COMPONENTS OF COTTON MIX VARIABILITY:

The different components of citron mix variability and the various factors influencing this variability are including:

- **Type of bale picking**
- **Bale arrangement**
- **Population variability**
- **Category breakpoint location (or category variance)**
- **Number of categories**
- **Number of bales in the mix.**

COTTON MIX VARIABILITY:

Total mix variability = between mix variability + within mix variability

In practice, these two components of variability are commonly displayed using the familiar **control charts.** Top chart displays the mean values of micronaire of each bale laydown, and the top chat displays the corresponding variability (standard deviation).

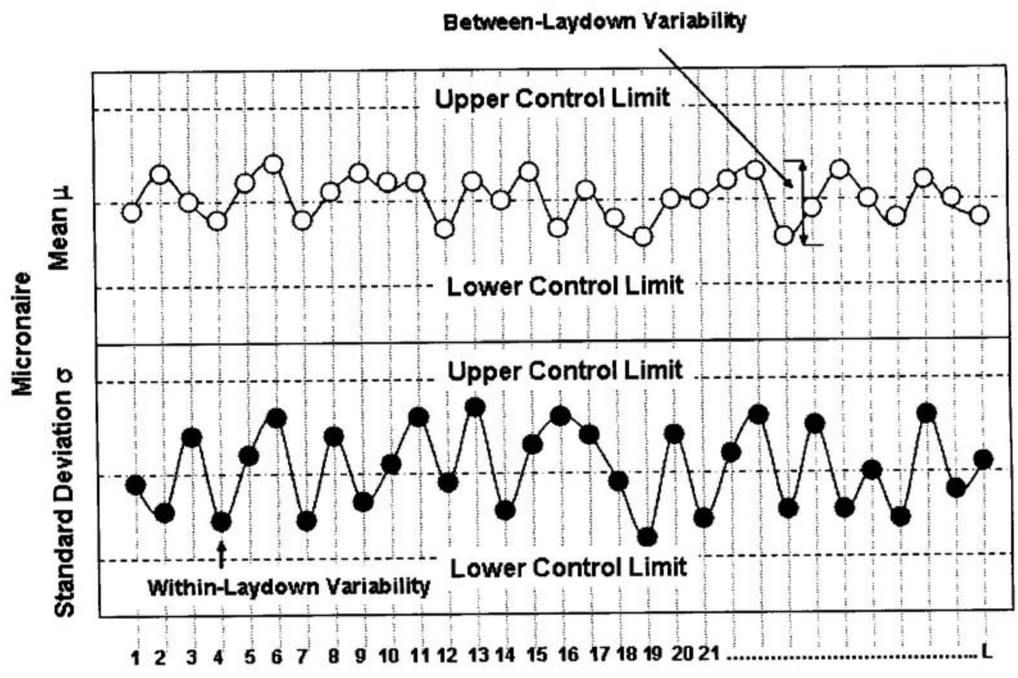

Figure 4.1: Between lay-down and within lay-down variability

The total mix variability can be partitioned as follows:

$$\sigma^2_{total} = \frac{(L-1).n}{(Ln-1)}\sigma^2_{BLD} + \frac{(n-1)}{(Ln-1)}\sum_{i=1}^{L}\sigma^2_{WLD}$$

Where L is the number of bale lay-downs, n is the number of bales per lay-down, σ^2_{BLD} is the between lay-down variance, and σ^2_{WLD} is the within lay-down variance.

The **within lay-down** variability mainly consists of two components: **within bale and between bale** variability.

Within mix variability = (between bale variability) + (within bale variability)

Accordingly, the total mix variability can be expressed by the following general relationship:

Total mix variability = (Between mix variability)+(within mix variability) = (Between mix variability)+(between bale variability)+ (within bale variability)

$$\sigma^2_{WLD} = \frac{(L-1).m}{(nm-1)}\sigma^2_{BB} + \frac{(m-1)}{(nm-1)}\sum_{i=1}^{n}\sigma^2_{WB}$$

Where n is the number of bales in the mix or lay-down, m is the number of samples taken from each bale, σ^2_{BB} is the between bale variance, and σ^2_{WB} is the within bale variance.

From the two equations settled before, the following general guidelines may be drawn:

1. The total variability of a fiber characteristic in a cotton mix or bale lay-down is equal to the sum of between lay-down variability σ^2_{BLD}, and within lay-down variability, σ^2_{WLD}.
2. Reducing of total mix variability necessities reducing both components of variability
3. The only possible way to reduce both components simultaneously is to pick bales from population that has minimum variability.
4. In a static bale population, in which no changes occur over short time, and all bales must be consumed to satisfy inventory constraints, the total variability (σ^2_{total}) in a fiber characteristic will be more or less constant.

   In this case, any attempt to reduce one component, will result in an increase of the second component.

5- The within mix variability is equal to the sum of the between bale variability σ^2_{BB}, and within bale variability σ^2_{WB} should be reduced.

6- Practically, each bale is identified by a set of average values of its fiber characteristics, and no information regarding within bale variability is normally issued, and thus fiber selection strategy ignores this source of variability, considering the between bale variability is typically much greater than within bale variability.

From the above discussion, it follows that a practical fiber strategy will typically rely on the following information:

- ❖ Between bale variability, σ_{BB}^2
- ❖ Within mix variability, σ_{WLD}^2 ,
- ❖ Between mix variability σ_{BLD}^2

Accordingly the total mix variability is reduced to be:

Total mix variability = Between mix variability + within mix variability
= Between mix variability + Between bale variability.

One of the common problems that often result from high between mix variability is the so-called **"fabric barré".** This problem is described by periodic variation in the weft direction of the woven fabrics or the course direction of knit fabric. Figure 4.2 illustrates a case of knit fabric barré that has resulted from between mix variability in the fiber micronaire.

