Part-Whole Relations
(Mereology, Meronymy, Partonomies, Functions, Aggregation & Composites)
v. 2

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Is-a and Part-of

- Foundational Ontologies require two kinds of relations - "is a" and "is part of".

- Both structure semantic or logical space in a hierarchical manner.

- PreuB and Cavegn suggest that these are processing differently by humans – the former by checking commonalities and the latter involves more complex processing of spatio-temporal and functional correlations. (Gerstl & Pribbenow)

Mereology

- μέρος (méros, "part")

- As Odell notes aggregation or composition of a whole from parts reduces complexity by treating many objects as one object.[11]

- A collection of formal systems...with variations in how they are formulated that is the choice of primitives, axioms and logic.[12]

- It is "Topic Neutral"

- A replacement for Set Theory?

Part-Whole Disciplinary Views

- Mereology – Mathematical Logic
- Meronymy – Linguistics, Psycholinguistics
- Partonomies – Psychology, Cognitive Science
- Sub-Functions – Engineering
- Aggregation – Object-Oriented Modeling
- Ontologies – Philosophy, Phenomenology, Domain Engineering, Conceptual Modeling Computer Science
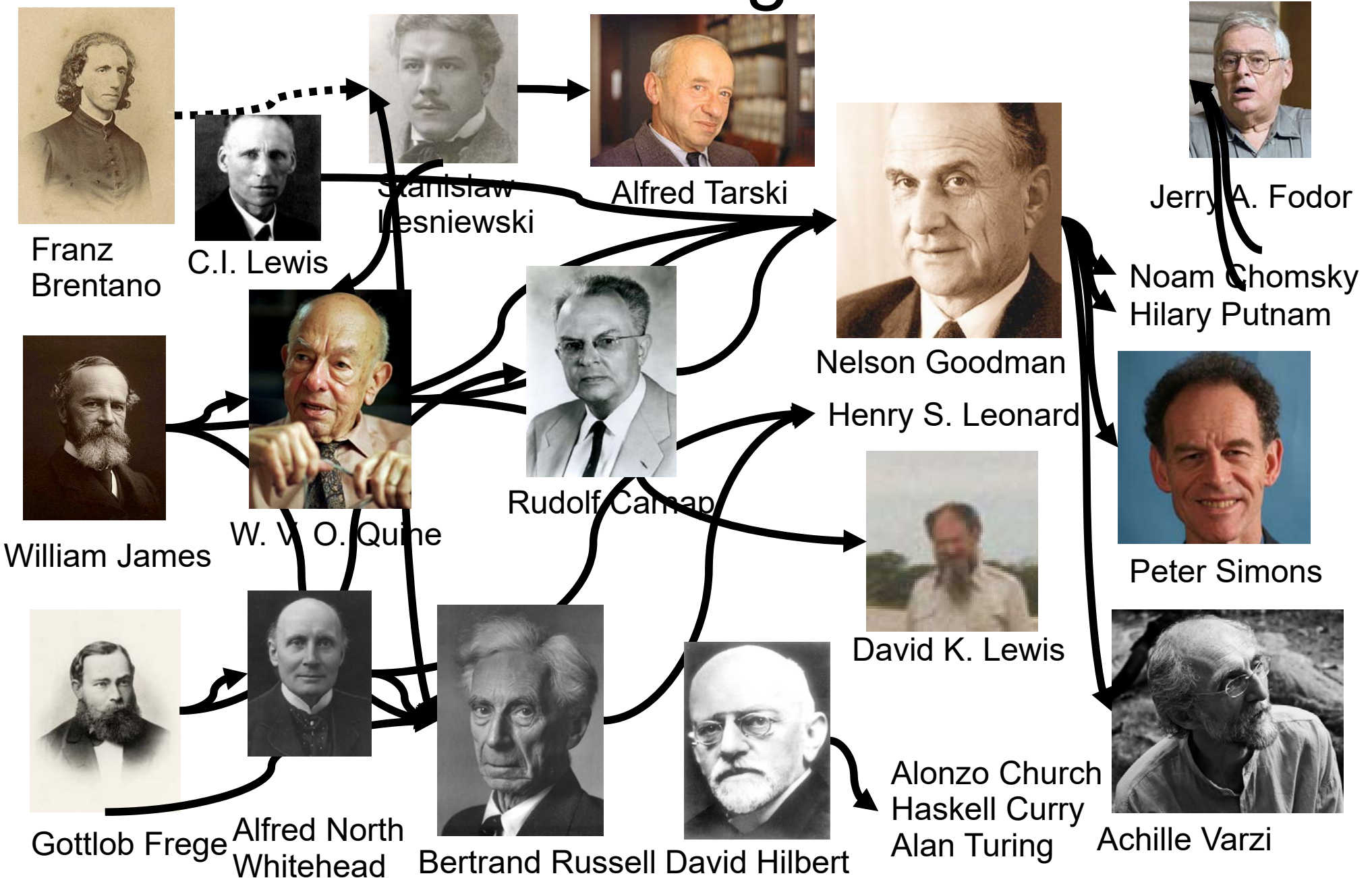- Systems Science

Mereology: Part-Whole Relations

- Mathematical Logic Influences

- Ontologists

- Parthood as a Partial Ordering

- Axioms of Core Mereology (M)

- Core Property Examples

- Proper Parthood as a Strict Partial Order

- Supplementation – Minimal Mereology (MM)

