DIMENSIONALITY

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ABSTRACT

New definitions of "dimension" and "dimensionality" are set forth that dominate older definitions. Among the benefits of the new interpretations of these terms are: greater effectiveness in describing and understanding systems, ability to accomodate quantitative and qualitative factors in the same framework, and the possibility of disciplining the design and management of complex systems, to avoid calamities of the type that occur too frequently in modern society.

Bringing Discipline to Management and Design

Recent, highly visible disasters involving technology include the Three-Mile Island and Chernobyl nuclear incidents, the Bhopal chemical plant incident, and the Challenger space shuttle explosion. These events had in common rapid occurrence, international visibility, complex systems including people and technology, and investigations as followups.

These one-shot events are only the tip of the sociotechnical iceberg. Other issues linger on and on (like environmental issues), but the combined impact of the one-shot events and the lingering problems still does not seem to be viewed as part of a pattern, but rather as a set of isolated, independent situations, correctible by reorganization, reassignment, improved procedures and practices that are locally specific. Return to "normalcy" is the goal, but from another point of view, normalcy is the reason for these disasters.

As emphasized by Kemeny [1] in his description of the Three-Mile Island situation, that event is merely illustrative of a whole class of situations that are brought about by bad practices in design and management.

From this alternative perspective, most of the undesired events or issues that have appeared and will appear that involve complex sociotechnical systems either can be prevented altogether or made much less destructive, once they are regarded as part of a generic pattern that can respond to generic systems management and design principles and practices.

In this paper, the concept of dimensionality of a system is proposed as a potential disciplinary force that can have a profound influence on the practice of management and design, once accepted and put into practice.

Dimensionality as a Potential Disciplinary Force

One of the reasons for speculating that the concept of dimensionality can be a powerful disciplinary force on management and design is that it has already played that role in situations where it is well-understood and well-defined. With the knowledge that is now available about motion in space and time, no respectable engineer or scientist could ever be expected to design an airplane or other vehicle with the idea that it would operate only in a geometric plane as a two-dimensional body. Yet there must have been a time before the advent of flight when vehicle designs were viewed as two-dimensional, and even today no one expects a skate-board to fly at several thousand feet of altitude. Today no manager could impose a requirement that a jet aircraft should be stabilized only in two of its several modes of motion due to (let's say) insufficient funds to provide for its control in other modes.

Yet, it is asserted, sociotechnical systems having high dimensionality are designed or managed as though their dimensionality was unknown, but low [2]. Thus dimensionality has little or no positive influence on such designs and where it is influential, it is in some subsystem where dimensionality is well understood.

In order to achieve the potential benefits of dimensionality in a large class of complex sociotechnical systems, it is necessary to do several things. First, the concept must be defined in a way that enables designers or managers to determine what the system dimensions are, so that they can determine the system dimensionality. Second, once the dimensions are known, a means for applying the knowledge to design and management must be clarified. And finally, given these accomplishments, it is necessary for designers and managers to become educated about these developments, their meaning, their significance, and their necessity, in order that they will then apply them correctly in meaningful situations.

Guiding Factors

Certain factors can be expected to guide the development of a broader concept of dimensionality than now prevails. We would expect, for example, that dimensionality would be associated with a mathematical space, inasmuch as it has this characteristic in all scientific applications. Secondly, we would expect that one cannot assess dimensionality without working with a mathematical set. The reason for this is that we know from past experience in working with this concept that while dimensionality appears to be a property of the system, the dimensions themselves are intellectual constructs constructed by the observer or scientists, and can take a multiplicity of forms. It is well-known that it is possible to formulate a problem in such a way that one or more of its variables is linearly dependent upon (determinable from) some subset of the variables. Given this, unless we are willing to assume infallibility in construction of a set of dimensions, the better part of wisdom is to adopt the posture that in seeking dimensions an initial set will be developed that is considered to be a candidate set from which the ultimate set of dimensions will be proclaimed. The ultimate set may be the same as the candidate set, may be a subset of the candidate set, or may be a different set altogether, depending on how the conceptualization is completed.

Thirdly, we would expect that as part of the process of choosing the ultimate set there would be an exploration of the independence or dependence that may hold among members of any candidate set that is surfaced, for reasons just mentioned.

While we have been speaking in the foregoing as though a system can only have one set of dimensions (even though it may take any of several forms), we should not automatically assume that every system will meet that description. In fact, if we assume that a system includes identifiable subsystems, there is reason to suppose that there might be collections of dimensions, some of which would apply to particular subsystems, and some of which would apply and have meaning only when the total system is assessed. All would be pertinent to the system.

