Cybernetics And Human Knowing. Vol. 10, nos. 3-4, pp. xx-xx

Eigenforms — Objects as Tokens for Eigenbehaviors

by Louis H. Kauffman[1]

This essay is a discussion of Heinz von Foerster's concept of an eigenform, wherein an object is seen to be a token for those behaviors that lend it (the object) its apparent stability in a changing world.

I. Introduction

This essay is a contemplation of the notion of eigenform as explicated by Heinz von Foerster in his paper [4]. In that paper Heinz performs the magic trick of convincing us that the familiar objects of our existence can be seen to be nothing more than tokens for the behaviors of the organism that create stable forms. This is not to deny an underlying reality that is the source of these objects, but rather to emphasize the role of process and the role of the organism in the production of a living map that is so sensitive that map and territory are conjoined. Von Foerster's papers [4,5,6] in the book [3] were instrumental in pioneering the field of second order cybernetics.

The notion of an eigenform is inextricably linked with second order cybernetics. One starts on the road to such a concept as soon as one begins to consider a pattern of patterns, the form of form or the cybernetics of cybernetics. Such concepts appear to close around upon themselves, and at the same time they lead outward. They suggest the possibility of transcending the boundaries of a system from a locus that might have been within the system until the circular concept is called into being, and then the boundaries have turned inside out. As Ranulph Glanville has pointed out "The inside is the outside is the inside is the..."

Forms are created from the concatenation of operations upon themselves and objects are not objects at all, but rather indications of processes. Upon encountering an object, after that essay of Heinz, you are compelled to ask: How is that object created? How is it designed? What do I do to produce it? What is the network of productions? Where is the home of that object? In what context does it exist? How am I involved in its creation?

Taking Heinz's suggestion to heart we find that an object in itself is a symbolic entity, participating in a network of interactions, taking on its apparent solidity and stability from these interactions. We ourselves are such objects, we as human beings are "signs for ourselves," a concept originally due to the American philosopher C. S. Peirce [9]. In many ways Heinz's eigenforms are mathematical companions to Peirce's work. We will not follow this comparison in the present essay, but the reader familiar with Peirce is encouraged to do so.

---

1. Math UIC, Chicago, IL. Email: kauffman@uic.edu.

Heinz performed a creative act that invites each of us into an unending epistemological investigation. The key to this act is the stance of an observing system. In an observing system, what is observed is not distinct from the system itself, nor can one make a separation between the observer and the observed. These stand together in a coalescence of perception. From the stance of the observing system all objects are non-local, depending upon the presence of the system as a whole. It is within that paradigm that these models begin to live, act and converse with us. We are the models. Map and territory are conjoined.

In this paper, sections 2 and 3 discuss the nature of object, leading from the descriptions and assumptions that we use in the everyday world to the somewhat different concepts of object that occur in scientific work, particularly in the physics of the very small. We indicate how the concept of object arising as eigenform (i.e. as token for the interaction and mutual production of the observer and the observed) intermediates among these points of view.

Section 4 gives an exposition of the form of Heinz's model. Once this mathematical model is on the table, one can discuss how it is related to our concept of "object." This section contains examples of fractal and geometric eigenforms.

Section 5 is a recounting of a conversation of the author and Ranulph Glanville in which Ranulph asked "Does every recursion have a fixed point?" In the language of the present essay this is the question "Does every process have an eigenform?" The obvious answer is no, but the answer that comes from Heinz's model is yes! There is always an ideal eigenform. The challenge is to integrate that form into the context of one's living.

In section 6 we point out that the construction of eigenforms in the sense of Heinz's model can be accomplished without an idealized excursion to infinity. The method was invented by Alonzo Church and Haskell Curry in the 1930's. This method is commonly called the "lambda calculus." The key to lambda calculus is the construction of a self-reflexive language, a language that can refer and operate upon itself. In this way eigenforms can be woven into the context of languages that are their own metalanguages, hence into the context of natural language and observing systems.

In section 7 we consider naming and self-reference and return to Heinz's definition of "I." The concept of section 7 is essentially related to the lambda calculus where names can act on names and their referents. We discuss how self-reference occurs in language through an indicative shift welding the name of a person to his/her (physical) presence and shifting the indication of the name to a metaname. More could be said at this point, as the indicative shift is a linguistic entry into the world of Godelian sentences and the incompleteness of formal systems. We emphasize the natural occurrence of eigenforms in the world of our linguistic experience and how this occurrence is intimately connected to our structure as observing systems.