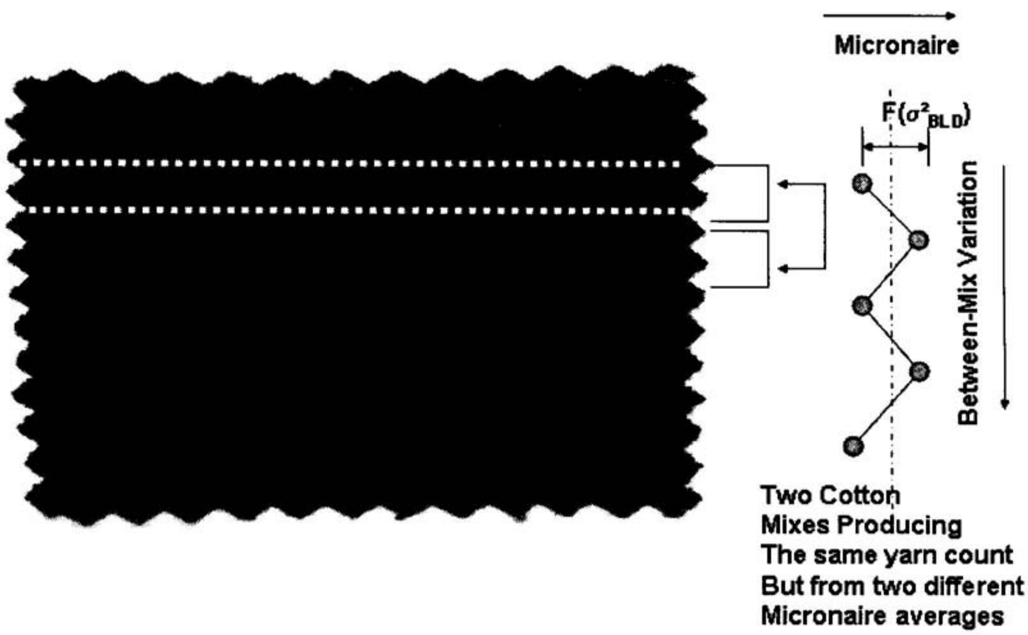

Figure 4.2: Fabric Barré resulting from between mix variation

We should point out that between-mix variation is not the only cause of fabric barré. Figure 4.3 shows many other causes of fabric barré. High values of between-mix variation will increase the probability of occurring fabric barré. However, even the best fiber selection strategy can do very little if other machine-related factors not optimized.

In practice, minimization of within mix variability may be achieved using one of the two approaches illustrated by the hypothetical cases of figure 4.4.These approaches apply only when category bale picking is implemented.

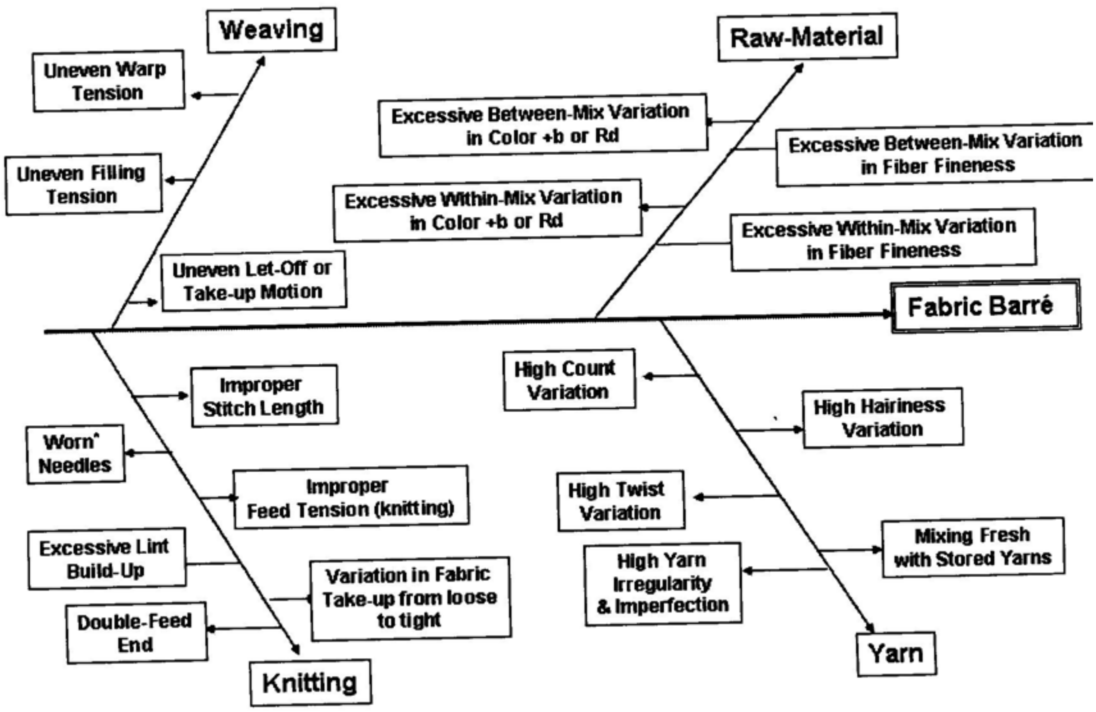

Figure 4.3: Possible causes of fabric Barré

The difference between the two approaches lies in the way bales are picked from different categories at particular period of time. The graph on the lift side demonstrates the order of bale picking from different categories. The graph on the right hand side illustrates the expected trend of both between and within variability corresponding to each approach.

In case (A) of figure 4.4, bale picking is initiated at the extreme end categories and over time it progresses towards the center categories of the population distribution. Accordingly, initial bale lay-down will consist of extreme low and high values of fiber characteristics. The average will be more or less equal to the population average (low between mix variability) but within mix variability will initially be very high. As bale picking progresses towards the center of the population distribution, within mix variability will gradually decrease.

In case (b) of figure 4.4, bale picking is initiated at the center categories and over time it progresses towards the extreme ends of the distribution. In this case, within lay-down variability is expected to be at its minimum level in the beginning, and increases as picking progresses towards the ends of the distribution.

