Soru-1

$\lim\limits_{x\to 0^-} \dfrac{\sin x}{\sqrt{1-\cos x}}$ değerini bulunuz.

Çözüm

$$\lim_{x\to 0^-} \frac{\sin x}{\sqrt{1-\cos x}} = \lim_{x\to 0^-} \frac{\sin x\cdot\sqrt{1+\cos x}}{\sqrt{1-\cos x}\cdot\sqrt{1+\cos x}}$$
$$= \lim_{x\to 0^-} \frac{\sin x\cdot\sqrt{1+\cos x}}{\sqrt{1-\cos^2 x}}$$
$$= \lim_{x\to 0^-} \frac{\sin x\cdot\sqrt{1+\cos x}}{\sqrt{\sin^2 x}}$$
$$= \lim_{x\to 0^-} \frac{\sin x\cdot\sqrt{1+\cos x}}{|\sin x|}$$
$$= \lim_{x\to 0^-} \frac{\sin x\cdot\sqrt{1+\cos x}}{-\sin x}$$
$$= \lim_{x\to 0^-} (-\sqrt{1+\cos x})$$
$$= -\sqrt{2}$$

Soru-2

$\lim\limits_{x\to(\frac{\pi}{2})^+} \dfrac{\sin 2x}{\sqrt{1-\sin x}}$ değerini bulunuz.

Çözüm

$$\lim_{x\to(\frac{\pi}{2})^+} \frac{\sin 2x}{\sqrt{1-\sin x}} = \lim_{x\to(\frac{\pi}{2})^+} \frac{\sin 2x\cdot\sqrt{1+\sin x}}{\sqrt{1-\sin x}\cdot\sqrt{1+\sin x}}$$
$$= \lim_{x\to(\frac{\pi}{2})^+} \frac{\sin 2x\cdot\sqrt{1+\sin x}}{\sqrt{1-\sin^2 x}}$$
$$= \lim_{x\to(\frac{\pi}{2})^+} \frac{\sin 2x\cdot\sqrt{1+\sin x}}{\sqrt{\cos^2 x}}$$
$$= \lim_{x\to(\frac{\pi}{2})^+} \frac{\sin 2x\cdot\sqrt{1+\sin x}}{|\cos x|}$$
$$= \lim_{x\to(\frac{\pi}{2})^+} \frac{2\cdot\sin x\cdot\cos x\cdot\sqrt{1+\sin x}}{-\cos x}$$
$$= \lim_{x\to(\frac{\pi}{2})^+} (-2\cdot\sin x\cdot\sqrt{1+\sin x})$$
$$= -2\sqrt{2}$$

Soru-3

$\lim\limits_{x\to 0} \dfrac{1-\cos^3 x}{x\cdot\sin 2x}$ değerini bulunuz.

Çözüm

$$\lim_{x\to 0} \frac{1-\cos^3 x}{x\cdot\sin 2x} = \lim_{x\to 0} \frac{(1-\cos x)\cdot\left(1+\cos x+\cos^2 x\right)}{x\cdot 2\sin x\cdot\cos x}$$
$$= \lim_{x\to 0} \frac{\left\{1-\left[1-2\sin^2\left(\frac{x}{2}\right)\right]\right\}\cdot\left(1+\cos x+\cos^2 x\right)}{x\cdot 4\sin\left(\frac{x}{2}\right)\cdot\cos\left(\frac{x}{2}\right)\cdot\cos x}$$
$$= \lim_{x\to 0} \frac{2\sin^2\left(\frac{x}{2}\right)\cdot\left(1+\cos x+\cos^2 x\right)}{x\cdot 4\sin\left(\frac{x}{2}\right)\cdot\cos\left(\frac{x}{2}\right)\cdot\cos x}$$
$$= \lim_{x\to 0} \frac{2\sin\left(\frac{x}{2}\right)\cdot\left(1+\cos x+\cos^2 x\right)}{4\cdot x\cdot\cos\left(\frac{x}{2}\right)\cdot\cos x}$$
$$= \lim_{x\to 0} \frac{\sin\left(\frac{x}{2}\right)\cdot\left(1+\cos x+\cos^2 x\right)}{\left(\frac{x}{2}\right)\cdot 4\cdot\cos\left(\frac{x}{2}\right)\cdot\cos x}$$
$$= \frac{3}{4}$$

