# Reformatted PREFACE to *Structuring Complex Systems*

John N. Warfield, 1974

This monograph presents an approach to organizing thoughts about systems with assistance from a computer. This approach can help people who conscientiously seek to:

- apply logical reasoning to complex issues
- communicate their reasoning fully to others.

It is proposed as a logistical apparatus to enable them to do so more efficiently and effectively.

This is the third Battelle monograph that treats complex systems.

The first, *A Unified Systems Engineering Concept*, sought to appraise the strengths and deficiencies of methodology applicable to the planning phases of systems engineering.

The second, *An Assault on Complexity*, explored various philosophical and methodological approaches for organizing complex issues, presented short case studies, and delved into structural aspects of policy analysis and synthesis.

The experience gained in developing these monographs led to the conviction that it is necessary to find ways of improving human capacity to develop structures germane to complex systems and issues. In pursuing this idea, the name ‘structural modeling’ developed as an appropriate title for the knowledge and methodology that seemed to be needed.

Four excellent books (1-4) contain important contributions or background relevant to structural modeling. Each of them has strongly influenced the work leading to this monograph. While (1) concentrates on directed graphs ("digraph"), (2) uses non-directed graphs as a basis for system organization. Both types have appeared in the two prior Battelle monographs mentioned.

(1) Harary, Frank, Norman, Robert Z., and Cartwright, Dorwin, *Structural Models: An introduction to the Theory of Directed Graphs*, John Wiley & Sons, Inc., New York, 1965.
(2) Alexander, C., *Notes on the Synthesis of Form*, Harvard University Press, Cambridge, 1964.
(3) Hartmanis, J., and Stearns, R. E., *Algebraic Structure Theory of Sequential Machines*, Prentice-Hall, Englewood Cliffs, 1966.
(4) Klir, G., *An Approach to General Systems Theory*, Van Nostrand Reinhold Co.,New York, 1969.

In developing this monograph, it was useful to think of structural models of two generic types.

The first type, the basic structural models, are those whose theory has evolved out of mathematics. They are the graphs and digraphs which carry no empirical or substantive information. Much is known about their properties. Methods exist for performing operations upon them that permit extensive manipulation and structural insight.

The second type, the interpretive structural models, are those developed to help organize and understand empirical, substantive knowledge about complex systems or issues. Intent structures, DELTA charts, and decision trees illustrated in the earlier monographs, are examples of interpretive structural models. Other examples include interaction graphs, PERT diagrams, signal flow graphs, organization charts, relevance trees, state diagrams, and preference charts.

If the full knowledge of basic structural models could be brought to bear upon the development of interpretive structural models, a significant advance could be made in the rational analysis and synthesis of complex systems. Yet, it seems impractical to expect that those who are engaged in day-to-day interaction with complexity in human affairs would take the time to learn to apply such abstract concepts as mathematical logic, matrix theory, and the theory of graphs in their work. It also seems unlikely that mathematicians would take the time to become highly knowledgeable of complex real-world systems and issues. The dilemma of how to wed substantive issues and knowledge of complex systems to the mathematics seems significant. But, even if people had all the mathematics and understood the complex system or issue, still another problem would be present. That is the extreme tyranny of working systematically to establish relations among many elements in the form of an interpretive structural model, and the long time period required to do this by manual methods.

One approach shows promise of a way out of the mentioned difficulties. This approach is to introduce the digital computer to aid in problem definition. If the necessary mathematical knowledge as well as the logistical tyranny can be transferred to the computer, leaving to the developer of the interpretive structural model only the minimum, but critical, core of effort (providing the substantive knowledge of the system or issue), then the developer would not need to learn the associated mathematics. Nor would he have to absorb the tyranny associated with the extensive manipulation of ideas on paper that would otherwise be required. The computer could be a major factor in compressing the timescale for development of an interpretive structural model.

This monograph presents a method whereby the computer can carry out the necessary operations for those interpretive structural models that can be put in correspondence with digraphs. Since the monograph is largely limited to such models, it does not encompass all possible kinds of interpretive structural models. It is easy, both to underestimate and to overestimate, the significance of the capability that can be developed from the theory presented in this monograph. Since the structuring of systems has largely been done in an ad hoc way in the past, it may seem that a theory designed to permit this process to become much more explicit and to be carried out with machine assistance, would be superfluous.

Now, the author believes very strongly that such a view would greatly underestimate the significance of the theory presented in this monograph. Three thoughts seem especially relevant.

