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Preliminary Mapping of Natural Language Relationships to a Mathematical Relation Hierarchy

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A primary function of the structural modeling process is the mapping of a general natural language relationship to a specific mathematical relation. This fundamental mapping process is part of the structural integration modeling activities. Structural integration modeling activities and processes are the key to producing valid, reproducible and effective structural modeling outcomes. In this document the logical properties associated with a natural language relationship are chosen as the key properties used to map a given natural language relationship to a mathematical relation. The Augmented Model-Exchange Isomorphism (AMEI) demonstrates the mapping of a natural language relationship (*connected-to*) to a set of 27 logical property groups. Each of these logical property groups indicates a specific type of system-structuring relationship. See document at:
https://www.researchgate.net/publication/272238246_Augmented_Model-Exchange_Isomorphism_Version_11

The categories in the AMEI are documented in Figure 1, Logical Characteristics. One logical element is taken from each of the three columns, and combined into 27 three-element logical groups. These 27 logical groups are shown in Figure 2, Permutations of Relation Properties.

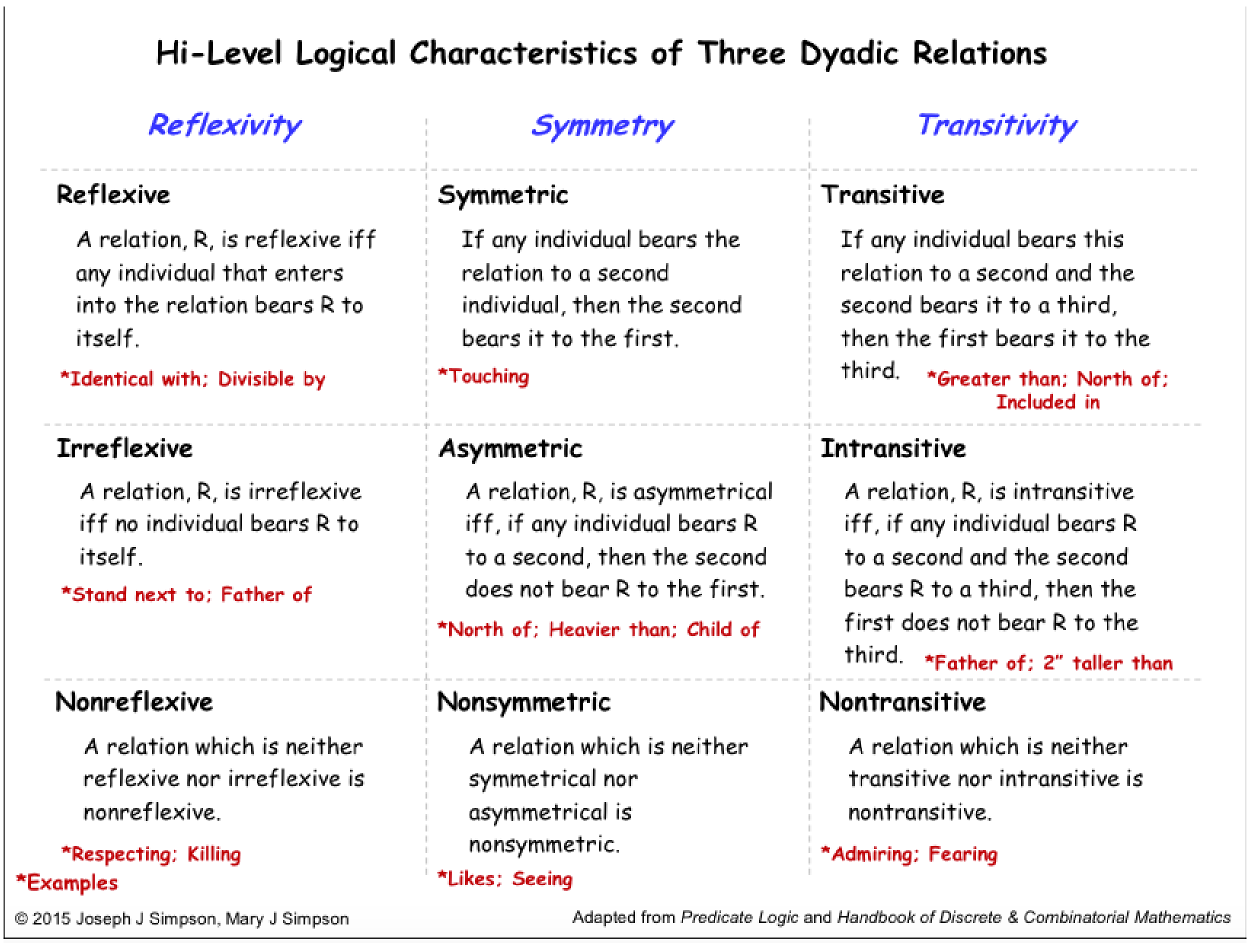

Figure 1. Logical Characteristics.