Section 8 is an appendix discussing the relationship between eigenforms and the structure of quantum mechanics. Eigenvectors in quantum mechanics are, in this view, particular examples of eigenforms. From the world of observing systems, quantum mechanics is a mathematical articulation of the non-locality and superposition of forms. This appendix will be unfolded in a sequel to this paper.

II. Objects

What is an object? At first glance, the question seems perfectly obvious. An object is, well... An object is a thing, a something that you can pick up and move and manipulate in three dimensional space. An object is three dimensional, palpable, like an apple or a chair, or a pencil or a cup. An object is the simplest sort of thing that can be subjected to reference. All language courses first deal with simple objects like pens and tables. La plume est sur la table.

An object is separate from me. It is “out there”. It is part of the reality separate from me. Objects are composed of objects, their parts. My car is made of parts. The chair is a buzzing whirl of molecules. Each molecule is a whirl of atoms. Each atom a little solar system of electrons, neutrons and protons. But wait! The nucleus of the atom is composed of strange objects called quarks. No one can see them. They do not exist as separate entities. The electrons in the atom are special objects that are not separate from each other and from everything else. And yet when you observe the electrons, they have definite locations.

The physicist's world divides into quantum objects that are subject to the constraints of the uncertainty principle, and classical objects that live in the dream of objective existence, carrying all their properties with them. The difference between the quantum level of objects and the classical level of objects is actually not sharp. From the point of view of the physicist all phenomena are quantum phenomena, but in certain ranges, such as the world of the very small, the quantum effects dominate. It is not the purpose of this essay to detail this correspondence, but a little more information can be found in section 8.

A classical object has a location at a given time. You can tell where it is. You can tell a story of where it has been. If the classical object breaks up into parts, you will be able to keep track of all the parts. Yet electrons and positrons can meet each other and disappear into pure energy! Should we allow objects to disappear? What sort of an object is the electromagnetic field of radio and television signals that floods this room?

Is my thought to be thought of as an object? Can I objectify my thought by writing it down on paper or in the computer? Am I myself an object? Is my body an object in the three dimensional space? Is the space itself an object? Objects have shape. What is the shape of space? What is the shape of the physical universe. What is the shape of the Platonic universe?

It seemed simple. Then, with more experience, the transformations of pattern that formed the space and the objects in it began to appear highly interwoven. In the physical microworld objects, if they were objects at all, did not have many of the properties of macroscopic objects like heads of cabbage and bowling balls. Give me a good macroscopic object any day, fully separate and useful. Don't confuse me with these subatomic fantasies of interconnectedness.

If a person (a thought, feeling and symbol object) were to read this section with the hope of finding a clear definition of object, he/she might be disappointed. Yet Heinz von Foerster in his essays [3,4,5,6] has suggested the enticing notion that "objects are tokens for eigenbehaviors." We want to see the meaning of this phrase of Heinz. The short form of this meaning is that there is a behavior between the perceiver and the object perceived and a stability or repetition that "arises between them." It is this stability that constitutes the object (and the perceiver). In this view, one does not really have any separate objects, objects are always "objects perceived," and the perceiver and the perceived arise together in the condition of observation.

III. Shaping a World

We, identify the world in terms of how we shape it. We shape the world in response to how it changes us. We change the world and the world changes us. Objects arise as tokens of that behavior that leads to seemingly unchanging forms. Forms are seen to be unchanging through their invariance under our attempts to change, to shape them.

Can you conceive of an object independent of your ability to perceive it? I did not say an object independent of your perception.

Lets assume that it is possible to talk of the tree in the forest where we are not. But how are we to speak of that tree? One can say, the tree is there. What does this mean? It means that there is a potentiality for that tree to appear in the event of the appearance of a person such as myself or yourself in the place called that forest. What is the tree doing when I am not in the forest?

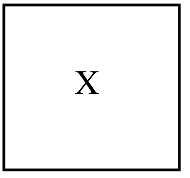
I will never know, but I do know that "it" obediently becomes treeish and located when "I" am "there." The quotation marks are indications of objects dissolving into relationships. Whenever "I" am present, the world (of everything that is the case) is seen through the act of framing. I imagine a pure world, unframed. But this is the world of all possibilities. As soon as we enter the scene the world is filtered and conformed to become the form that frame and brain have consolidated to say is reality.