While, in physical systems, the dimensions are specified by numbers, there is no inherent reason to impose this limitation on sociotechnical systems. We often hear orators talk about a certain dimension of a problem in which they identify a dimension that has no natural metric. While many engineers and physical scientists have been perhaps permanently impressed with Lord Kelvin's famous pronouncement to the effect that something couldn't really be understood unless it could be expressed in terms of numbers, Lord Kelvin also has gained some notoriety for stating publicly at the end of the nineteenth century that:

> "Radio has no future...X-rays will prove to be a hoax...heavier-than-air flying machines are impossible."

We should anticipate that in systems involving people we will often not be able to find an appropriate description in terms of dimensions if we insist in admitting only candidates that can be measured in terms of numbers.

Finally, barring a total upheaval that replaces all the older ideas, we would expect that the new concepts of dimensionality and dimension would dominate the older concepts in the sense that a new concept would incorporate the features of the older concept that are really critical to its survival, while including new features not found in the older concept.

An Extended Concept of Dimensionality

Let us now designate by T the "target" representation of a system, and let us suppose that the fundamental element of which this representation will be comprised is the "option" θ. Let us imagine that there is a set of options $\Theta = \{\theta\}$ and that the very first approximation to the representation is a simple two-level inclusion hierarchy of the form indicated in Figure 1, meaning that each option is included in the initial representation T. While there is only one target T, the cardinality

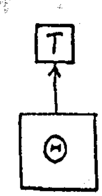

Figure 1. Inclusion Hierarchy

of the set of options may be quite large, so we indicate this in Figure 1 by using a larger box to represent the options set.

Now let us suppose that the options consist of simple, one-idea, descriptors. Further let us suppose that not all of the options will ultimately be required to describe the system. We can anticipate some process that chooses some options as being more appropriate than others; and also that certain options will not be competing for attention with certain others, but rather that the options can be grouped into similarity classes for purposes of comparison and choice.

Formally, then, let there be constructed a similarity relation $R_s(\Theta \times \Theta)$, [The means for carrying out such a construction have been described as part of the Generic Design Methodology (3, 4)].

Then such a relation induces a partition $\Pi_s(\Theta)$ on the set of options such that

$$\text{i)}\quad \Pi_s(\Theta) = d_1;\ d_2;\ \ldots;\ d_n$$

$$\text{where ii)}\quad \theta_i \equiv \theta_j\ [\Pi_s(\Theta)]$$

$$\text{iff}\ (\theta_i, \theta_j) \in R_s(\Theta \times \Theta)$$

where the notation $\equiv$ is read "is in the same block as" in the partition identified in brackets.

Then we call the set

$$D_s = \{d_i\},\ i \in I_n,\ I_n = [1..n]$$

the set of <u>dimensions</u> that corresponds to Θ. (We are omitting here certain refinements that have been discussed previously [5], and which allow for any necessary editing and testing.) The <u>dimensionality</u> corresponding to the set Θ is identified as $|D_s|$, the cardinality of the set D_s.

The partition $\Pi_s(\Theta)$ also induces an inclusion relation $R_h(\Theta \times D_s)$ whereby each member of Θ is included in precisely one member of D_s. This allows us to interpose a new level in Figure 1, as shown in Figure 2, which now represents a 3-level

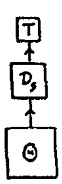

Figure 2. Second Inclusion Hierarchy

inclusion hierarchy. The first level contains only one member. The width of the second level is the same as the <u>dimensionality</u>, and we may call the width of the third level the <u>optionality</u>, these two terms corresponding to the cardinalities of sets identified herein.

Now let there be constructed another inclusion relation based on dependence and interdependence of dimensions. Specifically let a relation $R'(D_s \times D_s)$ be constructed, which induces a partition $\Pi'(D_s)$ on the dimensions, such that

$$\text{i)}\quad \Pi'(D_s) = c_1;\ c_2;\ \ldots;\ c_m$$

$$\text{where ii)}\quad d_i \equiv d_j\quad [\Pi'(D_s)]$$

$$\text{iff}\ (d_i, d_j) \in R'(D_s \times D_s)$$

and this latter condition will be met only if dimensions d_i and d_j are dependent (for more details see [5]).

Then we call the set

$$C = \{c_i\},\ i \in I_m,\ I_m = [1..m],\ m \leq n,$$

the set of <u>clusters</u> associated with Θ. The <u>clusterality</u> that corresponds to Θ is given by $|C|$, the cardinality of set C. The partition $\Pi'(D_s)$ also induces an inclusion relation $R_i(D_s \times D)$ whereby each member of D_s is contained in precisely one member of C. This allows us to interpose an additional level in Figure 2, leading to Figure 3.

