdot\left(\overrightarrow{BC} - \overrightarrow{AB}\right)$

$\Rightarrow \|\overrightarrow{AC}\|^2 = \|\overrightarrow{BC}\|^2 + \|\overrightarrow{AB}\|^2 + 2\overrightarrow{AB}\cdot\overrightarrow{BC}$ (1)

$\|\overrightarrow{BD}\|^2 = \|\overrightarrow{BC}\|^2 + \|\overrightarrow{AB}\|^2 - 2\overrightarrow{AB}\cdot\overrightarrow{BC}$ (2)

(1) ve (2) taraf tarafa toplanırsa

$\|\overrightarrow{AC}\|^2 + \|\overrightarrow{BD}\|^2 = 2\|\overrightarrow{BC}\|^2 + 2\|\overrightarrow{AB}\|^2$

$\Rightarrow e^2 + f^2 = 2a^2 + 2b^2$ bulunur.

28. $\overrightarrow{AD} = \dfrac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{AC}\right)$

$\Rightarrow \overrightarrow{AD}\cdot\overrightarrow{AD} = \dfrac{1}{4}\left(\overrightarrow{AB} + \overrightarrow{AC}\right)\left(\overrightarrow{AB} + \overrightarrow{AC}\right)$

$\Rightarrow \overrightarrow{AD}\cdot\overrightarrow{AD} = \dfrac{1}{4}\left(\overrightarrow{AB}\cdot\overrightarrow{AB} + \overrightarrow{AC}\cdot\overrightarrow{AC} + 2\overrightarrow{AB}\cdot\overrightarrow{AC}\right)$

$\Rightarrow {v_a}^2 = \dfrac{1}{4}\left[c^2 + b^2 + 2\cdot\dfrac{1}{2}\left(b^2 + c^2 - a^2\right)\right]$

$\Rightarrow {v_a}^2 = \dfrac{b^2 + c^2}{2} - \dfrac{a^2}{4}$ elde edilir.

29. $\left.\begin{array}{l} \vec{a}\cdot\vec{c} = \vec{a}\cdot\left(\|\vec{a}\|\cdot\vec{b} + \|\vec{b}\|\cdot\vec{a}\right) \\ \vec{b}\cdot\vec{c} = \vec{b}\cdot\left(\|\vec{a}\|\cdot\vec{b} + \|\vec{b}\|\cdot\vec{a}\right) \end{array}\right\}$

$\Rightarrow \left.\begin{array}{l} \|\vec{a}\|\cdot\|\vec{c}\|\cdot\cos\alpha = \|\vec{a}\|\cdot\vec{a}\cdot\vec{b} + \|\vec{a}\|^2\cdot\|\vec{b}\| \\ \|\vec{b}\|\cdot\|\vec{c}\|\cdot\cos\beta = \|\vec{a}\|\cdot\|\vec{b}\|^2 + \|\vec{b}\|\cdot\vec{a}\cdot\vec{b} \end{array}\right\}$

$\Rightarrow \left.\begin{array}{l} \|\vec{c}\|\cdot\cos\alpha = \vec{a}\cdot\vec{b} + \|\vec{a}\|\cdot\|\vec{b}\| \\ \|\vec{c}\|\cdot\cos\beta = \vec{a}\cdot\vec{b} + \|\vec{a}\|\cdot\|\vec{b}\| \end{array}\right\}$

$\Rightarrow \cos\alpha = \cos\beta \Rightarrow \alpha = \beta$

30. a. $\vec{A}=(1,3)$ vektörüne dik vektörlerden ikisi $\overrightarrow{A_1}=(3,-1)$ ve $\overrightarrow{A_2}=(-3,+1)$'dür.