- Weak Supplementation Principle

- Supplementation → Antisymmetry

Mereology: Part-Whole Relations

- Strong Supplementation
- Strong Supplementation Example
- Extensional Mereology (EM)
- Extensional Mereology Example
- Mereological Theories
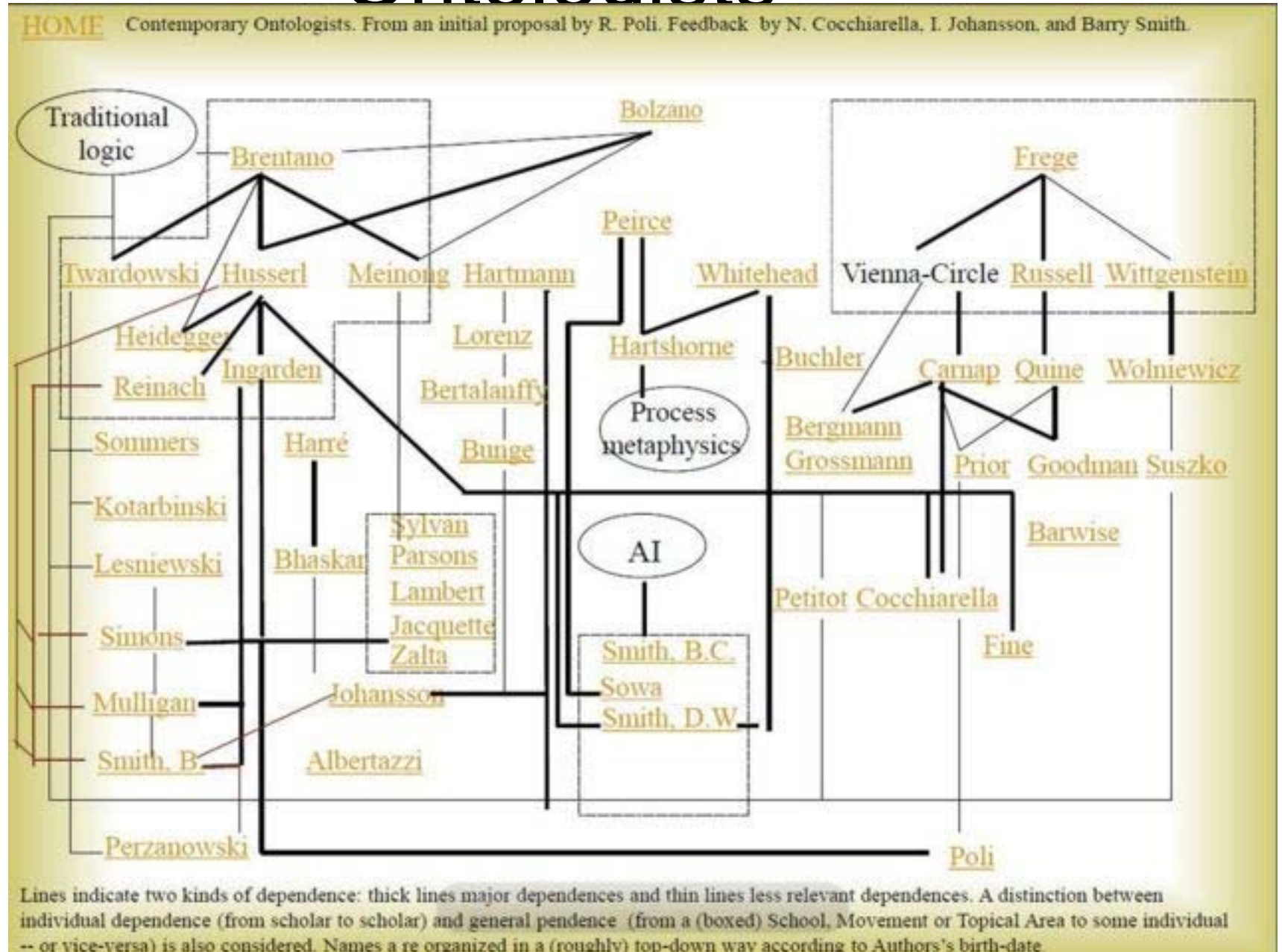- (GEM)
- Mereology & Boolean Algebra

Mathematical Logic Influences

Franz Brentano

C.I. Lewis

Stanislaw Lesniewski

Alfred Tarski

Jerry A. Fodor

Nelson Goodman

Noam Chomsky
Hilary Putnam

William James

W. V. O. Quine

Rudolf Carnap

Henry S. Leonard

Peter Simons

David K. Lewis

Gottlob Frege

Alfred North Whitehead

Bertrand Russell

David Hilbert

Alonzo Church
Haskell Curry
Alan Turing

Achille Varzi

Ontologists

Edmund Husserl

HOME Contemporary Ontologists. From an initial proposal by R. Poli. Feedback by N. Cocchiarella, I. Johansson, and Barry Smith.

Lines indicate two kinds of dependence: thick lines major dependences and thin lines less relevant dependences. A distinction between individual dependence (from scholar to scholar) and general pendence (from a (boxed) School, Movement or Topical Area to some individual -- or vice-versa) is also considered. Names a re organized in a (roughly) top-down way according to Authors's birth-date.

https://www.ontology.co/ontologists.htm
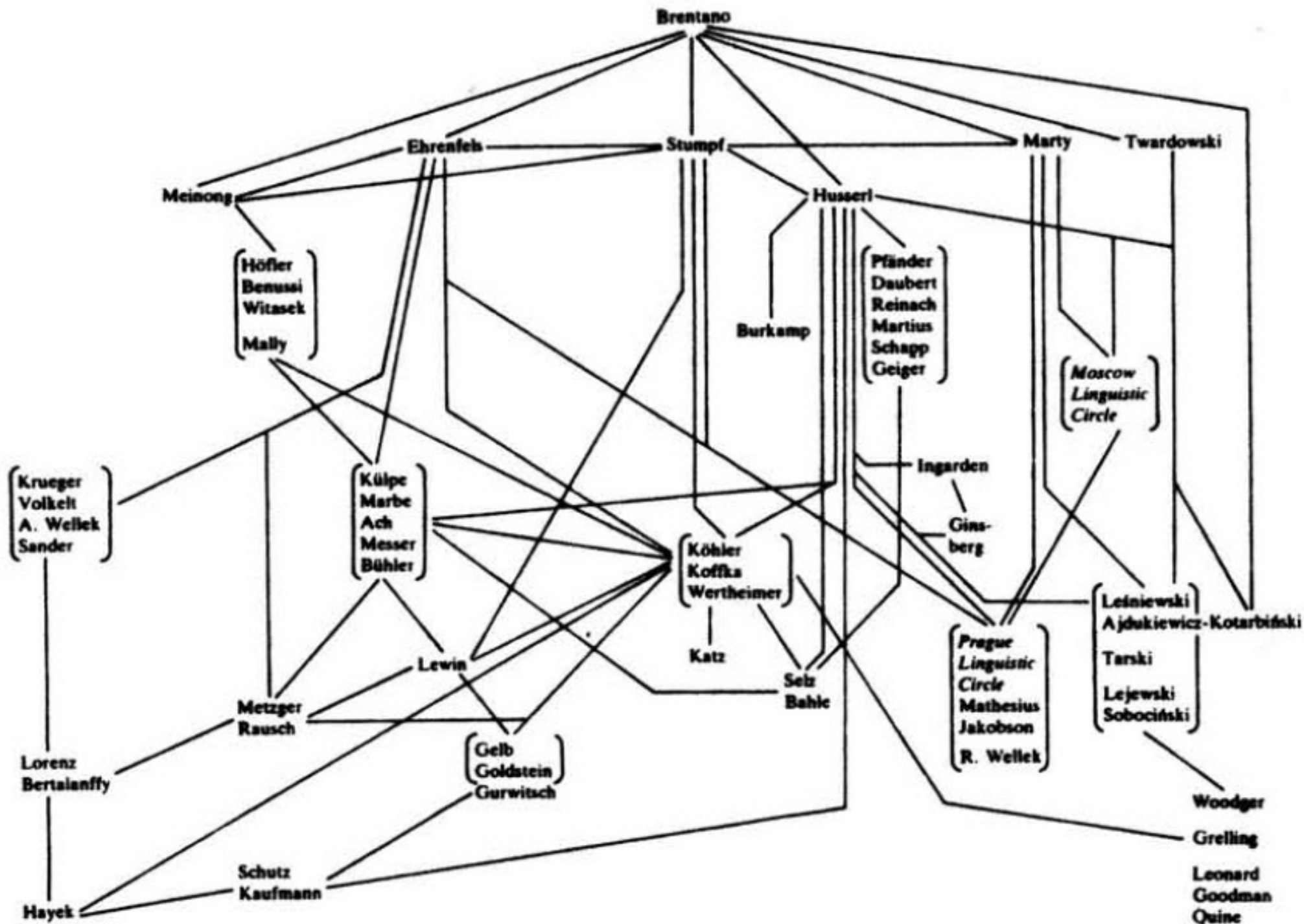
Barry Smith, Annotated Bibliography of Writings on Part-Whole Relations since Brentano
<http://ontology.buffalo.edu/smith/book/P&M/bibliography.pdf>

Parthood as a Partial Order

A Part-of relation is generally considered to be a Partial Order - reflexive, transitive, antisymmetric[1].