Permutations of Relation Properties, with Unique Identifiers					
RST-[1,1,1]	**R**eflexive, **S**ymmetric, **T**ransitive	RSI-[1,1,2]	**R**eflexive, **S**ymmetric, **I**ntransitive	RSN-[1,1,3]	**R**eflexive, **S**ymmetric, **N**ontransitive
IST-[2,1,1]	**I**rreflexive, **S**ymmetric, **T**ransitive	ISI-[2,1,2]	**I**rreflexive, **S**ymmetric, **I**ntransitive	ISN-[2,1,3]	**I**rreflexive, **S**ymmetric, **N**ontransitive
NST-[3,1,1]	**N**onreflexive, **S**ymmetric, **T**ransitive	NSI-[3,1,2]	**N**onreflexive, **S**ymmetric, **I**ntransitive	NSN-[3,1,3]	**N**onreflexive, **S**ymmetric, **N**ontransitive
RAT-[1,2,1]	**R**eflexive, **A**symmetric, **T**ransitive	RAI-[1,2,2]	**R**eflexive, **A**symmetric, **I**ntransitive	RAN-[1,2,3]	**R**eflexive, **A**symmetric, **N**ontransitive
IAT-[2,2,1]	**I**rreflexive, **A**symmetric, **T**ransitive	IAI-[2,2,2]	**I**rreflexive, **A**symmetric, **I**ntransitive	IAN-[2,2,3]	**I**rreflexive, **A**symmetric, **N**ontransitive
NAT-[3,2,1]	**N**onreflexive, **A**symmetric, **T**ransitive	NAI-[3,2,2]	**N**onreflexive, **A**symmetric, **I**ntransitive	NAN-[3,2,3]	**N**onreflexive, **A**symmetric, **N**ontransitive
RNT-[1,3,1]	**R**eflexive, **N**onsymmetric, **T**ransitive	RNI-[1,3,2]	**R**eflexive, **N**onsymmetric, **I**ntransitive	RNN-[1,3,3]	**R**eflexive, **N**onsymmetric, **N**ontransitive
INT-[2,3,1]	**I**rreflexive, **N**onsymmetric, **T**ransitive	INI-[2,3,2]	**I**rreflexive, **N**onsymmetric, **I**ntransitive	INN-[2,3,3]	**I**rreflexive, **N**onsymmetric, **N**ontransitive
NNT-[3,3,1]	**N**onreflexive, **N**onsymmetric, **T**ransitive	NNI-[3,3,2]	**N**onreflexive, **N**onsymmetric, **I**ntransitive	NNN-[3,3,3]	**N**onreflexive, **N**onsymmetric, **N**ontransitive

Figure 2. Permutations of Relation Properties.

A basic analysis of the generality of a given natural language relationship can now be performed using the 27 logical property groups. The natural language relationship *connected-to*, can be viewed as very general because this natural language relationship can be mapped to all 27 logical property groups. The natural language relationship *north-of*, can be viewed as very specific because it maps directly to only one logical property group. Creating a rule that categorizes natural language relationships as very specific when they fit into only one logical property group may be useful. Other categorization rules that result in a specific number of logical property groups that can be occupied by a given natural language relationship may be useful in creating sets of natural language relationships that fit into the same number of logical property groups. The relationships in any given set would have a high probability of fitting into the same general type of mathematical relation.

The logical group categorization process would create sets on relationships that could be analyzed to determine exactly (1) what these sets had in common and (2) if common mathematical methods could be used to properly express the system structures created by the sets of relationships. The abstract relation type was developed to support the structured, detailed documentation and communication of the information necessary to support the analysis of these sets of natural language relationships.