> First, when the number of elements to be considered is large, the number of interactions to be considered is at least comparable to the square of the number of elements. The logistics of dealing with so many interactions is by itself an inhibiting factor in conducting a studied structuring exercise and in manipulating the perceived relations.
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> Second, in the absence of assistance in developing this information, it seems likely that the most fundamental thinking that goes into model development will be lost, and with it considerable capacity to communicate to other material that may be of much importance in establishing credibility of a model.
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> Third, if at the time the structure of a model is being developed, it becomes feasible for the developer to concentrate primarily upon the substantive rather than the logistical aspects of model development, it should be much easier for model structuring to become a group activity. Thereby, a variety of people could become involved in model development and evolution. It is easy to imagine that several groups construct a structural model based on the same element set, and that in comparisons of differences among these models, significant insight might be gained into differing perceptions. These might, in turn, lead to superior structuring from a conglomeration of views. Alternatively, structuring alone might induce redefinition of the matters being considered, and alter the course of future events in useful way.

The author believes that there will be applications wherein the structural model is a desirable end in itself, and other applications wherein the structural model will be an intermediate step in the development of more sophisticated types of models. It will not always be easy to decide in a particular situation what will follow the development of a structural model. There is probably a danger that the mere construction of a structural model will be thought to be a very desirable end, in situations where the greatest value from the development would be dependent upon considerable follow-on activity. In such instances, it is easy to overestimate the benefits that could flow from use of the theory developed herein. Some of the content of Chapter 3 is intended to add more detail to illustrate the foregoing comments on significance of structural modeling.

This monograph is written, necessarily, for the reader who has or will acquire a grasp of the elementary parts of set theory, matrix theory, mathematical logic, and graph theory. In only a few spots is additional mathematical sophistication called upon, and in those instances references are provided. Thus this monograph is addressed primarily to those in a position to place this work in the service of non-mathematicians. It is hoped later to make available a non-mathematical monograph that will illustrate a variety of applications of structural models in various fields. This later monograph would be addressed primarily to potential users who are not mathematicians.

The Summary explains how the chapters are organized and can be read to gain a rough overview of the monograph. An abstract appears at the beginning of each chapter.

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SUMMARY

The problem considered in this monograph may be described as one of finding useful graphical descriptions of single contextual relations germane to a system. The problem is addressed as one of developing a process that minimizes the logistical burden on the developer of the product. The product is called an interpretive structural model. The latter is structurally isomorphic to a digraph, but contains both substantive and structural information supplied by the developer. All of the mathematical operations required by the process would be carried out by a computer. The critical, substantive system knowledge is supplied by the developer; the partnership of developer and computer gives the process its strength.

By a system, it is understood as a complex of entities $S$ with interactions $R$. Knowledge concerning $(S, R)$ can be structural or substantive or both, and is the consequence of perceptions of the system. Some knowledge of the system is presumed to be available. Total knowledge of a system is thought normally to be unattainable. A description of a system is approximate.

Let <$S$> be represented by a finite set $S = \{s_1, s_2, ..... s_n\}$ and let <$R$> be represented by a finite set <$R$> = {<$R_1$>, <$R_2$>, ... <$R_m$>}. Call $s_i$ an element of the system, and call <$R_j$> a contextual relation of the system. A contextual relation is a phrase in a colloquial language.

Let <$R_j$>, be represented by $R_j$, where $R_j$ is a binary relation on $S$ x $S$. Let $A_{ij}$ be a binary matrix indexed by $(S, S)$ which portrays both $R_j$ and its complement, not $R_j$. Let $M_{ij}$ be the transitive closure of $A_{ij} + I$, where $I$ is the identity matrix.

Then, there exists a permutation matrix $P$ such that the matrix $M_j = (1/P)M_{ij}P$ induces a unique partition on $S$ into subsets called levels, and each sub-matrix of $M_j$ indexed by a level is symmetric. For any sub-matrix indexed by a level, there exists a permutation such that its block diagonal sub-matrices form a minimum set of universal matrices. Each nontrivial index set of one or more of these is called a maximal cycle set. If an

index set contains only one element, the set is trivial and the element is an isolate within the level, though it may connect to other levels. If appropriate modifications are made to $M_j$, it can be converted to a matrix $M'_{j'}$ such that every maximal cycle set is replaced by a proxy element, and reachability is preserved by following a principle of condensation.