- *Everything is part of itself.*

- *Any part of any part of a thing is itself part of that thing.*

- *Two distinct things cannot be part of each other.*

- Simpson Relation #?

Axioms of Ground Mereology (M)

(P.1)   Reflexivity: Pxx

(P.2)   Transitivity: (Pxy ∧ Pyz) → Pxz

(P.3)   Antisymmetry: (Pxy ∧ Pyx) → x=y

"Pxy" is read "x is a part of y", "x" and "y" are "objects"

"∧" is "conjunction" ("and")

"→" is "implies" (if...then)

Standard first-order language with identity, supplied with a distinguished binary predicate constant, 'P', to be interpreted as the parthood relation. P is a Binary Predicate Constant (Varzi). The parthood relation is often represented by non-alphabetic symbols supporting infix notation, such as '≤' or '<' (the notation used in Leonard and Goodman 1940). Taking the underlying logic to be the classical predicate calculus with identity. Simplify notation by dropping all initial universal quantifiers. Unless otherwise specified, all formulas are to be understood as universally closed.[1]

Core Property Examples

- **Reflexivity** - Every object of the universe of discourse is part of itself. For instance, the European Union (EU) is part of the EU.

- **Transitivity** - If x is part of y, and y is part of z, then x is part of z. For instance, Madrid is part of Spain, and Spain is part of EU, therefore, Madrid is part of EU.

- **Antisymmetry** - If an object x is part of y, and y is part of x, then x and y are the same object. For instance, if the territory T1 is part of the territory T2, then the only way for T2 to be part of T1 is by being T1 and T2 the same territory.[2]

Proper Parthood: Strict Partial Order

A predicate by definition[1]

Proper Parthood: PPxy =df Pxy ∧ ¬x=y

(where "¬" *means "not")* or

(Strict) Proper Parthood PPxy =df Pxy ∧ ¬Pyx

PP is irreflexive and asymmetric, and transitive—so they are strict partial orderings. IAT-[2,2,1]

Proper part. A proper part is a part that is other than the individual itself. For example, Spain is a proper part of EU, since Spain is part of EU and they are different entities.[2]

Theories of Mereology

- Not every Partial Order qualifies as parthood. (Varzi) M is extended with additional principles which take us from wholes to parts, or decomposition, or from parts to wholes, or composition[1].

Weak Supplementation Principle

The decomposition principle of Supplementation* adds the intuition that *a whole cannot be decomposed into a single proper part[1]*.

(P.4)   Supplementation PPxy → ∃z(Pzy ∧ ¬Ozx)

where O is defined as "Overlap":

Oxy =df ∃z(Pzx ∧ Pzy)

∃ means "there exists"

Every proper part must be "supplemented" by another, disjoint part (the notion of a remainder)

* Also known as "Weak Supplementation"

Weak Supplementation Example

- Every object x with a proper part y has another part z that is disjoint of y[1].

- The domain of territories fulfills this principle.

- For example, given that Spain is a proper part of the European Union (EU), then EU has other parts that are disjoint of Spain: Portugal, France, Italy, etc..[2]

Minimal Mereology (MM)

- Irreflexive

- Asymmetric

- Transitive

- Weak Supplementation[1]

Supplementation → Antisymmetry

- Supplementation turns out to entail Antisymmetry so long as parthood is transitive and reflexive[1].

- If x and y were proper parts of each other, contrary to (P.3), then every z that is part of one would also be part of—hence overlap—the other, contrary to (P.4)

- (Overlap. The relation overlap is defined as a sharing part. That is, x and y overlap if and only if there is a z such that z is part of x and part of y. ie. Spain and Africa overlap, since Spain has territories in Africa (Canaries, Ceuta, Melilla))[2]

Strong Supplementation

Are there any stronger ways of expressing the supplementation intuition besides (P.4)?

(P.5)   Strong Supplementation[1]

¬Pyx → ∃z(Pzy ∧ ¬Ozx)

If an object fails to include another among its parts, then there must be a remainder, something that makes up for the difference.

"if an individual has a proper part, it has more than one"

Strong Supplementation Example

- Strong supplementation. If y is not part of x, then there is a part of y that does not overlap with x[1].

- For example, given that Spain is not part of Africa, there is a part of Spain (e.g. Madrid) that is not part of Africa.[2]

Extensional Mereology (EM)

Given M, (P.5) implies (P.4).

It is a theorem of EM that no composite objects with the same proper parts can be distinguished[1]:

$(\exists z PPzx \lor \exists z PPzy) \rightarrow (x=y \leftrightarrow \forall z(PPzx \leftrightarrow Ppzy))$

no two composite wholes can have the same proper parts[3]

"no two distinct objects can share the same proper parts"[4]

"an object is exhaustively defined by its constituent parts"[4]
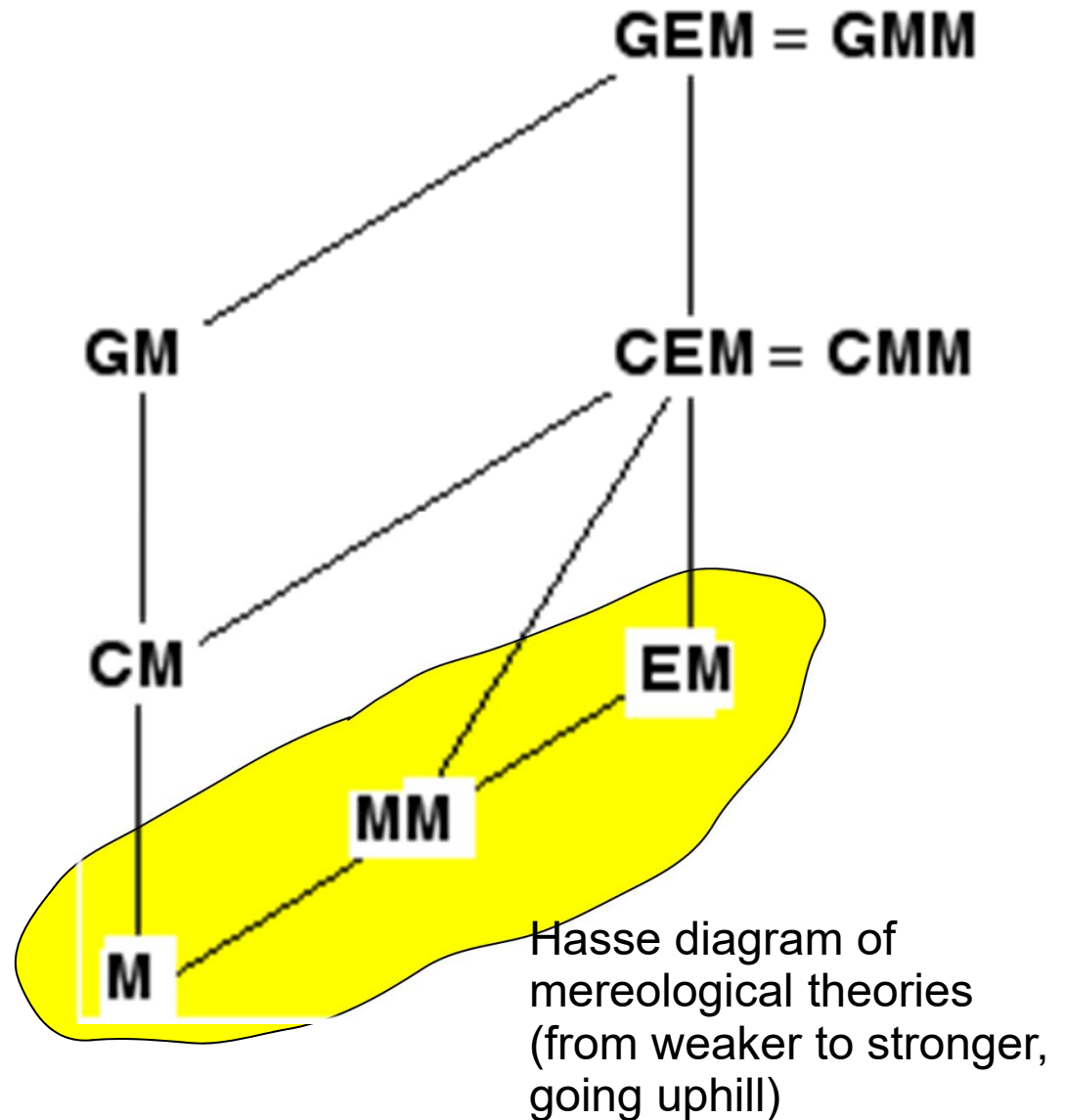
Extensional Mereology Example