A seemingly similar analysis activity, using logical property groups, is performed in the analysis of interoperable computing systems. The following graphic, Figure 3, A Hierarchy of Mathematical Theories, is from John F. Sowa's presentation at the Ontology Summit, 18 February 2016 (Revised 15 March 2016.) A copy of the presentation is available at: http://www.jfsowa.com/talks/interop.pdf A bold, blue line has been added to this graphic by the author to help in the mapping between these systems and the AMEI.

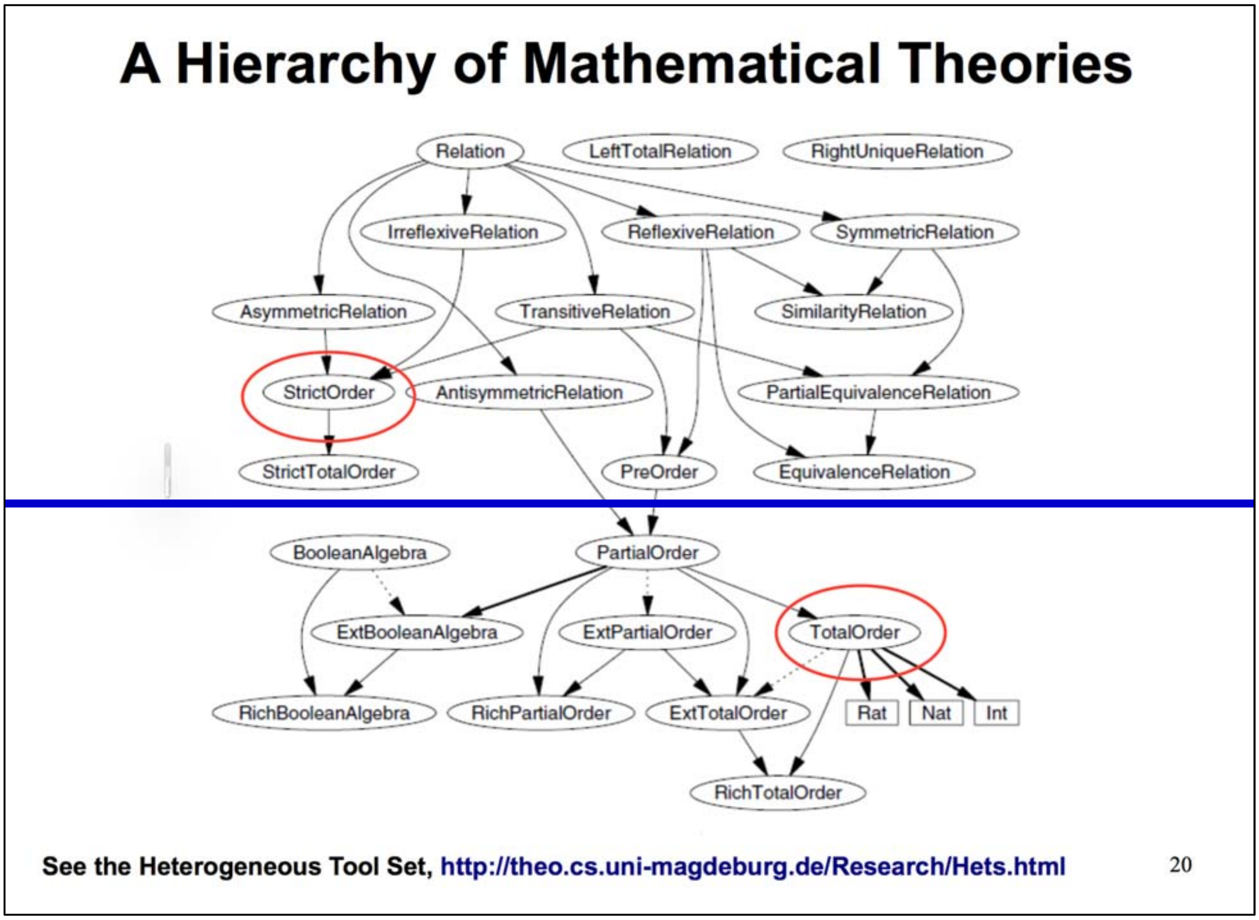

Figure 3. A Hierarchy of Mathematical Theories

In Figure 3, the items (in the ovals) near the top have fewer axioms than the items near the bottom of the figure. This type of hierarchal arrangement presents a very interesting contrast to the AMEI sets of natural language relationships. The Relation oval - on the left at the top - represents any mathematical relation, and is based on a small number of axioms.

