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This manuscript is written in a casual, expository style.

Dedication

To my wonderful wife, Anne, for years of patience and understanding.

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Preface

Structures of Intelligence in Contexts

“In his paper “On Defining Artificial Intelligence”, Pei Wang (2019) defines intelligence as “adaptation (to its environment) with insufficient knowledge and resources.” This highly compact definition of a term used to name a field of research, as well as some of its products, cuts to the heart of the natural phenomenon we call “intelligence” by addressing an issue that I will paraphrase as autonomous handling of novelty.”

...

“Wang points out – and rightly so – that definitions affect the way phenomena get studied in science. He also points out the side effect of premature definitions: They can lead us astray. Before we have a good scientific understanding of a particular phenomenon it is however next to impossible to come up with a good scientific definition – how could we possibly define something properly that we don’t understand well? And yet, to study any phenomenon scientifically requires making some assumptions about that phenomenon, especially its relation to better-understood ones. How can this conundrum be addressed?” – Kristinn R. Thórisson in *On Defining Artificial Intelligence – Author’s Response to Commentaries* [32], (additions by Pei Wang in [32]).

Caveat:

This book can be challenging on many fronts. I’m certain it will make some very intelligent and educated people upset, and some of its novel concepts

have done so, already¹. Rest assured, my objective is not to upset people – it is merely a logical consequence of a paradigm shift [20] caused by a different interpretation structural features of *intelligence* and understanding of the *science* that exhibits certain self-similar to that intelligence. This homology can be seen in a complex-coupled functional network comprised of logical chains of inference that can be described as a framework. That framework appears “less-wrong” than prior interpretations, if not perfectly correct.

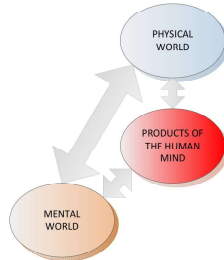
Furthermore, this paradigm shift has profound effects in our understanding of the Worlds of Artificial Intelligence and Science, as well. With that warning, this book is written for the domain of computer scientists, programmers and AI professionals – from the serious student to more advanced professionals. It’s probably not for the casual reader other than to provide a broad introduction to the field of study of intelligent systems.

0.1 A Brief Word About Science

It may seem strange to some that the idea for this book actually started from my reading of a book on physics. The book was written by the mathematical physicist, Roger Penrose, and entitled, *The Road to Reality - A Complete Guide to the Laws of the Universe* [35]. I have great admiration for Roger Penrose, but, frankly, the title of that book is misleading because it is not even close to being complete. I did, however, find Penrose was extremely enlightening from the synthesis of his philosophical, mathematical and scientific perspectives. From that synthesis, I found *something* that resembled *intelligence*.

Penrose starts his book with the chapter, *The Roots of Science*, a re-conceptualization of philosopher, Karl Popper’s *Three Worlds, Tanner Lecture* [38], in which Popper identifies the Physical World, the Mental World, and the World of Products of the Human Mind. I am not the only scientist who discovered that *something* relates to intelligence.

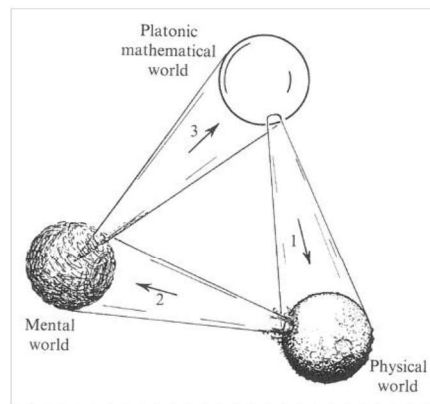
“This Capstone applies Popper’s Three-worlds paradigm to the academic discourse on Artificial General Intelligence (AGI). It intends to assess how this paradigm can be used to frame the opinions of scientists and philosophers on Artificial General Intelligence (AGI) ...” – Marta Ziosi, *Popper’s Theory of the Three Worlds Applied to Artificial General Intelligence* [1].



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Figure 1: Three Worlds, K. Popper [38]

Popper identified these worlds without identifying any particular relationships between them, however. Penrose took Popper's views as prior knowledge and re-conceptualized the third world much differently from Popper's *Products of the Human Mind*. Penrose goes further in identifying interesting relationships between his three worlds that he only identifies as “mysteries” [34] shown in Fig. 2.