The digraph of the condensed matrix is a hierarchy with the same number of levels as $M_{ij}$ and $M_j$. There exists a binary matrix $A_j$ for which $M'_{j'}$ is the transitive closure, called a skeleton matrix, such that there is no other matrix less than $A_j$ that has $M'_{j'}$ as its transitive closure. The digraph $D(A_j)$ is called the skeleton digraph of $M_{ij}$. It is the minimum-edge digraph that preserves condensed reachability or $A_{ij}$. To every maximal cycle set, there corresponds a minimum-edge digraph that preserves reachability within the cycle set. If every proxy vertex on $D(A_j)$ is replaced with such a minimum edge digraph, the resulting digraph is a minimum-edge approximation of $D(A_{ij})$. Because the matrix $A_{ij}$ is often difficult to develop, the matrix $M_{ij}$ is developed instead. In developing $M_{ij}$ transitivity is enforced. While it is impossible to determine $A_{ij}$ from $M_{ij}$, it is possible to determine $A_j$.

Since a minimum-edge representation of a maximal cycle is often an inadequate approximation to the structure of a maximal cycle, additional information concerning the structure of the cycle may be sought. Such structure is based upon a weighting matrix $W$ for the maximal cycle, wherein the intensity of some contextual relation is supplied on a specified scale. From such a weighting matrix, it is possible to compute a binary matrix that represents a threshold of intensity and preserves reachability within the maximal cycle. This binary matrix permits construction of a reachability-preserving digraph for the maximal cycle which normally has fewer edges than a complete graph for the cycle.

If those digraphs that result from application of an intensity threshold to weighting matrices of the various maximal cycles are substituted for the proxy elements in the skeleton digraph $D(A_{ij})$ while preserving overall reachability, it is believed a good approximation to the system structure results in so far as the relation $R_j$ is concerned. The digraph so formed may be complicated by the presence of many vertexes, edges, and crossings of the edges. If so, additional interpretive digraphs may be useful. A maximal geodetic cycle specified on $(s_i, s_j)$ is a maximal cycle consisting of a pair of (geodetic) paths, one path originating at vertex $s_i$ and terminating at $s_j$, the other originating at $s_j$ and terminating at $s_i$. It is possible to compute, for a given matrix $A_{ij}$, a set of maximal geodetic cycles. To each maximal geodetic cycle there corresponds a cycle set, which is a subset of $S$. The set of all such cycle sets can be placed in a hierarchical digraph using the relation of inclusion. A digraph whose vertices represent maximal cycle sets offers further opportunity for interpretation of the system, in connection with the particular contextual relation of interest.

For some systems and some situations, a structural model of a single contextual relation may suffice. At times more than one contextual relation will be of concern, consequently several different structural models may be prepared for a given system. Each can be thought of as one dimension of partial description of a system.

**Chapter 1**, titled "Probing Complexity", discusses the assumptions that underlie the work reported subsequently and the objectives of the research.

**Chapter 2**, titled "Binary Matrices in System Modeling", introduces the mathematical ideas that form the basis for the structural modeling process and briefly overviews the mathematical nature of the process.

**Chapter 3**, titled ”Surrounding Ideas and Background", presents certain links between this work and prior work. It is hoped to show, in this chapter, that the work reported herein occupies a reasonable place in relation to work by other investigators and to show linkages to various fields of study. The reader who develops a passion for the structural modeling process may want to become familiar with some of this work in other fields since it furnishes very useful collateral information in regard both to structuring and to substance.

**Chapter 4**, titled “Developing Subsystem Matrices in Structural Modeling", discusses the first phase of a two-phase process aimed at developing the data needed for construction of a structural model. The product of the first phase is a partially filled binary matrix representing some contextual relation among a set of system elements.

**Chapter 5**, titled "Developing Interconnection Matrices in Structural Modeling", discusses the second phase of the two-phase process, wherein the matrix development begun in Phase 1 is completed.

**Chapter 6**, titled “On Interpretation of Complex Structural Models“, summarizes the important partitions of reachability matrices, shows how to find condensation matrices and skeleton matrices, and discusses the special nature of cycles that occur in binary relations. It suggests one means for developing the fine structure of cycles. Geodetic cycles are defined and illustrated. It is shown that geodetic cycles can be placed in a hierarchy; such placement may be very useful in interpreting a cycle relation.

**Chapter 7**, titled "Correction Theory and Procedures“, indicates that structural modeling is inherently iterative, and that it is natural to expect that corrections will be required in initial models. In this chapter, a theory is given for making such corrections with machine assistance.

**Chapter 8**, titled ”The Process of Structural Modeling", seeks to summarize concisely the material presented in Chapters 4—6 from a process point of view.

**Chapter 9**, titled "Constructing Operational Value Systems for Proposed Two-Unit Coalitions”, illustrates one possible type of application of structural modeling. Also, it shows how the theory can be applied even when one of the contextual relations is not transitive.

**The Appendix** is a statement and proof of the "bordering theorem” that is applied developing the process and the correction theory.