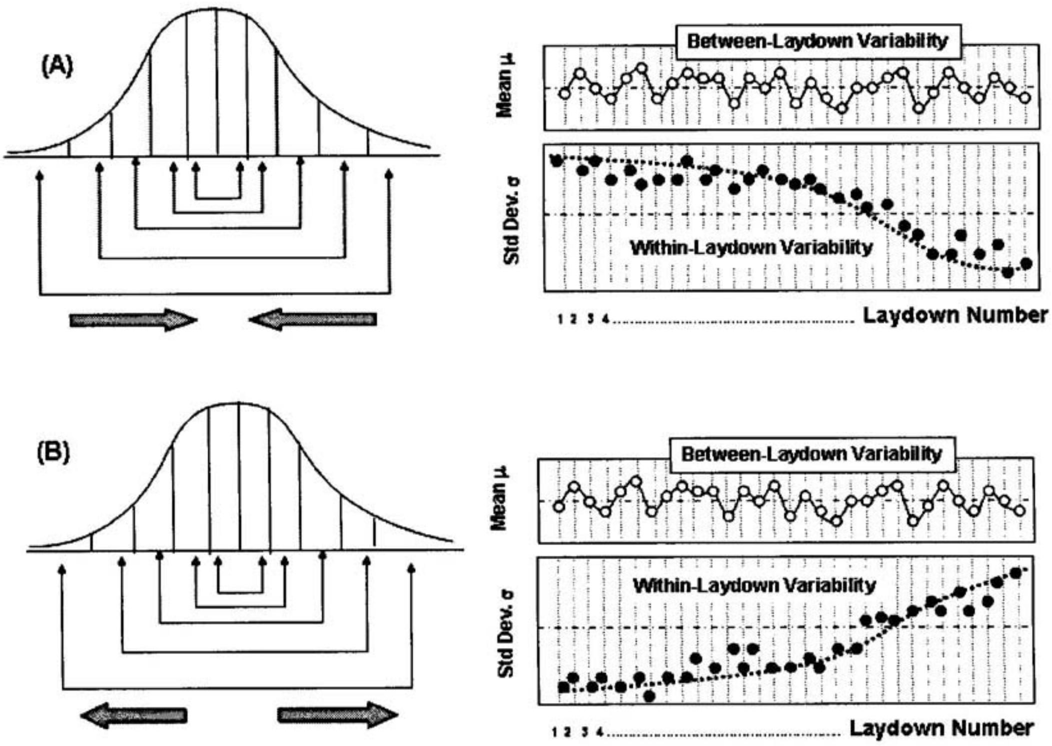

Figure 4.4: Hypothetical cases of category bale picking order

In practice, these two approaches may be implemented when the objective of the fiber selection strategy is to satisfy different levels of consistency associated with different products of yarn styles. In these situations, the two methods can be implemented simultaneously with the first approach being used for high variability mixes and the second for low variability mixes.

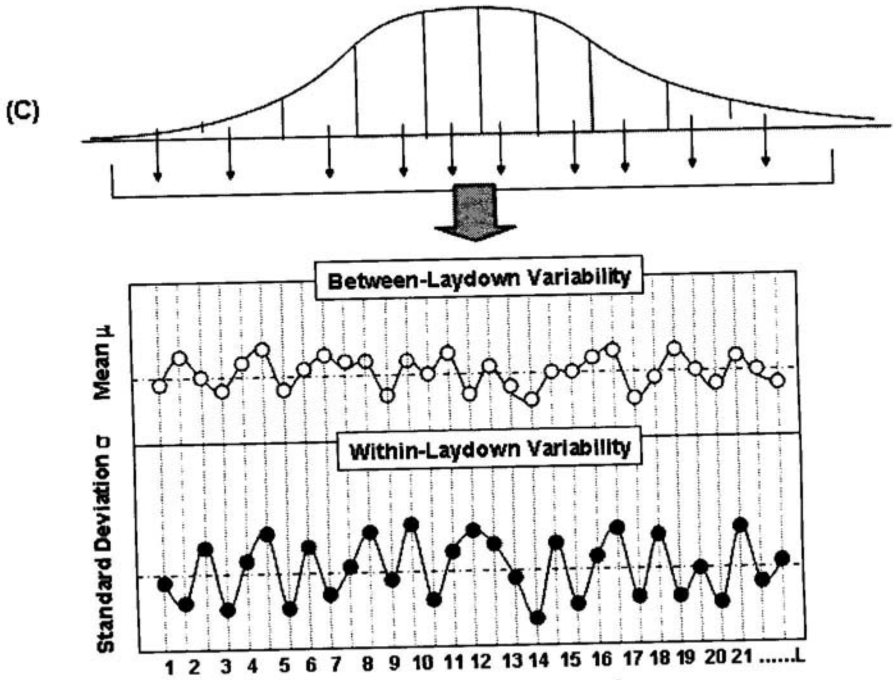

Figure 4.5: Hypothetical cases of category bale picking order (bales picked from all categories at once from the mix

When only one level of quality variability is required, it will be important to maintain consistent within mix variability over a long period of time. In this case, the approach should be as illustrated in figure 4.5. In this case, bales are picked from all categories at once to form a cotton mix. This approach yields mean values and within mix variability values that are more or less equal to those of bale population.

Effect of bale picking method on mix variability:

Bale picking systems which can generate consistent fiber profiles are:

- Random picking
- Category picking

Figure 4.6 illustrate these types of picking systems. The fundamental difference between these two methods lies in the way population heterogeneity (or variability) is manipulated.

In random picking, every cotton bale in a finite bale population will have the same chance of being selected in the cotton mix. A cotton mix of average values of fiber characteristics will have more or less equivalent to those of the population. **The between mix variability will depend on the number of bales in the mix n, and the overall population variability σ^2.** Within variability should directly reflect the variability of the parent population σ^2.

When category picking is used, the bale population is divided into a number of categories from which bales are randomly selected in proportion to their amounts in the categories. This should result in a cotton mix of average values of fiber characteristics that are more or less equivalent to those of population. **The between mix variability will depend on the number of bales in the mix n, the weight of each category in bale population W_i, and the variability of each category s_i^2.** Within mix variability should directly reflect the variability of parent population σ^2.

Effect of bale arrangement on mix variability:

Different bale picking schemes require bale arrangement in the ware house. In random picking scheme, it is important to store and arrange all population bales in such a way that allows picking any bale at any point of time (see Figure 4.7).