If the relationship *connected-to* is evaluated for placement on the hierarchy, then the outcome of this evaluation would place *connected-to* into many ovals. The version of *connected-to* that fits into the Relation oval at the top of the graphic would be very nonspecific. The versions of the connected-to relation would become more specific as the ovals further from the top are addressed.

The *north-of* relationship would fit into only two of the ovals. The difference between the two ovals is not associated with logical properties, but is associated with the contextual factor of the number of objects that are allowed at any level. The ovals above the blue line depend on (1) logical properties of the system structuring relationship and (2) the ordering of sets of equivalence categories. The ovals below the blue line allow for the mixing of equivalence relations and ordering relations in the same representation. The current structural modeling logical property groups include only those relationships found above the blue line. Structured techniques to address constructs below the blue line are in the process of being evaluated in the current structural modeling work.

A number of general factors need to be explored and evaluated to support a more detailed comparison between the permutations of logical properties given in Figure 2 and the configuration of logical properties given in the hierarchy of mathematical theories given in Figure 3. Three of these general factors are:

(1) the scope and content of logical property group;
(2) the role of a partial order; and
(3) the impact of contextual factors.

The scope and content of a logical property group is concerned with the configuration of the logical properties and the required number of logical properties. Can a natural language relationship have only one logical property? In the case of the AMEI, the answer is no. Each natural language relationship has a logical value assigned regarding symmetry, reflexivity and transitivity. Each logical value in the AMEI is selected from a closed set of three options. For example, consider the logical property of symmetry. It can have one of three values: (1) symmetric, (2) asymmetric and (3) non-symmetric. These three values for symmetry cover all instances of logical symmetry and provide full coverage for the needed logical scope of any relationship.

The role of a partial order is different. It includes two different logical property groups as well as a rule set to determine which group of logical properties to apply. Basically, a partial order is composed of two individual logical property groups which effectively transforms the partial order into a compound logical representation. Further, the uncertainty associated with the 'if' statements in a partial order increases the complexity of any operation that contains a partial order.

The general context for a structural modeling activity provides the additional information needed to encode the proper logical semantics in each specific case. When the *north-of* natural language relationship is used to order a set of objects, the specific type of ordering depends on the number of objects allowed at each level in the context. If this contextual problem is transformed to only address the ordering of the levels no matter how many objects are at each level, then the problem is transformed from one with compound logical groups to a problem with a single logical group. This transform reduces uncertainty and therefore complexity.

Each of the logical relation configurations above the blue line need to be identified and their associated logical property groups listed. This information is shown next.

The **Relation Oval** covers all 27 of the logical property groups.

The **Irreflexive Oval c**overs 9 logical property groups: IST, ISI, ISN, IAT, IAI, IAN, INT, INI, and INN.

The **Reflexive Oval** covers 9 logical property groups: RST, RSI, RSN, RAT, RAI, RAN, RNT, RNI, and RNN.

The **Symmetric Oval** covers 9 logical property groups: RST, RSI, RSN, IST, ISI, ISN, NST, NSI, and NSN.

The **Asymmetric Oval** covers 9 logical property groups: RAT, RAI, RAN, IAT, IAI, IAN, NAT, NAI, and NAN.

The **Transitive Oval** covers 9 logical property groups: RST, IST, NST, RAT, IAT, NAT, RNT, INT, and NNT.

The **Similarity Oval** covers 3 logical property groups: RST, RSI, and RSN.

The **Strict Order Oval** covers 1 logical property group: IAT.

The **Strict Total Order Oval** covers 1 logical property group: IAT.

The **Antisymmetric Oval** covers 10 logical property groups: RST, RAT, RAI, RAN, IAT, IAI, IAN, NAT, NAI, and NAN. Need to think about this some more. It is a good idea to create specific rules for inclusion into this category.

The **Partial Equivalence Oval** covers 3 logical property groups: RST, IST, and NST.

The **Equivalence Oval** covers 1 logical property group: RST.

The **PreOrder Oval** covers 3 logical property groups: RST, RAT, and RNT.

Regarding the ovals below the blue line on the mathematical hierarchy chart -

The ovals below the blue line seem to be strongly associated with specific types of constructs found in theoretical computer science.