IV. Heinz's Eigenform Model

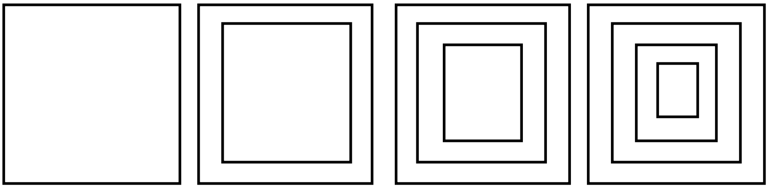
We have just moved through a discussion showing how the concept of an object has evolved to make what we call objects (and the objective world) processes that are interdependent with the actions of observers. The notion of a fixed object has changed to become a notion of a process that produces the apparent stability of the object. This process can be simplified in a model to become a recursive process where a rule or rules are applied time and time again. The resulting object of such a process is called by Heinz von Foerster the eigenform of the process, and the process itself is called the *eigenbehavior.*

In this way Heinz created a model for thinking about object as token for eigenbehavior. This model examines the result of a simple recursive process carried to its limit. For example, suppose that:

F(X) =



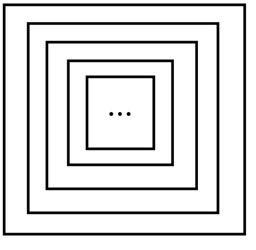
That is, each step in the process encloses the results of the previous step within a box. Here is an illustration of the first few steps of the process applied to an empty box X:



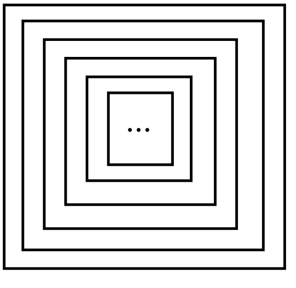
X F(X) F(F(X)) F(F(F(X)))

If we continue this process, then successive nests of boxes resemble one another, and in the limit of infinitely many boxes, we find that

X=F(F(F(...)))=



F(X) =



= X

the infinite nest of boxes is invariant under the addition of one more surrounding box. Hence this infinite nest of boxes is a fixed point for the recursion. In other words, if X denotes the infinite nest of boxes, then

$$X = F(X).$$

This equation is not meant to denote something arcane! It is a description of a state of affairs. Place your gaze on the infinite nest of boxes and note that if you add one more surrounding box, then there is no change to the resulting form. The form of an infinite nest of boxes is invariant under the operation of adding one more surrounding box. The infinite nest of boxes is one of the simplest eigenforms.

In the process of observation, we interact with ourselves and with the world to produce stabilities that become the objects of our perception. These objects, like the infinite nest of boxes, may go beyond the specific properties of the world in which we operate. They attain their stability through this process of going outside the immediate world. Furthermore, we make an imaginative leap to complete such objects that become tokens for eigenbehaviors. It is impossible to make an infinite nest of boxes. We do not make it. We *imagine* it. And in imagining that infinite next of boxes, we arrive at the eigenform that is the object for this process.

The leap of imagination to the infinite eigenform is akin to the human ability to create signs and symbols. In the case of the eigenform **X** with **X = F(X)**, **X** can be regarded as the name of the process itself or as the name of the limiting (imaginative) result of the process. Note that if you are told that

X = F(X),

then substituting **F(X)** for **X**, you can write

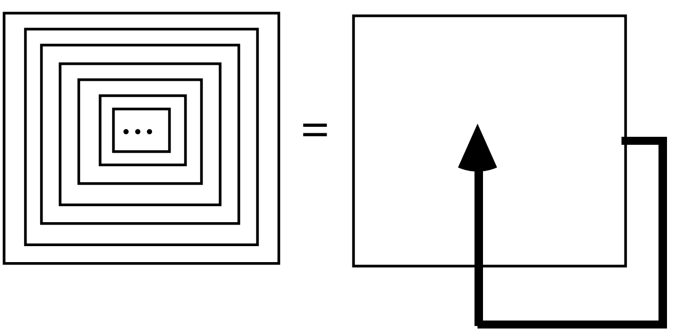
X = F(F(X)).

Substituting again and again, you have

X = F(F(F(X))) = F(F(F(F(X)))) = F(F(F(F(F(X))))) = ...