This theory is called 'extensional' because a theorem that can be demonstrated is

T1) for all x's and y's, such that x has proper parts or y has proper parts, x and y are identical if and only if x and y have the same proper parts, that is, for all z's , z is proper part of x if and only if z is part of y.

For example, the territory of the Community of Madrid is the same as that of the province of Madrid because both territories are composed of the same proper parts, that is, by the same municipalities.[2]

Mereological Theories

...but M, MM, and EM are not decidable and not a boolean algebra and can not be a substitute for set theory

GEM = GMM

GM

CEM = CMM

CM

EM

MM

M

Hasse diagram of mereological theories (from weaker to stronger, going uphill)

Varzi, A.C. Mereology. The Stanford Encyclopedia of Philosophy. (Fall 2004 Edition), Zalta, E.N. (ed.). <http://stanford.library.sydney.edu.au/archives/fall2008/entries/mereology/>

General Extensional Mereology (GEM)

Adding Composition Principles[1]

CEM=CMM - "any two objects have a sum"[4]

GEM – adds the Fusion Axiom

"every specifiable non-empty set of entities has a sum$_i$"

(P.15) Unrestricted Sum

$\exists w \varphi w \rightarrow \exists z S_i z \varphi w$

where φ is any formula in the language

$$(P8) \quad \exists x \phi \rightarrow \exists z \forall y (Oyz \leftrightarrow \exists x (\phi \wedge Oyx)).$$ [4]
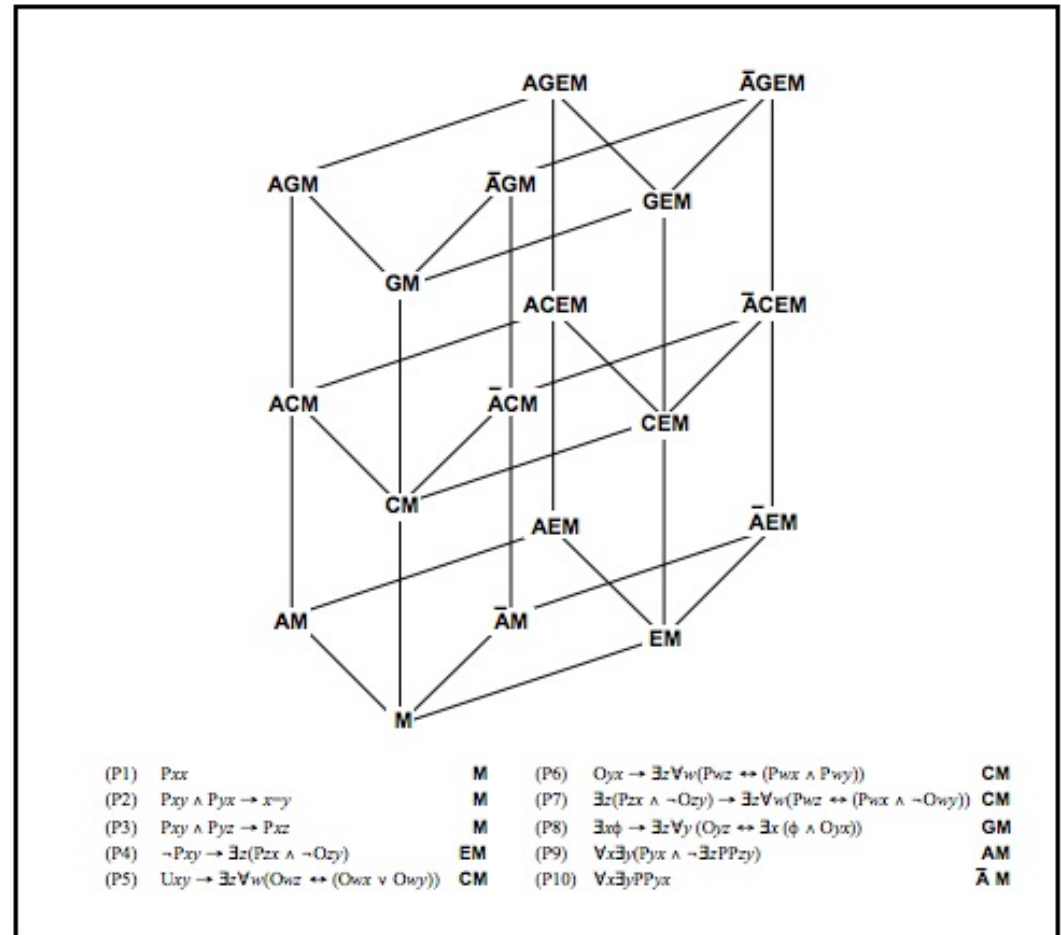
Mereological Theories

General Extensional Mereology
(+ Unrestricted Fusion)

**Closure Extensional Mereology =
Closure Minimal Mereology**
(+ Closure Operations, uniqueness condition)

Extensional Mereology
(+ Strong Supplementation,
Extensionality principle)

Minimal Mereology
(+ Weak Supplementation)

Ground Mereology
(Partial Order)