Soru-4

$\lim\limits_{x\to\infty} \left(\cos\sqrt{x+a}-\cos\sqrt{x}\right)$ değerini bulunuz.

Çözüm

$$\lim_{x\to\infty} \left(\cos\sqrt{x+a}-\cos\sqrt{x}\right)$$
$$= \lim_{x\to\infty} \left(-2\sin\frac{\sqrt{x+a}+\sqrt{x}}{2}\cdot\sin\frac{\sqrt{x+a}-\sqrt{x}}{2}\right)$$
$$= \lim_{x\to\infty} \left[-2\sin\frac{\sqrt{x+a}+\sqrt{x}}{2}\cdot\sin\frac{(\sqrt{x+a}-\sqrt{x})\cdot(\sqrt{x+a}+\sqrt{x})}{2(\sqrt{x+a}-\sqrt{x})}\right]$$
$$= \lim_{x\to\infty} \left[-2\sin\frac{\sqrt{x+a}+\sqrt{x}}{2}\cdot\sin\frac{a}{2(\sqrt{x+a}+\sqrt{x})}\right]$$
$$= \lim_{x\to\infty} \left[-2\sin\frac{\sqrt{x+a}+\sqrt{x}}{2}\right]\cdot\lim_{x\to\infty}\sin\frac{a}{2(\sqrt{x+a}+\sqrt{x})}$$
$$= \ell\cdot 0$$
$$= 0$$

Soru-5

$\lim\limits_{x\to 0} \dfrac{\sin x - \tan x}{x^3}$ değerini bulunuz.

Çözüm

$$\lim_{x\to 0} \frac{\sin x - \tan x}{x^3} = \lim_{x\to 0} \frac{\sin x \cdot \cos x - \sin x}{x^3 \cdot \cos x}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{\cos x - 1}{x^2 \cdot \cos x}$$

$$= 1 \cdot \lim_{x\to 0} \frac{-2\sin^2\left(\frac{x}{2}\right)}{x^2 \cdot \cos x}$$

$$= \lim_{x\to 0} \frac{-2\sin\left(\frac{x}{2}\right) \cdot \sin\left(\frac{x}{2}\right)}{2 \cdot \frac{x}{2} \cdot 2 \cdot \frac{x}{2} \cdot \cos x}$$

$$= -\frac{1}{2}$$

bulunur.

Soru-6

$\lim\limits_{x\to 0} \dfrac{\sqrt{1-\cos\left(x^2\right)}}{1-\cos x}$ değerini bulunuz.

Çözüm

$$\lim_{x\to 0} \frac{\sqrt{1-\cos\left(x^2\right)}}{1-\cos x} = \lim_{x\to 0} \frac{\sqrt{2 \cdot \sin^2\left(\frac{x^2}{2}\right)}}{2 \cdot \sin^2\left(\frac{x}{2}\right)}$$

$$= \lim_{x\to 0} \frac{\sqrt{2} \cdot \left|\sin\left(\frac{x^2}{2}\right)\right|}{2 \cdot \sin^2\left(\frac{x}{2}\right)}$$

$$= \lim_{x\to 0} \frac{\sqrt{2} \cdot \sin\left(\frac{x^2}{2}\right) \cdot \frac{x}{2} \cdot \frac{x}{2}}{2 \cdot \sin\frac{x}{2} \cdot \sin\frac{x}{2} \cdot \frac{x^2}{2} \cdot \frac{1}{2}}$$

$$= \sqrt{2}$$

bulunur.