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Figure 2: Three Worlds and Three Mysteries, R. Penrose [35][p. 18]

Penrose identified this world as *The Platonic World of Mathematical Forms* having a fractal structure in this way:

“Yet it (a fractal) has a robustness that is beyond any doubt;

for the same structure is revealed – in all its perceivable details, to greater and greater fineness the more closely it is examined – independently of the mathematician or computer that examines it. Its existence can only be within the Platonic World of Mathematical Forms.

I am aware that there will still be many readers who find difficulty assigning any kind of actual existence to mathematical structures. Let me make the request of such readers that they merely broaden their notion of what the term ‘existence’ can mean to them.” – Roger Penrose, *The Road to Reality - A Complete Guide to the Laws of the Universe* [35][p. 17].

I would loathe to ask any reader to simply broaden their notions of reality based on faith² – I am a scientist! – but these observations may serve as an axiom³, a starting point of reasoning, to ‘see’ intelligence as a very real but abstract concept that is exhibited through patterns by less abstract structures, all the way down to some physical embodiment. Perhaps, this is what Penrose meant by the fractal nature of a science of intelligence?

Fractals are mathematical constructs of structures having fractional dimensions [31]. I bring them up here in the Preface because they are often interpreted as a mysterious pattern – perhaps indicative even of Penrose’s mysteries. Humans tend to conceptualize in integer dimensions, so there are difficulties in understanding what fractal patterns represent. Fractals are the result of a view down patterns formed by iterated functions. For a simple example, a Sierpinski Triangle is a view on a *projection* of a 1.585-dimensional solid triangle mapped onto a 2-dimensional space. The result is characterized as Sierpinski’s Gasket because of the holes left in the mapping.

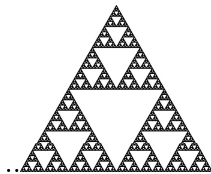


Figure 3: Sierpinski’s Gasket

I wanted to de-mystify Penrose’s mysteries which I found were means of translating (transforming) the ‘stuff’ in one world into ‘other stuff’ in the others. The ‘stuff’ turned out to be concepts at various levels of abstraction

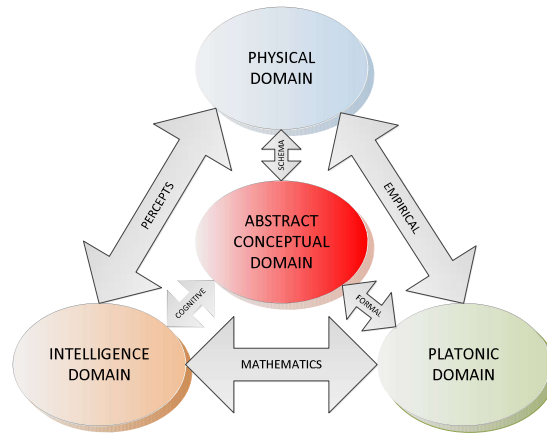


Figure 4: The Four Domains of a Scientific Framework, K. Lloyd

and realization, which will take some explaining to understand. Furthermore, it occurred to me that the strategies for explaining this came primarily from three sources – linguistics, cognitive science and mathematics⁴ A strategy for developing languages for communicating about intelligence came from these three fields which also avoids incompleteness, errors and paradox. The resulting languages (there are at least six) provide a road map for understanding, validating and communicating that understanding.

What originally appealed to me in Penrose’s Three Worlds was that, above the Physical World, each other World existed at a different level of abstraction. Furthermore, the Worlds were not hierarchically ordered, but serve as mutually-connected meta-worlds⁵ to the others mutually coupled by the mysteries.

Therefore, I hope to take scientific roads, whose paths are rather peripatetic, in conceptualizing intelligence. But, Penrose identified something he calls *structure* that seems common to almost anything anyone could think of – and beyond. This structure is a map to the roads to intelligence – showing the paths – we must travel in science. This road map of science can be seen in Fig. 4 which may lead us to knowledge of intelligence and intelligent systems. I’m sure the reader will notice the similarity of my Scientific Framework to Popper and Penrose. At our destination, we should find the workings of certain structures that exhibit Intelligence, and that becomes the Scientific focus of this book, as shown in Fig. 5 .