AGEM   ĀGEM
AGM   ĀGM   GEM
GM   ACEM   ĀCEM
ACM   ĀCM   CEM
CM   AEM   ĀEM
AM   ĀM   EM
M

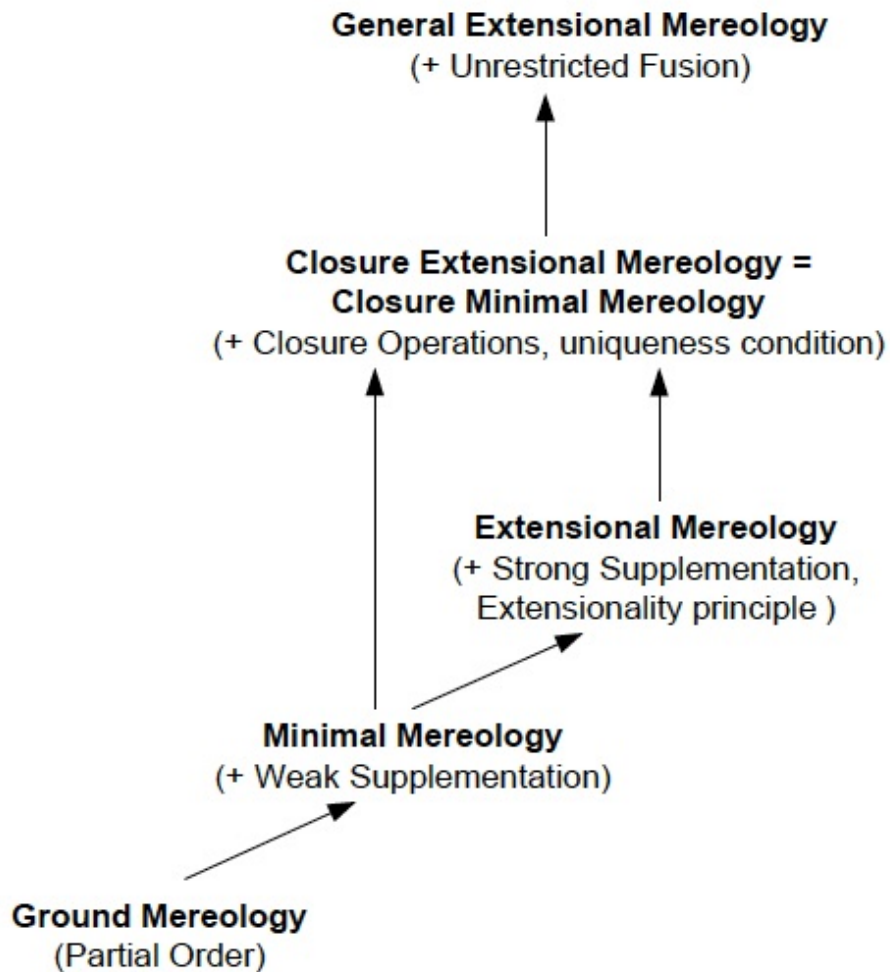|					
(P1)	Pxx	M	(P6)	$Oyx \rightarrow \exists z \forall w (Pwz \leftrightarrow (Pwx \wedge Pwy))$	CM
(P2)	$Pxy \wedge Pyx \rightarrow x=y$	M	(P7)	$\exists z (Pzx \wedge \neg Ozy) \rightarrow \exists z \forall w (Pwz \leftrightarrow (Pwx \wedge \neg Owy))$	CM
(P3)	$Pxy \wedge Pyz \rightarrow Pxz$	M	(P8)	$\exists x \phi \rightarrow \exists z \forall y (Oyz \leftrightarrow \exists x (\phi \wedge Oyx))$	GM
(P4)	$\neg Pxy \rightarrow \exists z (Pzx \wedge \neg Ozy)$	EM	(P9)	$\forall x \exists y (Pyx \wedge \neg \exists z PPzy)$	AM
(P5)	$Uxy \rightarrow \exists z \forall w (Owz \leftrightarrow (Owx \vee Owy))$	CM	(P10)	$\forall x \exists y PPyx$	Ā M

Figure 2: Hess diagram of the basic mereological systems (the inclusion relation goes uphill); in the listing, the characteristic axioms are matched with the lowest theory in which they appear.

Mereology & Boolean Algebra

PROPOSITION 1 (Tarski). Every model of (A)GEM is isomorphic to an (atomic) complete quasi-Boolean algebra (= Boolean algebra with the zero element removed). A model of A – GEM is given by a complete quasi-Boolean algebra on the set of regular open subsets of a Euclidean Space.[7]

"Tarski was the first to point out that Lesniewski's axioms determine a class of structures that strongly resemble complete Boolean lattices. More precisely, given any complete Boolean algebra, we can turn it into a model of classical mereology by, mutatis mutandis, deleting the zero element. And vice versa, any model of mereology can be turned into a complete Boolean lattice upon adding an element to serve as the zero of the structure."[3]

It (GEM) is isomorphic to the inclusion relation restricted to the set of all non-empty subsets of a given set, which is to say a complete Boolean algebra with the zero element removed—a result traced back to Tarski (1935: n. 4) first proved in Grzegorczyk (1955: §4).

Mereology & Set Theory

- Calculus of Individuals vs Calculus of Classes

- Parthood vs Subsethood

- Difference of Part-of relation and

  - Set Inclusion
  - Set Membership

Mereology & Set Theory

- The formal study of parthood relations took off in the foundations of mathematics, as an alternative to set theory. (Varzi)

- Set theory is based on two parthood relations, element $\in$ and subset $\subseteq$, and a corresponding type-theoretic distinction (if $\alpha \subseteq \beta$, then α and β must be sets; if $\alpha \in \beta$, then just β is required to be a set). Consequently, set theory distinguishes between singleton sets and their elements ($a \neq \{a\}$), and assumes an empty set.

Mereology & Set Theory

- Mereology was proposed by Leśniewski (1916, see Simons 2011) and Leonard & Goodman (1940) as a simpler alternative without such assumptions.

- It does not distinguish between elementhood and subsethood, and

- It does not assume abstract entities like sets.

- Consequently, it does not distinguish between singleton sets and their elements, and does not entertain the notion of the empty set.

Lucas Champollion & Manfred Krifka, Mereology, pg. 2.
ling.auf.net/lingbuzz/002099/v1.pdf

Mereology & Set Theory

	Property	CEM	Set Theory
1	Reflexivity	$x \leq x$	$x \subseteq x$
2	Transivity	$x \leq y \wedge y \leq z \rightarrow x \leq z$	$x \subseteq y \wedge y \subseteq z \rightarrow x \subseteq z$
3	Antisymmetry	$x \leq y \wedge y \leq x \rightarrow x = y$	$x \subseteq y \wedge y \subseteq x \rightarrow x = y$
4	Interdefinability	$x \leq y \leftrightarrow x \oplus y = y$	$x \subseteq y \leftrightarrow x \cup y = y$
5	Unique sum/union	$P \neq \emptyset \rightarrow \exists! z \, sum(z, P)$	$\exists! z \; z = \cup P$
6	Associativity	$x \oplus (y \oplus z) = (x \oplus y) \oplus z$	$x \cup (y \cup z) = (x \cup y) \cup z$
7	Commutativity	$x \oplus y = y \oplus x$	$x \cup y = y \cup x$
8	Idempotence	$x \oplus x = x$	$x \cup x = x$
9	Unique separation	$x < y \rightarrow \exists! z [x \oplus z = y \wedge \neg x \circ z]$	$x \subset y \rightarrow \exists! z [z = y - x]$

Lucas Champollion & Manfred Krifka, Mereology, pg. 4.
ling.auf.net/lingbuzz/002099/v1.pdf

Implications for ISM #1,2

- Proper Parthood

  - Never ask "Is A Part-of A"

- Asymmetry

  - Don't ask B->A if A->B is already confirmed

  - (But if A & B are synonyms for the same part then it will show up as a whole with one part.)