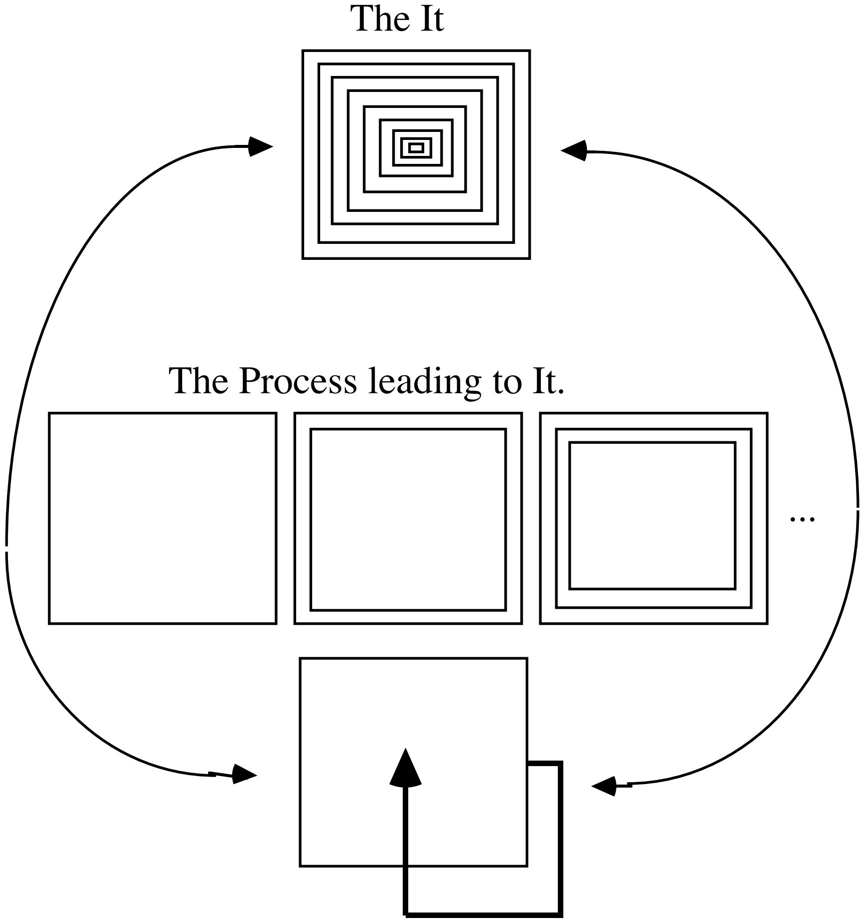
The process arises from the symbolic expression of its eigenform. In this view the eigenform is a kind of implicate order for the process that generates it.

Sometimes one stylizes the structure by indicating where the eigenform **X** reenters its own indicational space by an arrow or other graphical device. See the picture below for the case of the nested boxes.



Does the infinite nest of boxes exist? Certainly it does not exist in this page or anywhere in the physical world with which the writer or presumably the reader is familiar. The infinite nest of boxes exists in the imagination! In that sense it is a symbolic entity.

The key concept in the understanding of eigenform is its placement in the reciprocal relationship of the object (the "It") and the process leading to the object (the process leading to "It"). In the diagram below we have indicated these relationships with respect to the eigenform of nested boxes. Note that the "It" is illustrated as a finite approximation (to the infinite limit) that is sufficient to allow an observer to infer/perceive the generating process that underlies it.



Just so, an object in the world (cognitive, physical, ideal,...) provides a conceptual center for the investigation of a skein of relationships related to its context and to the processes that generate it. An object can have varying degrees of reality just as does an eigenform. If we take Heinz's suggestion to heart that objects are tokens for eigenbehaviors, then an object in itself is a entity, participating in a network of interactions, taking on its apparent solidity and stability from these interactions. An object in this view is an amphibian between the symbolic and imaginary world of the mind and the complex world of personal experience. The object when viewed as process is a dialogue between these worlds. The object when seen as a sign for itself, or in and of itself, is as imaginary as a pure eigenform.

Why are objects only apparently solid? Of course you cannot walk through a brick wall even if you think about it differently. I do not mean apparent in the sense of thought alone. I mean apparent in the sense of appearance. The wall appears solid to me because of the actions that I can perform. The wall is quite transparent to a neutrino, and will not even be an eigenform for that neutrino.
This example shows quite sharply how the nature of an object is entailed in the properties of its observer.

Heinz's model can be expressed (as indeed he did express it) in quite abstract and general terms. Suppose that we are given a recursion symbolically by the equation

$$\mathbf{X(t+1) = F(X(t))}.$$

Here **X(t)** denotes the condition of observation at time **t**. **X(t)** could be as simple as a set of nested boxes, or as complex as the entire configuration of your body in relation to the known universe at time t. Then **F(X(t))** denotes the result of applying the operations symbolized by **F** to the condition at time **t**. You could, for simplicity, assume that **F** is independent of time. Time independence of the recursion **F** will give us simple answers and we can later discuss what will happen if the actions depend upon the time. In the time independent case we can write