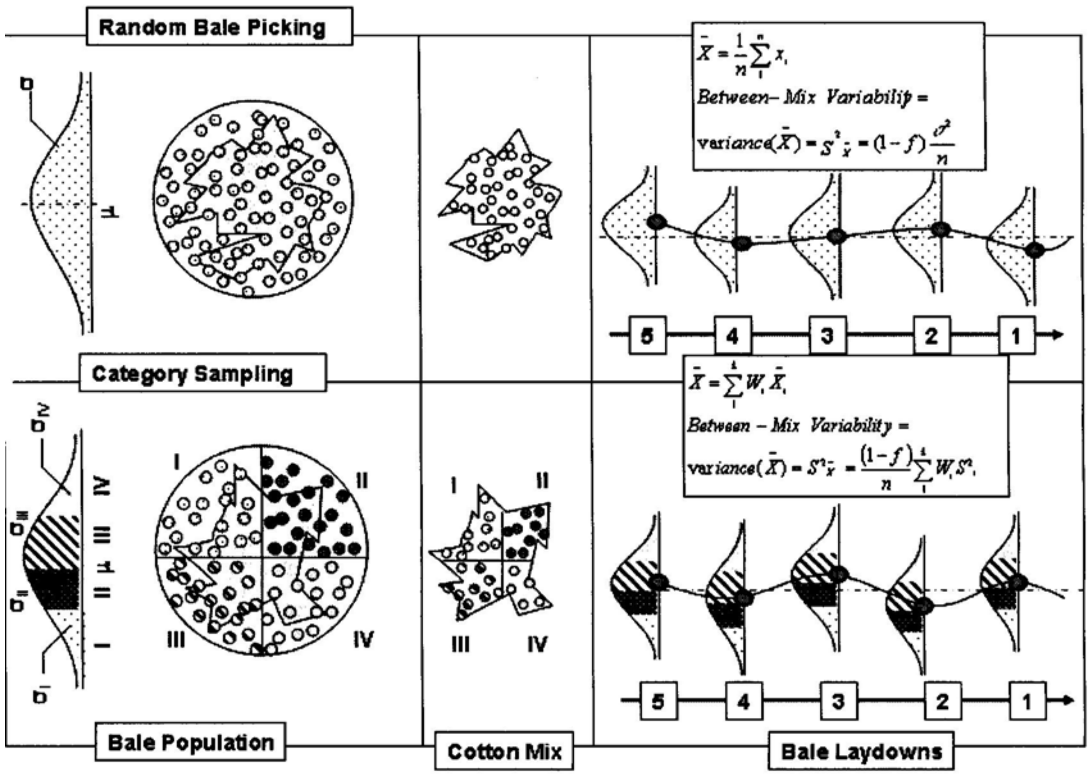

Figure 4.6: Illustration of random and category bale picking

If bales are retrieved by identification numbers, an ideal arrangement of the bales in the warehouse will be the one that involves minimum bale picking. Otherwise, retrieval of bale that is located at bottom of a high stack will be practically difficult and time consuming.

In a category picking scheme, the population is divided into k categories, and bales are picked randomly from each category. Typically category picking scheme provides better replication of population variability by ensuring representation in the mix of different values of population characteristics. However, category picking involves more bale arrangement in recognized cell in the warehouse. As stated before, **the larger the number of categories**

and/or the larger the number of fiber attributes, the larger the number of category combinations, which in turn result in large number of storage cells in the warehouse. It is important, therefore to minimize the number of category combinations to make category picking practically doable. Figure 4.8 shows a case of nine categories and retrieval policy of the mill.

Effect of population variability on mix variability:

In case of **random picking**, each bale has the same chance to be picked and processed. When the population size N, is large or selected mix contains a small number of bales, (i.e. $f = n / N$), random picking often **fails to replicate** population heterogeneity in mix. This failure is not due to deficiency of the fundamental concept of random picking scheme, but rather due to practical difficulty of physically of **finding and accessing** the selected bales. Accordingly, it is important to store and arrange all population bales in such a way that allows picking any bale at any point of time (see figure 4.7). **Retrieval of bales** located **at bottom** of high stack will practically difficult and time consuming.

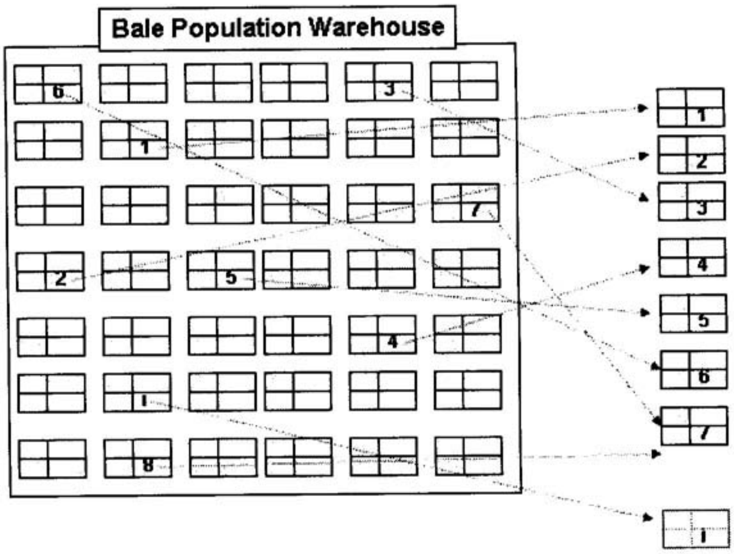

Figure 4.7: Bale arrangement for random picking

In **Category picking scheme,** the population is divided into k categories and bales are picked randomly from each category. Typically, category picking schemes provide better replication of population variability. However, category picking involves more bale arrangement, since bales from each category combination should be placed together in a recognized cell in the warehouse. Therefore, it is important to minimize the number of category combinations to make category picking practically doable. Figure 4.8 shows a case of nine category combinations 9three category/two fiber attributes.

The type of bale picking should be selected in view of storage and retrieval policy of the mill. Cotton bales that meet the selection criteria should be easily retrievable.