$\vec{A}$ ile 45° lik açı yapan vektörler $\vec{A}$ ile $\overrightarrow{A_1}$'nün ve $\vec{A}$ ile $\overrightarrow{A_2}$'nün belirttiği açıların açıortayları doğrultusunda olurlar. Bu açıortay vektörlerinden ikisi $\overrightarrow{a_1}=(x_1,1)$ ve $\overrightarrow{a_2}=(x_2,1)$ olsun.

$$\left.\begin{array}{l}\vec{A}\cdot\overrightarrow{a_1}=\|\vec{A}\|\cdot\|\vec{a}\|\cdot\cos 45^\circ\\ \overrightarrow{A_1}\cdot\overrightarrow{a_1}=\|\overrightarrow{A_1}\|\cdot\|\overrightarrow{a_1}\|\cdot\cos 45^\circ\end{array}\right\}$$

ve $\|\vec{A}\|=\|\overrightarrow{A_1}\|$ olduğundan $\vec{A}\cdot\overrightarrow{a_1}=\overrightarrow{A_1}\cdot\overrightarrow{a_1}$

$\Rightarrow (1,3)\cdot(x_1,1)=(3,-1)\cdot(x_1,1)$

$\Rightarrow x_1+3=3x_1-1\Rightarrow x_1=2$ bulunur.

$\overrightarrow{a_1}=(2,1)\Rightarrow \overrightarrow{u_{a_1}}=\left(\dfrac{2}{\sqrt5},\dfrac{1}{\sqrt5}\right)$ olur.

Aynı yolla; $\overrightarrow{u_{a_2}}=\left(\dfrac{-1}{\sqrt5},\dfrac{2}{\sqrt5}\right)$ bulunur.

b. $\vec{A}$ ile 22,5° lik açı yapan vektörler $\vec{A}=(3,1)$ ile $\overrightarrow{a_1}=(2,1)$'nün ve $\vec{A}$ ile $\overrightarrow{a_2}=(-1,2)$'nün açıortaylar doğrultusunda olurlar. Bu açıortay vektörlerinden ikisi $\overrightarrow{a_3}=(x_3,1)$ ve $\overrightarrow{a_4}=(x_4,1)$ olsun.

$\vec{A}\cdot\overrightarrow{a_3}=\|\vec{A}\|\cdot\|\overrightarrow{a_3}\|\cdot\cos 22{,}5^\circ$ (1)

$\overrightarrow{a_1}\cdot\overrightarrow{a_3}=\|\overrightarrow{a_1}\|\cdot\|\overrightarrow{a_3}\|\cdot\cos 22{,}5^\circ$ (2) olur.

(1) ve (2) taraf tarafa bölünürse,

$$\frac{\vec{A}\cdot\overrightarrow{a_3}}{\overrightarrow{a_1}\cdot\overrightarrow{a_3}}=\frac{\|\vec{A}\|}{\|\overrightarrow{a_1}\|}\Rightarrow\frac{(3,1)\cdot(x_3,1)}{(2,1)\cdot(x_3,1)}=\frac{\sqrt{10}}{\sqrt5}$$

$\Rightarrow \dfrac{3x_3+1}{2x_3+1}=\sqrt2\Rightarrow x_3=5+\sqrt2$ bulunur.

$\overrightarrow{a_3}=\left(5+\sqrt2,1\right)$

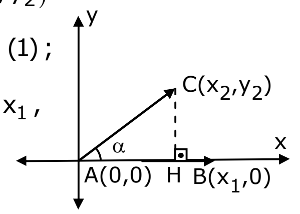
$\Rightarrow \overrightarrow{u_{a_3}}=\left(\dfrac{5+\sqrt2}{\sqrt{28+10\sqrt2}},\dfrac{1}{\sqrt{28+10\sqrt2}}\right)$ olur.

Aynı yolla;

$\overrightarrow{u_{a_4}}=\left(\dfrac{\sqrt2-1}{\sqrt{4-2\sqrt2}},\dfrac{1}{\sqrt{4-2\sqrt2}}\right)$ bulunur.