$$\mathbf{J = F(F(F(...)))}$$

the infinite concatenation of F upon itself. We then see that

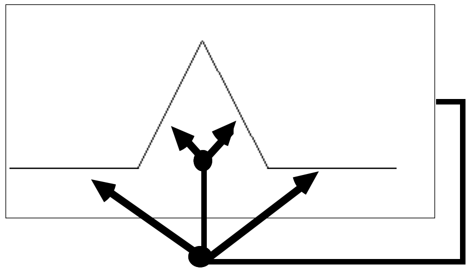
$$\mathbf{F(J) = J}$$

since adding one more **F** to the concatenation changes nothing. Thus **J**, the infinite concatenation of the operation upon itself leads to a fixed point for **F**. **J** is said to be the eigenform for the recursion **F**. It is just like the nested boxes, and we see that every recursion has an eigenform. Every recursion has an (imaginary) fixed point.

We end this section with one more example. This is the eigenform of the Koch fractal [10]. In this case one can write symbolically the eigenform equation

$$\mathbf{K = K\ \{\ K\ \ K\ \}\ K}$$

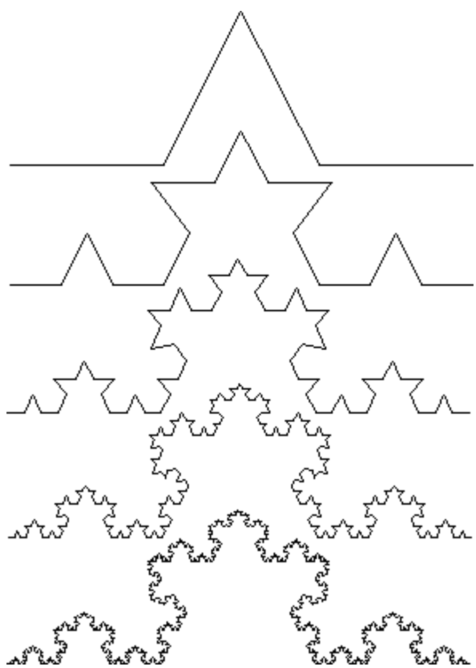
to indicate that the Koch Fractal reenters its own indicational space four times (that is, it is made up of four copies of itself, each one-third the size of the original. The curly brackets in the center of this equation refer to the fact that the two middle copies within the fractal are inclined with respect to one another and with respect to the two outer copies. In the figure below we show the geometric configuration of the reentry.



K = K { K K } K

In the geometric recursion, each line segment at a given stage is replaced by four line segments of one third its length, arranged according to the pattern of reentry as shown in the figure above.

The recursion corresponding to the Koch eigenform is illustrated in the next figure. Here we see the sequence of approximations leading to the infinite self-reflecting eigenform that is known as the Koch snowflake fractal.



Five stages of recursion are shown. To the eye, the last stage vividly illustrates how the ideal fractal form contains four copies of itself, each one-third the size of the whole. The abstract schema

K = K { K K } K

for this fractal can itself be iterated to produce a “skeleton” of the geometric recursion:

K = K { K K } K
= K { K K } K { K { K K } K K { K K } K } K { K K } K
= ...

We have only performed one line of this skeletal recursion. There are sixteen K's in this second expression just as there are sixteen line segments in the second stage of the geometric recursion. Comparison with this abstract symbolic recursion shows how geometry aids the intuition. The interaction of eigenforms with the geometry of physical, mental, symbolic and spiritual landscapes is an entire subject that is in need of deep exploration.

It is usually thought that the miracle of recognition of an object arises in some simple way from the assumed existence of the object and the action of our perceiving systems. What is to be appreciated is that this is a fine tuning to the point where the action of the perceiver and the perception of the object are indistinguishable. Such tuning requires an intermixing of the perceiver and the perceived that goes beyond description. Yet in the mathematical levels, such as number or fractal pattern, part of the process is slowed down to the point where we can begin to apprehend the process. There is a stability in the comparison, in the one-to-one correspondence that is a process happening at once in the present time. The closed loop of perception occurs in the eternity of present individual time. Each such process depends upon linked and ongoing eigenbehaviors and yet is seen as simple by the perceiving mind.

V. A Conversation with Ranulph Glanville

This essay has its beginnings in a conversation with Ranulph Glanville. Ranulph asked “Does every recursion have a fixed point?" hoping for a mathematician's answer. And I said first, “Well no, clearly not, after all it is common for processes to go into oscillation and so never come to rest.” And then I said, “On the other hand, here is the

Theorem: Every recursion has a fixed point.