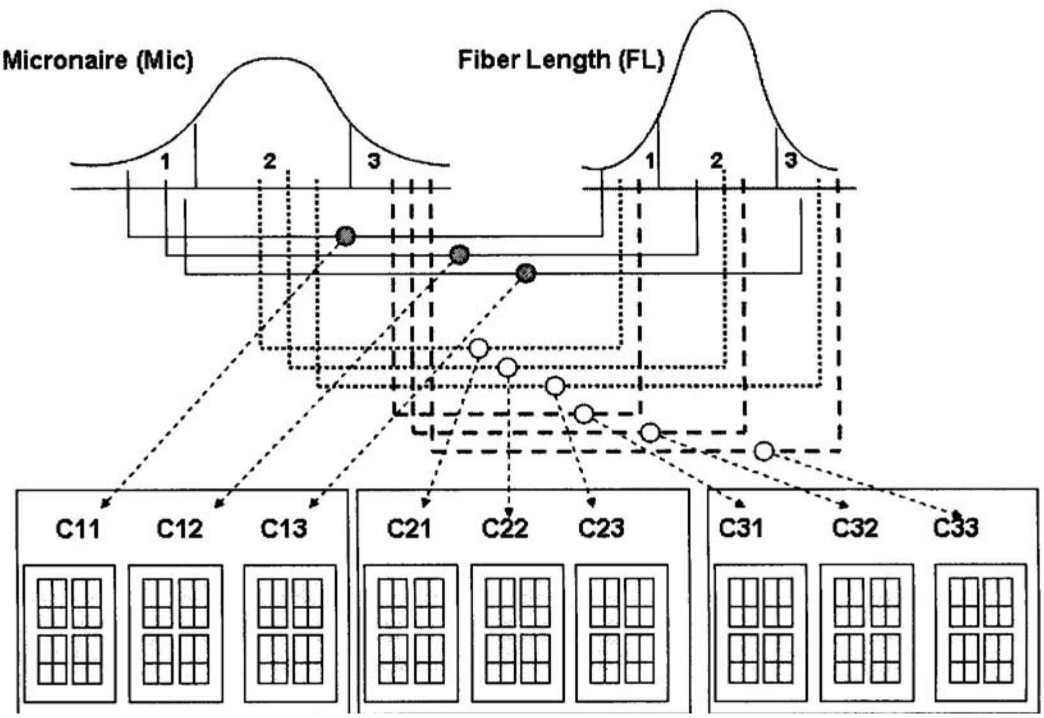

Figure 4.8: Bale arrangement for category picking (9category combinations)

Effect of population variability on mix variability:

For random picking, the effect of population variability σ^2 on between mix variance is expressed by the following equation:

$$\text{var}(\overline{X}) = s_{\overline{X}}^2 = (1-f)\frac{\sigma^2}{n}$$

In proportional weight category (PWC) picking, the effect of population variability on between lay-down variance is expressed by the sum of weighed variances of different categories as the following equation:

$$\text{var}(\overline{X}) = (1-f)\sum_{i=1}^{k} W_i s_i^2$$

Where $\overline{X}$ is the average of the values of a fiber characteristics in category i, W_i is the weight of category I (or $\frac{n_i}{N}$), f is a constant determined by $\frac{n_i}{N}$, and s_i^2 is the category variance.

In variance optimum category (OPC) picking, the following expression expresses is used in analyzing the variability of mix

$$\mathrm{var}_{\min}(\bar{X}) = \frac{1}{n}\left(\sum_{i=1}^{k} W_i S_i\right)^2 - \frac{1}{n}\sum_{i=1}^{k} W_i S_i^{\,2}$$

Accordingly, the performance of any picking scheme will largely depend on the population variability. In order to demonstrate this effect, two normally distributed populations of micronaire are randomly generated. Both populations had the same average $(\mu = 4.0)$, both different variability levels (population A: σ =0.098 Mic., population B: σ =0.786). These two distributions are in figure 4.9

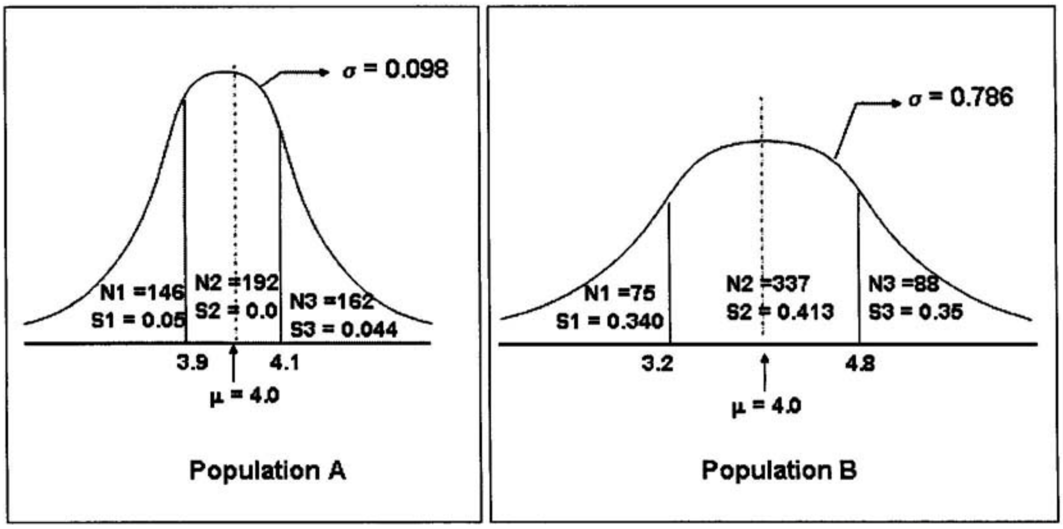

Figure 4.9: Examples of two populations of different variability levels of micronaire

From each population twenty lay-downs, each of 20 bales, using the picking schemes mentioned above are selected. Figure 4.10 shows the micronaire profiles (average values and standard deviations) resulting from these picking procedures.

The following remarks can be drawn:

- Using any bale picking scheme case B, will not help in decreasing the between mix variability $S_{\bar{X}}$, compared to case A. The variance optimum category (OPC) category picking scheme result in lowest between lay-down variability followed by the proportional weight category picking scheme.
- Using any bale picking scheme, high variability population (case B) results in higher within mix variability than low variability population (case A). Category picking scheme provide better consistency of the values of within lay-down standard deviations than random picking scheme, as indicated by the S_s value (the standard deviation of within lay-down standard deviation values).

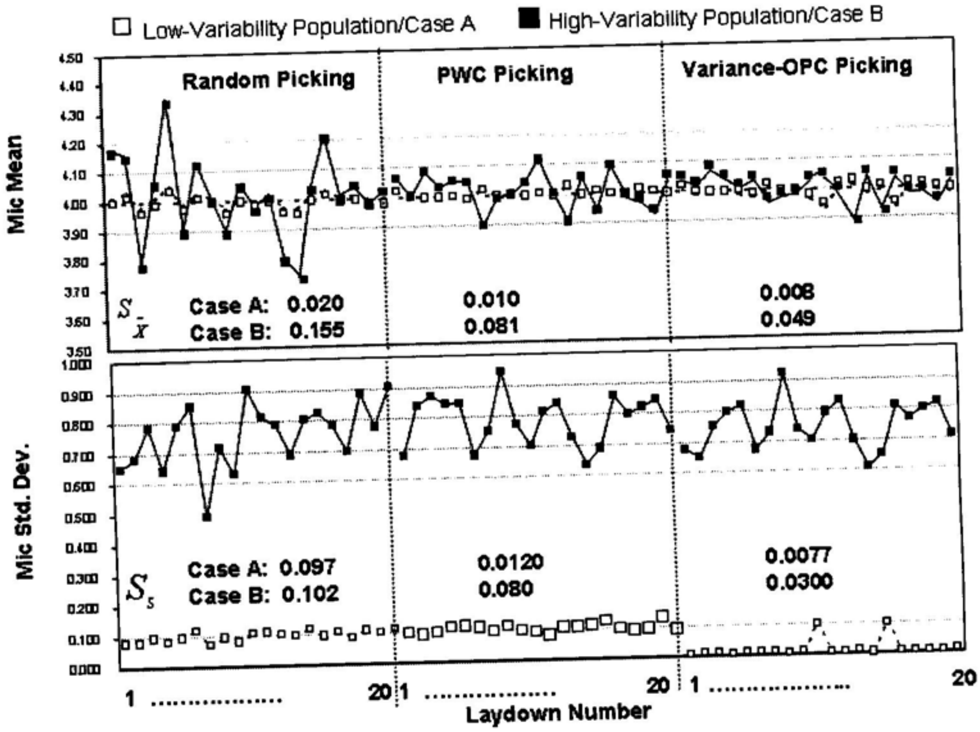

Figure 4.10: Profiles of micronaire mixes using different picking schemes and high/low levels of population variability

