(Preface -- Part 3 - Structuring Complex Systems - Warfield 1974)

SUMMARY

The problem considered in this monograph may he described as one of finding useful graphical descriptions of single contextual relations germain to a system. The problem is addressed as one of developing a process that minimizes the logistical burden on the developer of the product. The product is called an interpretive structural model. The latter is structurally isomorphic to a digraph, but contains both substantive and structural information supplied by the developer. All of the mathematical operations required by the process would be carried out by a computer. The critical, substantive system knowledge is supplied by the developer, The partnership of developer and computer gives the process its strength.

By a system is understood a complex of entities S with interactions R. Knowledge concerning (S, R) can he structural or substantive or both, and is the consequence of perceptions of the system. Some knowledge of the system is presumed to be available. Total knowledge of a system is thought normally to be unattainable. A description of a system is approximate.

Let <S> be represented by a finite set S = {s1, s2, ... sn} and let <R> be represented by a finite set <R> = {<R1>, <R2>, ... <Rm>}. Call si an element of the system, and call <Rj> a contextual relation of the system. A contextual relation is a phrase in a colloquial language.

Let <Rj>, be represented by Rj, where Rj is a binary relation on S x S. Let Aij be a binary matrix indexed by (S, S) which portrays both Rj and its complement, not Rj. Let Mij be the transitive closure of Aij + I, where I is the identity matrix.

Then, there exists a permutation matrix P such that the matrix Mj = (1/P)MijP induces a unique partition on S into subsets called levels, and each submatrix of Mj indexed by a level is symmetric.

For any submatrix indexed by a level, there exists a permutation such that its block diagonal submatrices form a minimum set of universal matrices. Each nontrivial index set of one or more of these is called a maximal cycle set. If an index set contains only one element, the set is trivial and the element is an isolate within the level, though it may connect. to other levels.

If appropriate modifications are made to Mj, it can be converted to a matrix M'j' such that every maximal cycle set is replaced by a proxy element, and reachability is preserved by following a principle of condensation.

The digraph of the condensed matrix is a hierarchy with the same number of levels as Mij and Mj. There exists a binary matrix Aj for which M'j' is the transitive closure, called a skeleton matrix, such that there is no other matrix less than Aj that has M'j' as its transitive closure. The digraph D(Aj) is called the skeleton digraph of Mij. It is the minimum-edge digraph that preserves condensed reachability or Aij.

To every maximal cycle set, there corresponds a minimum-edge digraph that preserves reachability within the cycle set. If every proxy vertex on D(Aj) is replaced with such a minimum edge digraph, the resulting digraph is a minimum-edge approximation of D(Aij).

Because the matrix Aij is often difficult to develop, the matrix Mij is developed instead. In developing Mij transitivity is enforced. While it is impossible to determine Aij from Mij, it is possible to determine Aj.

Since a minimum-edge representation of a maximal cycle is often an inadequate approximation to the structure of a maximal cycle, additional information concerning the structure of the cycle may be sought. Such structure is based upon a weighting matrix W for the maximal cycle, wherein the intensity of some contextual relation is supplied on a specified scale. From such a weighting matrix, it is possible to compute a binary matrix that represents a threshold of intensity and preserves reachability within the maximal cycle. This binary matrix permits construction of a reachability-preserving digraph for the maximal cycle which normally has fewer edges than a complete graph for the cycle.

If those digraphs that result from application of an intensity threshold to weighting matrices of the various maximal cycles are substituted for the proxy elements in the skeleton digraph D(Aij) while preserving overall reachability, it is believed a good approximation to the system structure results insolar as the relation Rj is concerned. The digraph so formed may be complicated by the presence of many vertexes, edges, and crossings of the edges. If so, additional interpretive digraphs may be useful.

A maximal geodetic cycle specified on (si, sj) is a maximal cycle consisting of a pair of (geodetic) paths, one path originating at vertex si and terminating at sj, the other originating at sj and terminating at si. It is possible to compute, for a given matrix Aij, a set of maximal geodetic cycles. To each maximal geodetic cycle there corresponds a cycle set, which is a subset of S. The set of all such cycle