Proof. Let the recursion be given by an equation of the form

$$\mathbf{X' = F(X)}$$

where **X'** denotes the next value of **X** and **F** encapsulates the function or rule that brings the recursion to its next step. Here **F** and **X** can be any descriptors of actor and actant that are relevant to the recursion being studied. Now form

$$\mathbf{J = F(F(F(F(...)))),}$$

the infinite concatenation of F upon itself. Then we see that

$$F(J) = F(F(F(F(F(...))))) = J.$$

Hence J is a fixed point for the recursion and we have proved that every recursion has a fixed point.//

Ranulph said “Oh yes I remember that! Can I quote your proof?” and I said “Certainly, but you will have to make your attribution to Heinz and his paper ‘Objects: Tokens for (Eigen-)Behaviors’ [4], for that is where I came to appreciate this result, although I first understood it via the book *Laws of Form* [2] by G. Spencer-Brown.”

And I went on to say that this theorem was in my view a startling magician's trick on Heinz's part, throwing us into the certainty of an eigenform (fixed point) corresponding to any process and at the same time challenging us to understand the nature of that fixed point in some context that is actually relevant to the original ground of conversation. Ranulph agreed, and our emails settled back into the usual background hum.

VI. Church and Curry

Alonzo Church and Haskell Curry [1] showed (in the 1930's, long before Heinz wrote his essays) how to make eigenforms without an apparent excursion to infinity. Their formalism is usually called the “lambda calculus.” The key concept in their work is the use of a domain where there is (in a well-defined sense) no distinction between the language and metalanguage. They use a language that can talk about itself and operate upon itself.

In the Church-Curry language (the lambda calculus), there are two basic rules: Naming and Reflexivity.

1. Naming.
If you have an expression in the symbols in lambda calculus then there is always a single word in the language that encodes this expression. The application of this word has the same effect as the application of the expression itself. For example, suppose we consider the expression

“Form the square of a number N and add one.”

Then in the lambda calculus there will be a word, lets say it is “**Squatch**” that has the same effect as this operation. So “**Squatch N**” means “square N and add one.” We have:

Squatch 3 = 3x3 + 1 = 10.

2. Reflexivity.
*Given any two words **A** and **B** in the lambda calculus, there is permission to form their concatenation **AB**, with the interpretation that **A** operates upon or qualifies **B***. In this way, every word in the lambda calculus is both an operator and an operand. The calculus is inherently self-reflexive.

We have given an example of reflexivity with the equation **Squatch N = NxN + 1**. But lambda calculus will also allow

Squatch Squatch = Squatch x Squatch + 1.

One then has to ask what it means to multiply Squatch by itself, but the language allows you to write this equation and then ask the question!

Here is an example. Let **GA** denote the process that creates two copies of **A** and puts them in a box.

$$GA = \boxed{AA}$$

Then in lambda calculus we are allowed to apply G to itself. The result is two copies of G next to one another, inside the box.

$$GG = \boxed{GG}$$

This equation about GG exhibits GG directly as a solution to the eigenform equation

$$X = \boxed{X}$$

thus producing the eigenform without an infinite limiting process.

More generally, we wish to find the eigenform for a process **F**. We want to find a **J** so that **F(J) = J**. Church and Curry admonish us to create an operator **G** with the property that

GX = F(XX)

So **GG** is a fixed point for **F**.

We have solved the eigenvalue problem without the customary ritual excursion to infinity. If you reflect on this magic trick of Church and Curry you will see that it has come directly from the postulates of Naming and Reflexivity that we have discussed above. These notions, that there should be a name for everything, and that words can be applied to the description and production of other words, allow the language to refer to itself and to produce itself from itself. The Church-Curry construction was devised for mathematical logic, but it is fundamental to the logic of logic, the linguistics of linguistics and the cybernetics of cybernetics.

I like to call the construction of the intermediate operator **G**, the "gremlin" (See [9].) Gremlins seem innocent enough. They just duplicate entities that they meet, and set up an operation of the duplicate on the duplicand. But when you let a gremlin meet a gremlin then strange things can happen. It is a bit like the story of the sorcerer's apprentice. A recursion may happen whether you like it or not.

A formal eigenform must be placed in a context in order for it to have human meaning. The struggle on the mathematical side is to control recursions, bending them to desired ends. The struggle on the human side is to cognize a world sensibly and communicate well and effectively with others. For each of us, there is a continual manufacture of eigenforms (tokens for eigenbehavior). Such tokens will not pass as the currency of communication unless we achieve mutuality as well. One can say that mutuality itself is a higher eigenform. Achieving mutual understanding will be recognized. As with all eigenforms, the abstract version exists. Realization happens in the course of time.