EFFECT OF CATEGORY BREAKPOINT LOCATION (OR CATEGORY VARIANCE) ON MIX VARIABILITY:

In category picking, breakpoint at which the population distribution of a fiber characteristic is divided into categories determines the variance of each category. In proportional weight category picking (PWC), as the range of or the variance within categories decreases, the sum term $\sum W_i S_i^2$ decreases, resulting in smaller between laydown variability. In variance optimum category picking (OPC), the first term of the second equation will always be greater than the second term. Therefore a decrease in category variance will result in more enhanced reduction in between laydown variance than the PWC picking scheme. The effect of category breakpoint location will be explained for one factor/three category system.

One factor/three category system:

In one factor/three category system, one fiber property is considered in the selection and three categories of its value are established. In this case, the distribution of fiber characteristic is divided by two breakpoints at a distance from the center of $\pm z\sigma$. In figure 4.11, equal proportions of bales in the three categories can be achieved at $z = \pm 0.41$.

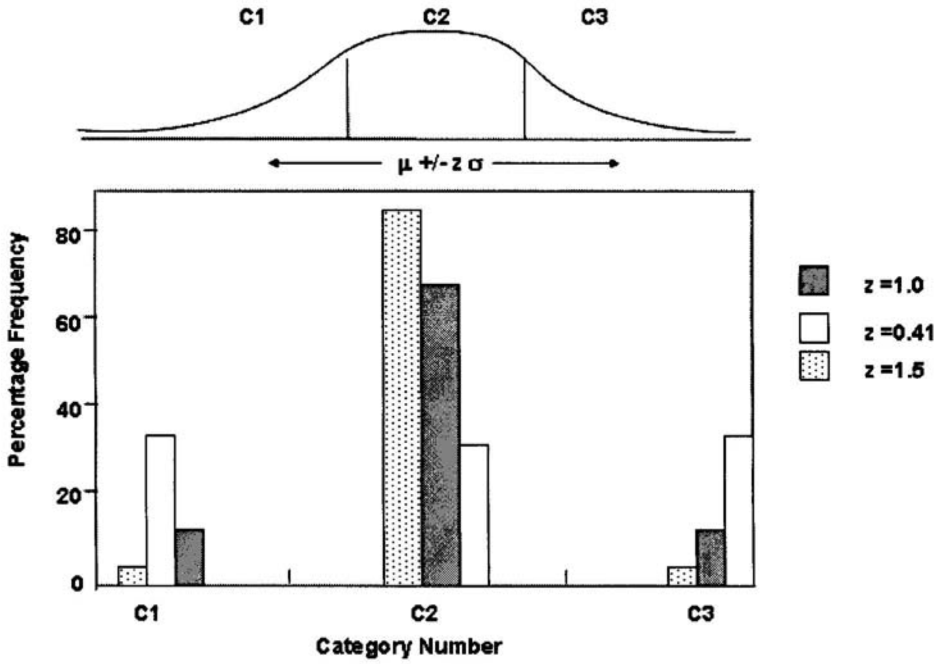

Figure 4.11: Percent distribution of bales over three categories at different break points (one factor/ three category system)

In practice, a z value of 1 is often used. This value corresponds to about 68% of population bales being located in middle category. When population variability is high, a value of ± 0.41 will yield better results.

Three factors/three category system:

In a three factor/three category system, the total number of category combinations is 27. Figure 4.12 illustrates these different combinations. The effect of breakpoint location is shown in figure 4.13. Using ± 0.41 breakpoint, will results in approximately equal proportions of bales in the twenty seven category combinations. It follows that the 0.41 value gives better uniformity.

In practice, however, we often use $\pm 1\sigma$. The main reason for this choice is the limited size of populations and the tendency to allocate bales of extreme values of fiber attributes in separate groups.

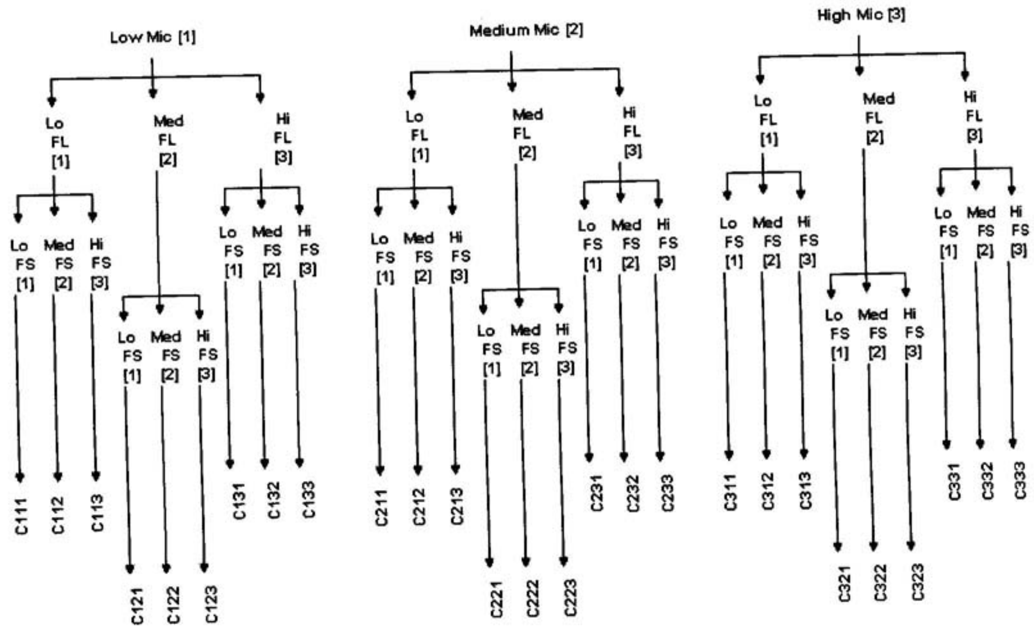

Figure 4.12: Different category combinations in three fiber attributes / three category system

In practice, however, we often use $\pm 1\sigma$. The main reason for this choice is the limited size of populations and the tendency to allocate bales of extreme values of fiber attributes in separate groups.