  So – don't ask both ways on the diagonal nor on the complement across the diagonal.

  These are apriori, before inquiry restrictions.

Implications for ISM #3

- Supplementation – *No wholes with one part*
  - If there is an A with only part B then ask:
  - Is there another object we did not yet ask about which may be Part-of A?
  - If not, are A & B identical - just two different names for the same object?
  - If not can we identify another part?
  - If not, then condense the matrix and create synonyms for the object and/or leave this inquiry open.

  This is a post inquiry check which initiates another inquiry procedure.

Implications for ISM #4, 5

- Extension:
  - If after inquiry there are two objects which are composed of the same parts then see if they can be distinguished. Otherwise consider them synonyms.

 Also a post inquiry check which initiates another inquiry procedure.

- Composition/Fusion/Unrestricted Sum
  - *Stopping Rule*: Inquiry is not complete until all pairwise combinations (sans restrictions on Proper Partood/Refexivity & Asymmetry) are asked about.

Implications for ISM #6

- Transitivity

  - If B is part-of (reachable from) A then do not ask if A is part-of (reachable from) B.

  - (Otherwise it violates asymmetry.)

  - Example: a piston is a part of an engine, and thus a part of a car but a car is not part of a piston[13].

(Apriori inquiry restriction.)

Non-Strict Partial Order Mereologies

- Non-IAT-[2,2,1] Part-of relations are proposed[28]

- "The Metaphysics of Non-Classical Mereologies"[25]

- Mereologies in which transitivity fails[26]

- "non-wellfounded" mereologies obtained by dropping the asymmetry postulate[27]

- Note also - different extensionality and composition treatments[1, 29]

Meronymy

- Concern with intransitivity motivated the linguistic investigation[16].

  - ie. an arm is part of a musician, and a musician is part of an orchestra, but the arm is not part of the orchestra

Meronymy

- Primary kinds of semantic relations expressed by normal English use of the term "part-of"[16]:
    - Component/Integral Object E.g./ handle-cup
    - Member/Collection E.g. tree-forest
    - Portion/Mass E.g. slice-pie
    - Stuff/Object E.g. steel-bike
    - Feature/Activity E.g. paying-shopping
    - Place/Area E.g. oasis-desert

Meronymy

- Each of these kinds of parthood have different semantic properties. According to Winston et al there are about "40 such terms which are narrower in scope than 'part' but with a fairly wide range of application," and a much larger number of specialized terms Roget's Theasaurus listing approximately 400 synonyms of "part"[16].

Semantic Relations

- The meronymic view of parthood is sometimes confused with similar notions such as possession, ownership, attachment, and attribution, as well as spatial, temporal, and class inclusion[16].
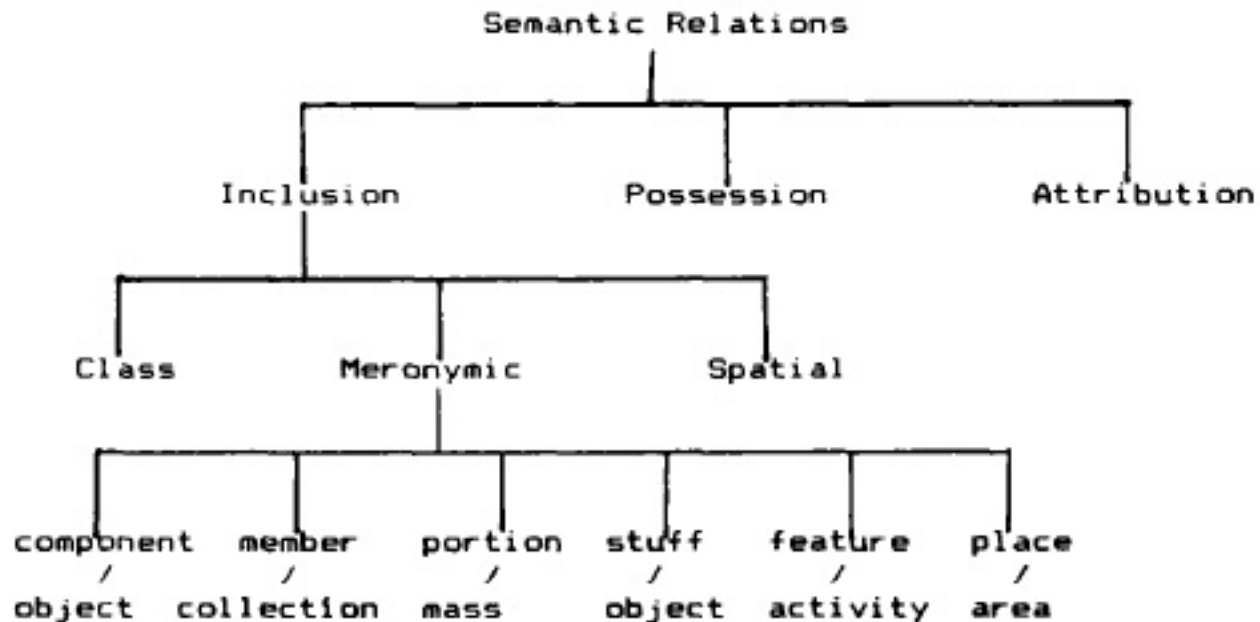
Figure 1. Partial Classification of Semantic Relations