VII. The Form of Names

The simplicity of a thought, the apparent clarity of distinction is mirrored in the sort of eigenforms that come from the Church-Curry realm as described in the last section. Consider a linguistic example: Each person has a name (at least one). In the course of time we are introduced to people and come to know their names. We know that name not as an item to look up about the person (and this applies to certain objects as well) but as a direct property of the person. That is, if I meet Heinz he appears to me as Heinz, not as this person with certain characteristics, whose name I can find in my social database if I care to do so. It is like this only when we are first introduced. At the point of introduction there is this person and there is his name separate from him. Once learned, *the name is shifted* and occurs in space right along with the person. Heinz and his name are in the same cognitive space which is also in the same place as the apparent physical space. We can observe this shifting process in the course of learning a name. We can also observe how physical and cognitive spaces are superimposed. The many classical optical illusions illustrate these matters vividly.

Now we have Heinz with his name inseparable from his presence, and this is true even if he is not physically present, for the shift has occurred and will not be undone. But we also have his name Heinz separate from him, and able to be pinned upon another. And we have his name not quite separate from him, but rather this Heinz is the name of the name we have attached to him! This is Heinz's metaname. How do we distinguish among all these different names for Heinz? We use the same symbols for them, yet they are different. Lets choose a way to indicate the differences.

We start with the reference:

Heinz -----> Cybernetic Magician

(The arrow will indicate that the entity on the left is the name of the entity on the right.)

We get to know him and shift the reference.

#Heinz -----> Cybernetic Magician Heinz

Now the name is in the cognitive space of Heinz, and the metaname #Heinz refers to that conjunction. We shall call this **the indicative shift**.

name -----> object
#name-----> object name

The indicative shift occurs, constantly weaving the apparent external reality with the linguistic reality. *Self-reference occurs when one calls up (names) the metanaming operator.*

At first the metanaming operator is not marked and no name has been chosen for it. But then its name is chosen (as #). We have

 # ----->

That is, # refers at first to the singular place where there is an absence of naming, a void in the realm of distinctions.

Then the shift occurs. We have the reference of the meta-naming operator to itself (as the operator enters a space formerly void!).

 # -----> #

Suppose that the meta-naming operator has another name, say M. Then we have

M -----> #

which shifts to a self-reference at the second articulated level of meta naming

#M -----> #M

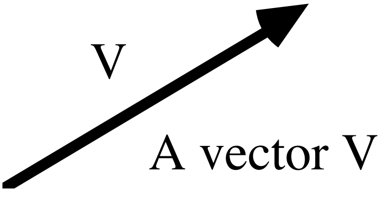
These are the eigenforms of self consciousness in the realm of names.

Heinz said [5]: **"I am the observed link between myself and observing myself."** Self-reference at this level, the action of a domain upon itself, leading to cognition, is the beginning of the realm of eigenforms in Heinz's world.

VIII. Appendix - Eigenform, Eigenvalue and Quantum Mechanics

There are two reasons for including a discussion of quantum mechanics in this essay. On the one hand the quantum mechanics of the twentieth century has been a powerful force in asking us to rethink our notions of objects and causality. On the other hand, Heinz von Foerster's notion of eigenform is an outgrowth of his background as a quantum physicist. We should ask what eigenforms might have to do with quantum theory and with the quantum world.

In this section we meet the concurrence of the view of object as token for eigenbehavior and the observation postulate of quantum mechanics. In quantum mechanics observation is modeled not by eigenform but by its mathematical relative the eigenvector. The reader should recall that a vector is a quantity with magnitude and direction, often pictured as an arrow in the plane or in three dimensional space.



In quantum physics [11], the state of a physical system is modeled by a vector in a high-dimensional space, called a Hilbert space. As time goes on the vector rotates in this high dimensional space. Observable quantities correspond to (linear) operators **H** on these vectors **v** that have the property that the application of **H** to **v** results in a new vector that is a multiple of **v** by a factor λ.