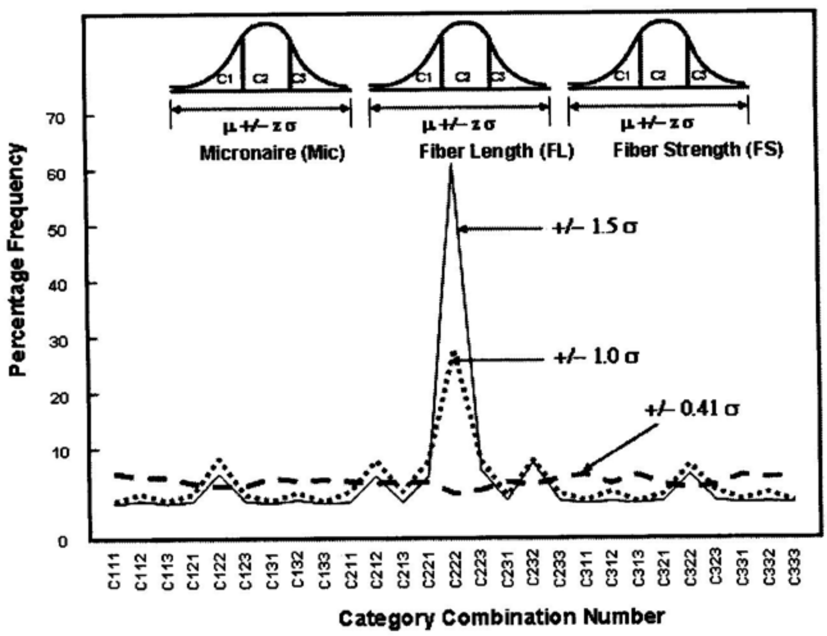

Figure 4.13: Percent distribution of bales in a 27 category combination system and at different break point location

EFFECT OF NUMBER OF CATEGORIES ON MIX VARIABILITY:

In general, the larger the number of categories, the smaller the category sizes, consequently, the smaller the within category variance (S_i^2). Figure 4.14 shows between laydown variability plotted against laydown size at three different numbers of categories.

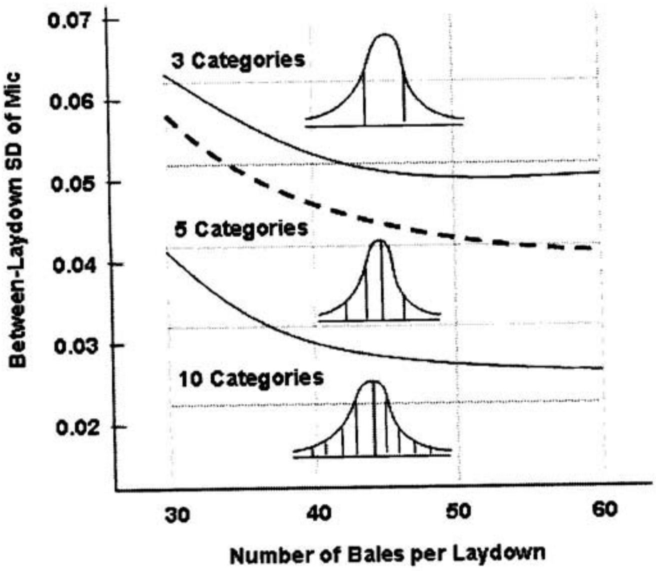

Figure 4.14: Effect of number of categories ob between lay down variability
(PWC picking system)

Effect of bale Lay-down size on mix variability:

It was concluded before, the larger the number of bales per lay-down, the smaller the between lay-down variability. In practice, the number of bales per laydown is determined by technological criteria including machine capacity, space and production rate.

In relation to blend consistency, the minimum number of bales per laydown is determined on the basis of statistical condition that:

$$P\left[\left|\mu - \bar{X}\right| > d\right] \le \alpha$$

Where μ, is the population mean of a fiber characteristic, $\bar{X}$ is the corresponding laydown average, d is some prescribed value of the difference between population and laydown average and α is the value of probability of the difference.

According to the above condition, the question of the minimum number of bales should be stated in terms of the precision required to reproduce the population average in each selected laydown in a certain percentage of time. Using basic features of the normal distribution, the above inequality may be rewritten as follows: $P\left[\frac{\left|\mu-\bar{X}\right|}{S_{\bar{X}}}>\frac{d}{S_{\bar{X}}}\right]\leq\alpha$

Based on the central limit theorem, the distribution of bale laydown averages drawn from normally distributed population is also normal distribution with mean μ and variance $S_{\bar{X}}^2$.

Thus, the above inequality requires that $\frac{d}{S_{\bar{X}}}\geq z_{\alpha}$

Where z_{α} is a statistic corresponding to a certain value of the probability α.

Accordingly, the minimum number of bales per laydown should satisfy the inequality:

$$S_{\bar{X}}^2\leq\left(\frac{d}{z_{\alpha}}\right)^2$$

Since different bale picking schemes have different values of variance of average, the minimum number of bales per laydown n, will be depending on the bale picking scheme. For the three schemes, the values of minimum n are given as follows:

For random picking:

$$n\geq\frac{\sigma^2}{S_{\bar{X}}^2}\left[1+\frac{1}{n}\frac{\sigma^2}{S_{\bar{X}}^2}\right]^{-1}$$

When the size of the population is substantially large, a first approximation of the required number of bales may be given by:

$$n_o\geq\frac{\sigma^2}{S_{\bar{X}}^2}$$

For proportional weight category (PWC) picking:

$$n_o\geq\frac{1}{S_{\bar{X}}^2}\left(\sum_{i=1}^{k}W_i^2S_i^2\right)$$

and

$$n\geq n_o\left(1+\frac{n_o}{N}\right)^{-1}$$

For optimum category (OPC) picking:

$$n_o\geq\frac{1}{S_{\bar{X}}^2}\left(\sum_{i=1}^{k}W_iS_i\right)^2$$

and,

$$n \geq n_o \left(1 + \frac{1}{NS_{\bar{X}}^2} \sum_{i=1}^{k} W_i S_i^2 \right)^{-1}$$

A precise value of minimum number of bales per laydown should be estimated on the basis of the desired value of between laydown variance S^2 (or the difference d), the population variability (σ^2 in case of random picking, and the sum terms in case of both PWC and OPC picking scheme), and the size of the population N.