When it comes to describing the phenomena of intelligence and their abstractions, using the framework of science with all its abstractions, I recall Systems Scientist Kenneth E. *Boulding's Backward Basis*: “Things are the way they are because they got that way”. Well, OK. But I always like to add a question posed by Physicist and Mathematician James Clerk Maxwell: “What’s the go o’ that?”

In that regard, I’ll admit this book is quite different and some will find it controversial. I will also say this is the hardest book I’ve ever attempted to write.

Notes

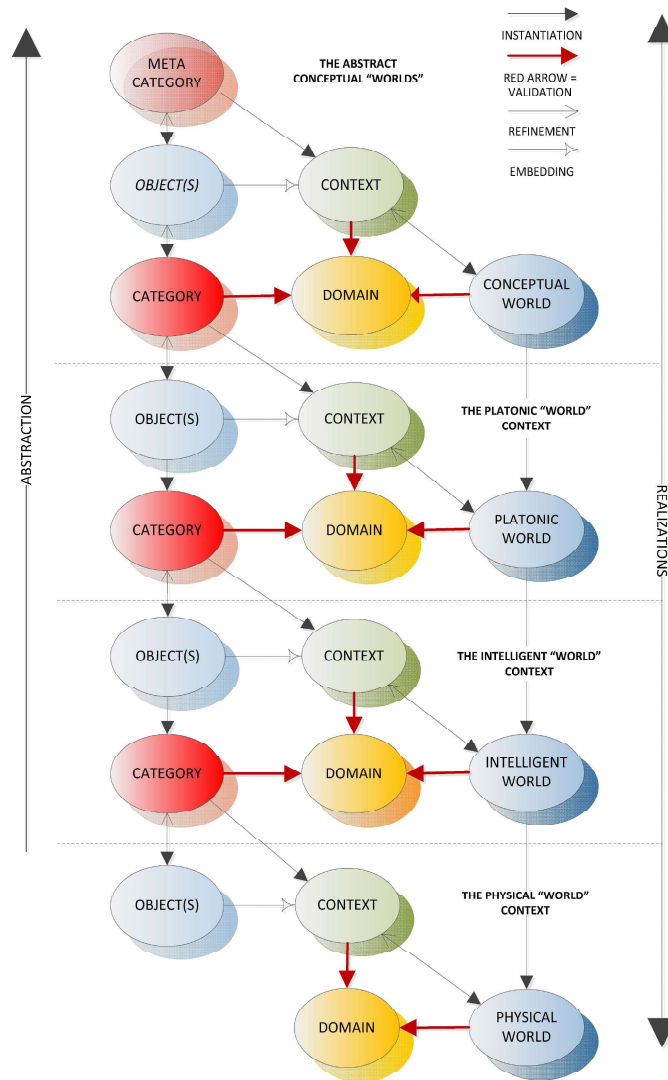
¹Upsetting other scientists seems to be a common theme in science. For example, when David Hilbert heard of Kurt Gödel’s Incompleteness Proof from Paul Bernays that proved the impossibility of Hilbert’s *Mathematical Formalism Program*, he was “somewhat angry” [40]. When Niels Bohr and Max Planck identified quantum phenomenon and introduced Quantum Mechanics, which eventually overthrew the widely accepted orthodoxy of Newtonian Mechanics, the whole world of science was “quite upset”. Many scientists *of a certain age* have been indoctrinated by Bourbakian Set Theory. Category Theory seems to be anathema to everything they know. “So, is Theory of Sets by Nicolas Bourbaki as outdated and obsolete as A. R. D. Mathias suggests?”

²However, Karl Popper “strongly advocates not only the existence of the products of the human mind, but also their being real rather than fictitious.” [1]

³“An axiom was originally a statement that is obvious and needs no justification, but the meaning of this word has changed. It is still a starting point, but one that is carefully chosen to facilitate the development of a particular body of abstract theory.” – Paul Taylor [46]

⁴Here, I owe a great debt of gratitude to physicist, mathematician, cognitive scientist and polyglot, Douglas Hofstadter – probably the most intelligent person I have ever met face to face.

⁵My use of the term *meta-* merely means beyond, not necessarily above.



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 Figure 5: A Homological Chain Complex of the Scientific Framework, K. Lloyd