31. a. $\overrightarrow{AB}\cdot\overrightarrow{AC}=(x,0)\cdot(x_2,y_2)$

$\Rightarrow \overrightarrow{AB}\cdot\overrightarrow{AC}=x_1\cdot x_2$ (1) ;



$\|\overrightarrow{AB}\|=|x_1|\Rightarrow\|\overrightarrow{AB}\|=x_1$,

$\|\overrightarrow{AC}\|=\sqrt{x_2^{\,2}+y_2^{\,2}}$,

$\cos\alpha=\dfrac{\|\overrightarrow{AH}\|}{\|\overrightarrow{AC}\|}\Rightarrow\cos\alpha=\dfrac{x_2}{\sqrt{x_2^{\,2}+y_2^{\,2}}}$

$\Rightarrow\|\overrightarrow{AB}\|\cdot\|\overrightarrow{AC}\|\cdot\cos\alpha$

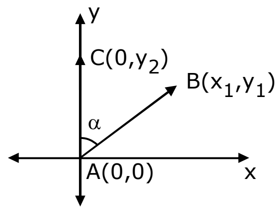
$=x_1\cdot\sqrt{x_2^{\,2}+y_2^{\,2}}\cdot\dfrac{x_2}{\sqrt{x_2^{\,2}+y_2^{\,2}}}$

$\Rightarrow\|\overrightarrow{AB}\|\cdot\|\overrightarrow{AC}\|\cdot\cos\alpha=x_1\cdot x_2$ (2) olur.

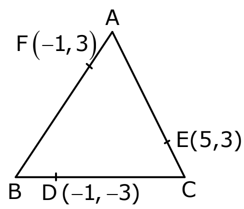
(1) ve (2) den;

$\overrightarrow{AB}\cdot\overrightarrow{AC}=\|\overrightarrow{AB}\|\cdot\|\overrightarrow{AC}\|\cdot\cos\alpha$ bulunur.

b. Aynı yolla yapınız.



32. a. $\overrightarrow{DE}$ ve $\overrightarrow{FE}$'nü $\overrightarrow{AB}$ ve $\overrightarrow{AC}$ türünden ifade edelim.



$\overrightarrow{DE}=\overrightarrow{DC}+\overrightarrow{CE}$

$\Rightarrow\dfrac23\overrightarrow{BC}+\dfrac13\overrightarrow{CA}=(6,6)$

$\Rightarrow\dfrac23\left(-\overrightarrow{AB}+\overrightarrow{AC}\right)-\dfrac13\overrightarrow{AC}=(6,6)$

$\Rightarrow-\dfrac23\overrightarrow{AB}+\dfrac13\overrightarrow{AC}=(6,6)$; (1)

$\overrightarrow{FE}=\overrightarrow{FA}+\overrightarrow{AE}$

$\Rightarrow -2/\quad -\dfrac13\overrightarrow{AB}+\dfrac23\overrightarrow{AC}=(6,0)$ (2)

(1) ve (2) den, $\overrightarrow{AB}=(-6,-12)$ ve $\overrightarrow{AC}=(6,-6)$ bulunur.

$\overrightarrow{BC}=\overrightarrow{BA}+\overrightarrow{AC}=(6,12)+(6,-6)$

$\Rightarrow\overrightarrow{BC}=(12,6)$ olur.

b. $\overrightarrow{FA} = \frac{1}{3}\overrightarrow{BA} \Rightarrow \vec{A} - (-1,3) = \frac{1}{3}(6,12)$

$\Rightarrow \vec{A} = (1,7) \Rightarrow A(1,7)$;

$\overrightarrow{BD} = \frac{1}{3}\overrightarrow{BC} \Rightarrow (-1,-3) - B = \frac{1}{3}(12,6)$

$\Rightarrow \vec{B} = (-5,-5) \Rightarrow B(-5,-5)$;

$\overrightarrow{DC} = \frac{2}{3}\overrightarrow{BC} \Rightarrow C - (-1,-3) = \frac{2}{3}(12,6)$

$\Rightarrow \vec{C} = (7,1) \Rightarrow C(7,1)$