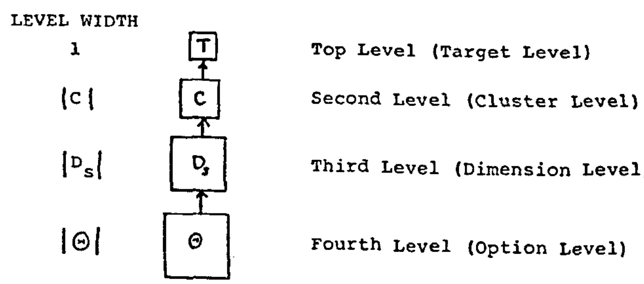

Figure 3. Quad

This inclusion hierarchy in Figure 3 is fundamental to much of the remaining discussion. We call this structure a <u>quad</u>, and denote it Q(T).

With this structure, we can now speak of various properties of the quad. For example, we may call $S = \{T, \Theta, C, D_s\}$ the <u>sets of the quad</u>, and we can speak of the clusterality, dimensionality, and optionality <u>of the quad</u> Q(T). The latter represent the <u>widths of</u> the respective hierarchical levels in Figure 3. Since the width of a given level can never exceed (and will usually be less than) the width of the level below it (if any), we can also portray the quad somewhat more suggestively as a triangle, as shown in Figure 4.

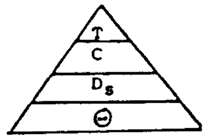

Figure 4. Quad Symbol

We can also introduce the concept of "ports" of a quad. By introducing this concept, we suggest the possibility that there may be multiple targets which may be nested, instead of a single target as supposed up to now. Consider that in Figure 4 the top level consists of a single element, T. If T happens to represent a subsystem of a larger system, we may view the top level in Figure 4 as a port which, when joined to a different quad, may become a part of a larger description. Similarly, with the bottom level having a number of elements, we can say that there are as many ports at the bottom of the quad in Figure 4 as the optionality of the quad, namely $|\Theta|$.

We can also introduce the concept of quad overlap. In contrast to many networks, where an element of one component connects to elements of another component, what we have in quad overlap is a situation where the top level element of a lower quad is identified with and, in effect, welds to or joins with a bottom level element of an upper quad which it overlaps. For this purpose let us adopt the term "knit" as a way to describe the joining of two quads in this way, as illustrated in Figure 5.

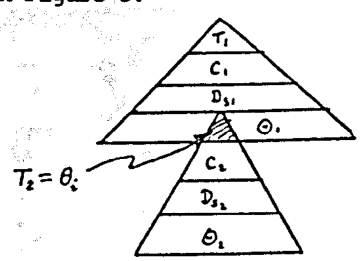

Figure 5. Two Quads Knot Together by the Identification of T_2 with θ_i.

We may call the drawing in Fig. 5 a tapestry of two quads and, more generally, we can say that an arbitrary system can be represented, following this pattern, as a tapestry of quads.

At this point it becomes possible to define the dimensionality of a system. Specifically, it is

$$D = \sum_i D_{si}$$

where the sum is taken over all the quads in the tapestry.

Let us now consider the application of the foregoing ideas. It appears that the application depends on certain key factors:

- The capacity to construct a suitable set of options for each quad
- The capacity to construct a similarity relation for a set of options that provides suitable groupings into dimensions
- The capacity to determine dimensional interdependence, allowing identification of the clusters
- The knowledge of when to close each of the foregoing activities (i.e., availability of "stopping rules")
- The ability to dispense with any information that becomes irrelevant or not useful at any time, in order to avoid being swamped with excessive information

Each of these factors can be dealt with satisfactorily using methods described elsewhere [5].

Acknowledgments

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References

1. John Kemeny, "Saving American Democracy: The Lessons of Three-Mile Island," Technol. Rev. 83(7), 67-75, 1980.

2. N. R. Glass and K. Watt, "Land Use, Energy, Agriculture, and Decision Making," Institute of Ecology, University of California, Davis, May 28, 1971.

3. J. N. Warfield, "Education in Generic Design," Proc. Annual Meeting, Society for General Systems Research, Salinas: Intersystems, May, 1986, H22-H33.

4. ____________, "Developing a Design Culture in Higher Education," Proc. of the Society for General Systems Research, Salinas: Intersystems, 1985, 725-729.

5. ____________, "Organizations and Systems Learning," General Systems, Vol. 27, 1982, 5-74.

6. ____________, "Some Principles of Knowledge Organization," IEEE Trans. Syst., Man & Cybern., June, 1979, 317-325.