Figure 4.15 provides a simple graph for determining the minimum number of bales per laydown for one factor/three category system, the micronaire being the selection factor. Both high and low variability populations are represented in the plot, and each at two different sets of category break point $(\pm 0.41\sigma, and \pm 1\sigma)$. The higher the number of bales per lay down is, the smaller the difference between $\mu - \bar{X}$. The effect of break points is more pronounced for high variability population.

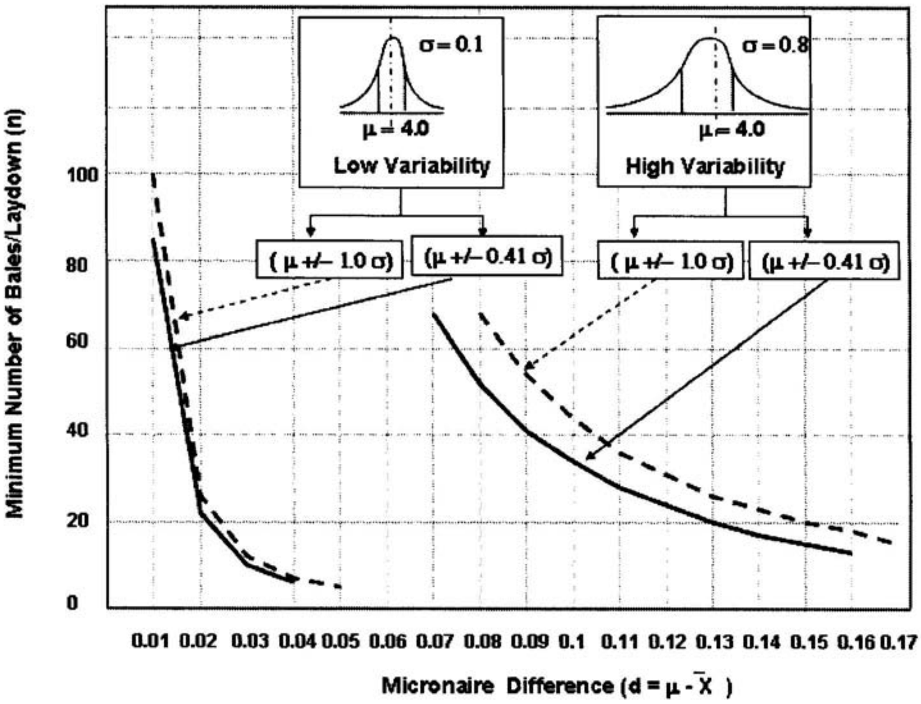

Figure 4.15: Minimum number of bales per laydown in a category system

(Normal distribution, 95% confidence interval, $N \geq 1000$)

In three factor/three category system, computation of the minimum number of bales per laydown becomes more complex, which is attributed to different levels of population variability associated with each factor of fiber property. In order to overcome this difficulty, the difference between population and laydown average is standardized with respect to population variability. Figure 4.16 shows a graph in which the minimum number of bales per laydown n is plotted against the value of $\mu - \overline{X} / \sigma_{pop}$.

In order to estimate the minimum number of bales per laydown, the desired difference for each fiber property is established, and this difference is divided by the corresponding population standard deviation. The minimum number of bales per lay down is determined from the graph using the lowest standardized difference. Assuming that the values of population standard deviation of micronaire, fiber length and fiber strength are 0.8, 0.08, and 2.0 respectively. Id the desired corresponding differences are specified to be less than or equal 0.1, 0.02 and 0.5, respectively, the corresponding standardized differences will be 0.125, 0.25 and 0.25, respectively. The minimum number of bales per laydown will be that corresponding to a standardized difference of 0.125, which is about 40 bales for $\pm 0.41\sigma$ breakpoint, and 45 for $\pm 1\sigma$, breakpoint. This method is known as composite sample size

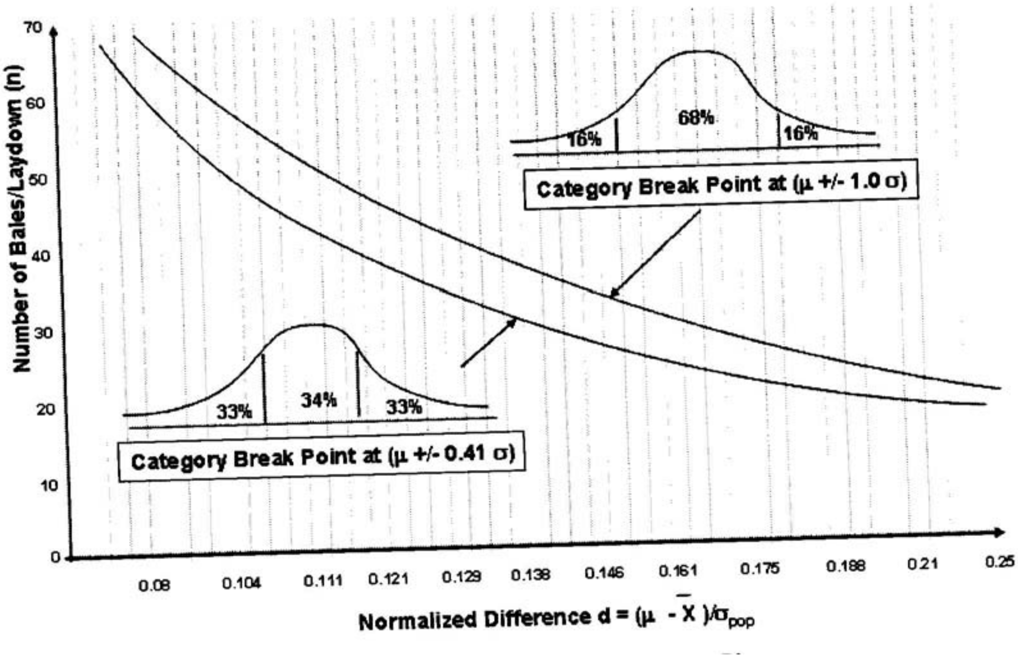

Figure 4.16: composite sample size