The Partial Order Oval

The Boolean Algebra Oval

The Extended Boolean Algebra Oval

The Rich Boolean Algebra Oval

The Extended Partial Order Oval

The Total Order Oval

The Extended Total Order Oval

The Rich Total Order Oval

It is interesting to note the absence of some specific single logical properties in the mathematical theories chart. These missing logical properties are:

- Intransitive
- Non-transitive
- Non-symmetric
- Non-reflexive

When a mathematical relation or a natural language relationship is based on a 'compound set' of logical properties, a determination must be made related to when one kind of logical property is applied, rather than the other kind. The contextual information associated with the situation of interest is the main source of data upon which the selection of the correct logical property set is made.

However, there are other issues associated with the use of the mathematical concepts of *antisymmetry* and *partial order*. These issues may be addressed using the basic system structure of hierarchy. The first step in the evaluation of these issues is the discussion of the difference between a sequence and a hierarchy. Len Troncale provided the following insights and information in an email message on May 23rd, 2016.

> "CAN HIERARCHIES EXIST WITHOUT A SEQUENCE? (Joe Simpson)"
>
> **[LT]** "Classical, or early literature on hierarchies often refer to supersystems made up of systems made up of sub-systems. The minimal hierarchy is described as at least three levels (N+1)(N)(N-1). Even in our making of a hierarchical outline, one of the rules is that there should at least be two sub-headings for a heading or it becomes a list, not a hierarchy. So these criteria suggest that a sequence that has the characteristics of subsumption, or as in a structural hierarchy of decomposable subunits, is a hierarchy. But then a simple list is not a hierarchy. The same goes for the other types of hierarchy beyond the structural. In classification hierarchies, there has to be several species for a genus, etc. up the taxonomy. And in control hierarchies, there would have to be several levels of control with numbers of the controlled for each controller. Both classification and control H's then have sequential relations. But clearly just a listing of species is not a taxonomic hierarchy.
>
> Notice also that "sequence" inherent in H's indicates origins and/or development. So there are also development or origins hierarchies (not often included in old literature). My Unbroken Sequence of Origins (ca. 1972) is thus a hierarchy of levels that emerge or come out or unfold from each other. This view has profound ontological and ontogeny implications often missed. But it is an ontology from science-based empirical studies not philosophy or computer science-based history."
>
> "IS A SEQUENCE A PREREQUISITE FOR HIERARCHY? (Joe Simpson)"
>
> **[LT]** "From the above it is assumed (but little in hierarchies are proven empirically) that there has to be some kind of relation between the higher levels and the lower, in fact, consistently across all the levels of a specified hierarchy. This is a way of saying there has to be a "sequence" of relations. But then how is this related to the word "list?" Well, perhaps it depends on the rationale for the listing. It may or may not contain some of the features of hierarchy.
>
> To date, in my research papers for SPT, I have not included "sequence" per se as an Identifying Feature. Perhaps I should. Certainly I should discuss these questions in my upcoming book. I have published and presented at ISSS conferences, several papers on empirically testing the concept of structural (and therefore origin) hierarchies in biological systems from molecular to ecosystemic using real data from those disciplines statistically analyzed and tested for inherent clustering (that means testing that the levels are real in nature and not just human invention or classification). These are (will be) included in my personal website; look under hierarchies button or systems allometry button.
>
> It might also be useful to pin down what we usually mean by "sequence." ((unfortunately, many of the words we use in systems, and especially for putative isomorphs, also have popular meanings (such as sequence, flows, hierarchies, chaos, cycles, etc. and these confound our dialogues))."

Given the restrictions of set theory, a hierarchy is a sequence with a varying (increasing?) number of elements in the equivalence classes that are being ordered. The concept of partial order greatly restricts the number of candidate logical property groups that may be assigned to any given hierarchical system structure. The augmented model-exchange isomorphism greatly expands the number of allowable logical property groups that may be assigned to this type of system structure.

Systems engineering and systems science research appears to have a difficult time in establishing and reusing basic foundational concepts and ideas. In the realm of computer science, the problem has been described as:

> "Indeed, one of my major complaints about the computer field is that whereas Newton could say, "If I have seen a little farther than others, it is because I have stood on the shoulders of giants," I am forced to say, "Today we stand on each other's feet." Perhaps the central problem we face in all of computer science is how we are to get to the situation where we build on top of the work of others rather than redoing so much of it in a trivially different way. Science is supposed to be cumulative, not almost endless duplication of the same kind of things".
>
> Richard Hamming 1968 Turning Award Lecture

In the area of systems engineering and systems science, it could be said that "Today we deny the fact that others have feet or ground to stand on."