(An operator is said to be linear if H(av +w) = aH(v) + H(w) for vectors v and w, and any number a. Linearity is usually a simplifying assumption in mathematical models, but it is an essential feature of quantum mechanics.) In symbols this has the form:

$$Hv = \lambda v.$$

One says that v is an eigenvector for the operator H, and that λ is the eigenvalue. The constant λ is the quantity that is observed (for example the energy of an electron). These are particular properties of the mathematical context of quantum mechanics. The λ can be eliminated by replacing H by G = H/λ (when λ is non zero) so that

$$Gv = (H/\lambda)v = (Hv)/\lambda = \lambda v/k = v.$$

Thus

$$Gv = v.$$

In quantum mechanics observation is founded on the production of eigenvectors **v** *with* **Gv=v** *where* **v** *is a vector in a Hilbert space and* **G** *is a linear operator on that space.*

Many of the strange and fascinating properties of quantum mechanics emanate directly from this model of observation. In order to observe a quantum state, its vector is projected into an eigenvector for that particular mode of observation. By projecting the vector into that mode and not another, one manages to make the observation, but at the cost of losing information about the other possibilities inherent in the vector. This is the source, in the mathematical model, of the complementarities that allow exact determination of the position of a particle at the expense of nearly complete uncertainty about its momentum (or vice versa the determination of momentum at the expense of knowledge of the position).

Observation and quantum evolution (the determinate rotation of the state vector in the high dimensional Hilbert space) are interlocked. Each observation discontinuously projects the state vector to an eigenvector. The intervals between observations allow the continuous evolution of the state vector. This tapestry of interaction of the continuous and the discrete is the basis for the quantum mechanical description of the world.

Heinz was certainly aware, as a practicing physicist, of this model of observation in quantum theory. His theory of eigenforms is a sweeping generalization of quantum mechanics that creates a context for understanding the remarkable effectiveness of that theory. If indeed the world of objects is, in fact, a world of tokens for eigenbehaviors, and if physics demands forms of observations that give numerical results, then a simplest example of such observation is the observable in the quantum mechanical model.

This is a reversal of epistemology, a complete turning of the world upside down. Heinz has tricked us into considering the world of our experience and finding that it is our world, generated by our actions and that it has become objective through the self-generated stabilities of those actions. He has convinced us to come along with him and see that all of cybernetics confirms this point of view. He has left the corollaries to us. He has not confronted the physicists and the philosophers head on. He has brought us into his world and let us participate in the making of it. And he has pointed to the genesis and tautological nature of quantum theory to those of us who might ask the question.

But he has also left the consequence of the question to us. For if the world is a world of eigenforms and most of them are in time oscillatory and unstable, must we insist on stability at the level of our present perception of that world? In principle, there is an eigenform, but that form leads always outward into larger worlds and new understanding. In the case of quantum mechanics, the whole theory has the appearance of an elementary exercise confirming the view point of objects as tokens for eigenbehaviors in a special case. Heinz leaves us with the conundrum of finding the more general physical theory that confirms that special case.

This dilemma is itself a special case of the dilemma that Heinz has given us. He said it himself many times. If you give a person an undecidable problem, the action of that person in attempting to solve the problem reveals the identity of the person and the nature of his/her creativity

References

[1] Barendregt, H. P. (1985). *The lambda calculus - Its syntax and semantics*. New York: North Holland Pub. (Originally published in 1981)

[2] Spencer-Brown, G. (1969). *Laws of form*. London: Allen & Unwin, Ltd.

[3] Foerster, H. von (1981). *Observing systems*. The Systems Inquiry Series. Seaside, CA: Intersystems Publications.

[4] Foerster, H. von (1981). Objects: tokens for (eigen-) behaviors. In *Observing Systems* (pp. 274 - 285). The Systems Inquiry Series. Seaside, CA: Intersystems Publications.

[5] Foerster, H. von (1981). Notes on an epistemology for living things. In *Observing Systems* (pp. 258 - 271). The Systems Inquiry Series. Seaside, CA: Intersystems Publications.

[6] Foerster, H. von (1981). On constructing a reality. In *Observing Systems* (pp. 288 - 309). The Systems Inquiry Series. Seaside, CA: Intersystems Publications.

[7] Kauffman, L. H. Self-reference and recursive forms, Journal of Social and Biological Structures (1987), 53-72.

[8] Kauffman, L. H. (1995). Knot logic. In L. H. Kauffman (Ed.), *Knots and Applications* (pp. 1-110). Singapore: World Scientific Pub. Co.

[9] Kauffman, L. H. (2001). The mathematics of Charles Sanders Peirce. *Cybernetics and Human Knowing, 8*(1-2), 79-110.

[10] Mandelbrot, B. B. (1982). *The fractal geometry of nature*. New York: W. H. Freeman & Company (Originally published in 1977)

[11] Sakurai, J. J. (1985). *Modern quantum mechanics*. San Francisco: Benjamin/Cummings Publishing Company, Inc.


LEONOR FINI
GALERIE ALEXANDRE IOLAS
196, Boulevard Saint-Germain
VICTOR BRAUNER
GALERIE ALEXANDRE IOLAS
VON FOERSTER