Structured vs Unstructured
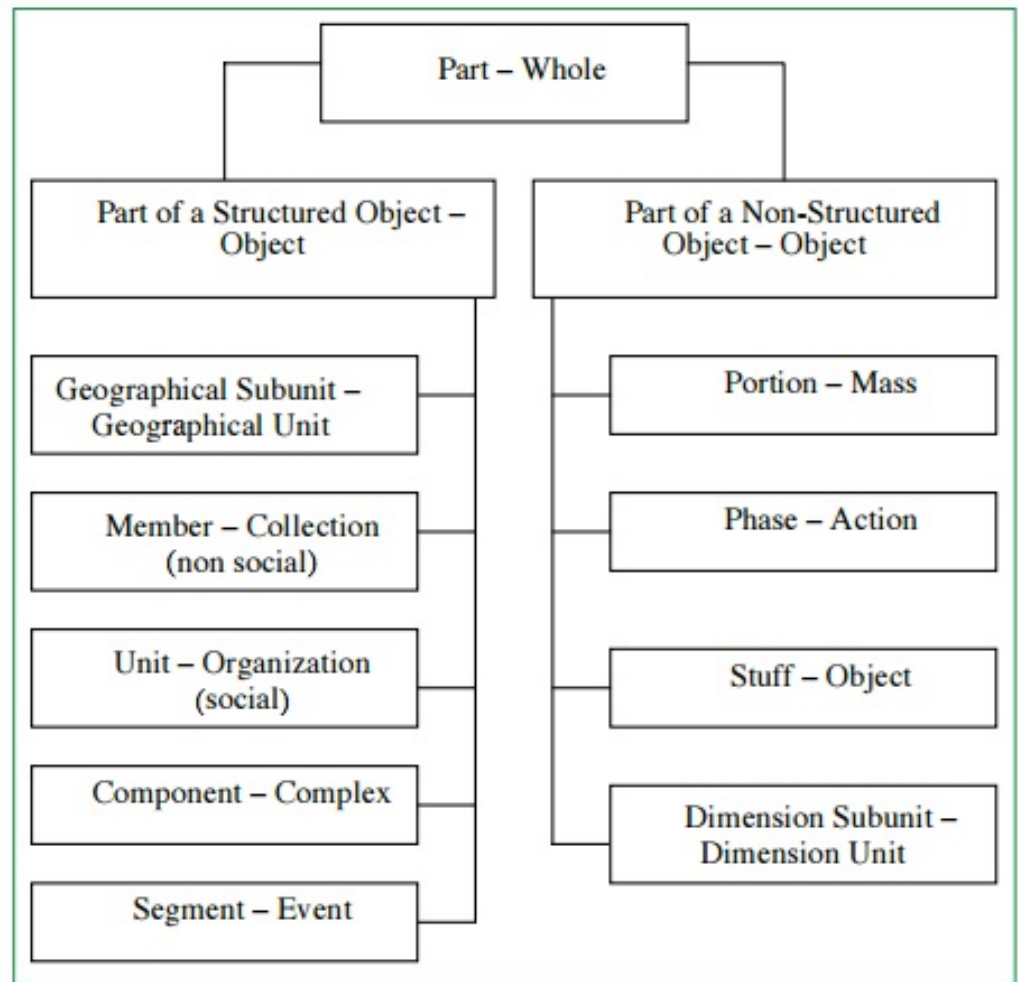
Figure 1. Hierarchy of meronymic relations.

Katrin Weller and Wolfgang G. Stock, Transitive Meronymy: Automatic Concept-Based Query Expansion Using Weighted Transitive Part-Whole Relations, NFD Information-Wissenschaft und Praxis, 59(2008)3, 165-170. https://www.phil-fak.uni-duesseldorf.de/fileadmin/Redaktion/Institute/Informationswissenschaft/stock/120937803 9transitive.pdf

Meronymic Semantics

- Three key characteristics/properties are used to distinguish each type of part-whole relation[16].

    - Configuration: functional roles such as 'an impeller is part-of a pump',

    - Homeomerous: the similarity of the parts with respect to the whole such as 'a molecule of water is a part of water,' and

    - Separabiity/Invariance - whether the parts are separable from the whole. (Jordan et al)

Meronymic Semantics[16]

TABLE 1
Six Types of Meronymic Relations with Relation Elements

Relation	Examples	Relation Elements		
		Functional	Homeomerous	Separable
Component/ Integral Object	handle-cup punchline-joke	+	−	+
Member/ Collection	tree-forest card-deck	−	−	+
Portion/Mass	slice-pie grain-salt	−	+	+
Stuff/Object	gin-martini steel-bike	−	−	−
Feature/Activity	paying-shopping dating-adolescence	+	−	−
Place/Area	Everglades-Florida oasis-desert	−	+	−

Functional (+)/*Nonfunctional* (−): Parts are/are not in a specific spatial/temporal position with respect to each other which supports their functional role with respect to the whole.

Homeomerous (+)/*Nonhomeomerous* (−): Parts are similar/dissimilar to each other and to the whole to which they belong.

Separable (+)/*Inseparable* (−): Parts can/cannot be physically disconnected, in principle, from the whole to which they are connected.

has-a and is-a

- "X is a meronym of Y if and only if sentences of the form Y has Xs / and an X is a part of Y are normal when the noun phrases an X, aY are interpreted generically."[17]

  - A wife has a husband.

  - ? A husband is a part of a wife.

  - A sound has a pitch and a volume.

  - ? A volume is a part of a sound.

  - Changing nappies is part of being a mother.

  - ? Being a mother has changing nappies.

  - A book has pages. A page is a part of a book.

Partonomies

- "These two ways of solving a problem point to two major ways the mind organizes information, partonomically, by dividing into components, and taxonomically, by grouping by kinds (e. g., Miller and Johnson-Laird, 1976; Tversky, 1985, 1990; Tversky and Hemenway, 1984).[18]"

- "Cognitive science has taught that insight into abstract concepts and processes can be gained from analysis of concrete ones."

Partonomies

- "Good parts are good figures, in the Gestalt sense of good."

- "Good parts are also those that have functional significance..."

- "the connection between appearance and action. For people, the appearance of objects often suggest or afford the actions appropriate for it."

- "This correspondence between appearance and function...promotes inferences..."[18]

Partonomies

- Tree Structure (a part can not be part of two different wholes).

- Three levels

- Objects

- Scenes

- Events

Separation of Concerns

"Let me try to explain to you, what to my taste is characteristic for all intelligent thinking. It is, that one is willing to study in depth an aspect of one's subject matter in isolation for the sake of its own consistency, all the time knowing that one is occupying oneself only with one of the aspects. We know that a program must be correct and we can study it from that viewpoint only; we also know that it should be efficient and we can study its efficiency on another day, so to speak. In another mood we may ask ourselves whether, and if so: why, the program is desirable. But nothing is gained —on the contrary!— by tackling these various aspects simultaneously. It is what I sometimes have called "the separation of concerns", which, even if not perfectly possible, is yet the only available technique for effective ordering of one's thoughts, that I know of. This is what I mean by "focusing one's attention upon some aspect": it does not mean ignoring the other aspects, it is just doing justice to the fact that from this aspect's point of view, the other is irrelevant. It is being one- and multiple-track minded simultaneously."[19] (Dijkstra)

Functions

- "The functional part-whole relationship described above is similarly a part-whole relationship directly between functions and not a functionally-defined structural part-whole relationship between technical systems and their structural parts.

- A wall, for instance, may be a functionally-defined structural part of a house but the wall is not a subfunction part of the function of the house; rather the function to support is a functional part of the function to provide shelter."[20]

Functions

- Systems

- Functions

- Objects & Processes (Endurants & Perdurants)

- In philosophy of science and epistemology literature on mechanistic explanations activities of mechanisms are analysed in terms of the organised objects and activities that make up mechanisms[20].

Functions

- "mereological language is not expressive enough to capture functional part-whole relationship"

- *Organization of sub-functions*[20]

- "x is part of y at time t"[21]

Object-Oriented Systems

- Originally modeled as a "has-a" attribute – parts were not "first class citizens" - the "belonging fallacy" (Wilensky)[22]

- But when it was treated as a first class modeling construct this concept remains somewhat vague in object modeling due to, in particular, the wide range of terms used in the literature Aggregation, Composition, Assembly, Containment, Membership[23]

Object-Oriented Systems

- But "It turns out that the basic principles of GEM are inappropriate when considering real domains of application of the theory."

- For example, "In both (the following) cases, GEM is not able to account for the subtle aspects related to identity principles."[22]

Object-Oriented Systems

- "Sometimes, the extensionality and sum principles are too strong."[22]
  - "How to deal with individuals described not only by means of their parts, but also by means of their properties?"
    - "Two individuals may have different identifying properties while being made of the same parts"
  - "...how to deal with individuals which maintain their identity while loosing or acquiring some part at different times"
    - "Two individuals may have the same identifying properties while being made of different parts" ie. A person that has lost hair

Object-Oriented Systems

- "GEM appears to be too weak to capture the notion of a whole as a one piece entity, as opposed to a scattered entity made up of several disconnected parts"[22]

  - the object composed by Johns' left hand and the door of your house does not make any sense

- non-extensional and/or topological frameworks

Object-Oriented Systems

- Implicit vs. Explicit Wholes

  – Reuseability, Understandability, Extendability

- Naming Parts

- Horizontal & Vertical Relationsips

  – Integrity

  – Rigid & Generic Dependency[22]

Implicit

$john : AGE : 35,$
  $SEX : Male,$
  $HEIGHT : 175,$
  $\underline{WIFE} : mary.$

$mary : AGE : 33,$
  $SEX : Female,$
  $HEIGHT : 165,$
  $\underline{HUSBAND} : john.$

Explicit

$john : AGE : 35,$
  $SEX : Male,$
  $HEIGHT : 175.$

$mary : AGE : 33,$
  $SEX : Female,$
  $HEIGHT : 165.$

$m\text{-}j\text{-}family : \underline{WIFE} : mary,$
  $\underline{HUSBAND} : john.$

Naming

```
Car : HAS-WHEEL : Wheel,
      HAS-ENGINE : Engine,
      HAS-COLOR : Red.
```

```
Car : HAS-PART : Wheel,
      HAS-PART : Engine,
      HAS-COLOR : Red.
```

Horizontal - the classical example of an arch which can be considered as a whole made out of inter-related parts. For an arch, its parts should satisfy the following constraints: the lintel is supported-by the uprights, and each upright is on-the-side-of and not-connected-with the other.

Vertical: According to Simons, we say that an individual is rigidly dependent on another individual if the former cannot exist unless the latter exists; further, an individual is generically dependent on a class if, in order the individual to exist, an instance of such class has to exist. As an example of rigid dependence (essential, inseparable) , consider the relationship between a person and its brain: if we change the brain, we cannot assume that the person is the same any more. On the other hand, we can assume that a person is only generically dependent (mandatory) on his/her heart, since this can be replaced.[22]

OO – Generic Dependence

- The whole is generically dependent on a particular class of parts: essential parts.

- A part is generically dependent on the whole: in this case: dependent parts.

- There exists at most one whole containing a particular part: such part is in this case exclusive for that whole.[22]

Vertical Dependence of Properties

- Properties which the whole inherits from its parts: in many cases, for instance, the whole is defective if one of its parts is defective.

- Properties which the parts inherit from the whole: for instance, within a certain granularity, certain locative properties (e.g., being on the table") of the whole hold also for its parts.

- Properties of the parts which are systematically related to properties of the whole: for instance, the region occupied by a single part is always inside the region occupied by the whole...[22].

Unified Modeling Language

- Shared Aggregation & Composite Aggregation
- "composite implies propagation semantics"[23, 24]

TABLE 1
Primary and Secondary Characteristics of Whole-Part

Primary Characteristics	Whole-Part, emergent property, resultant property, asymmetry at instance level, antisymmetry at type level.
Secondary Characteristics	Encapsulation, overlapping lifetimes (9 cases), transitivity, shareability, configurationality, separability, mutability, existential dependency.

Unified Modeling Language
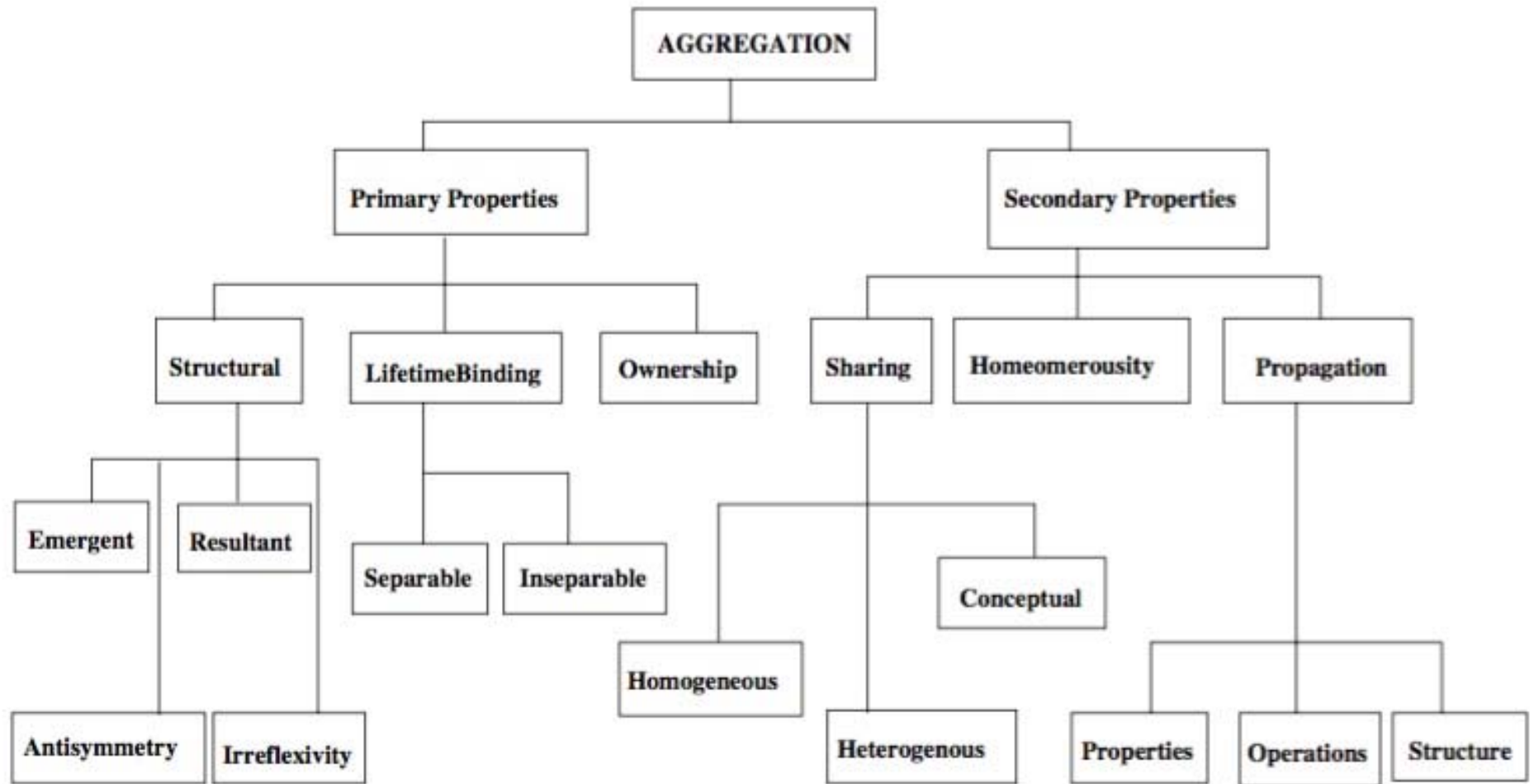
Fig. 1. Properties of Aggregation

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