Rectangular Coordinates

$$g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{R} = x\boldsymbol{e}_x + y\boldsymbol{e}_y + z\boldsymbol{e}_z$$

$$\boldsymbol{a} = a^x\boldsymbol{e}_x + a^y\boldsymbol{e}_y + a^z\boldsymbol{e}_z$$

$$\boldsymbol{A} = -k(\boldsymbol{a}\cdot\boldsymbol{R})\boldsymbol{R}^2\boldsymbol{e}_z = -k\left(a^xx^3 + a^xxy^2 + a^xxz^2 + a^yx^2y + a^yy^3 + a^yyz^2 + a^zx^2z + a^zy^2z + a^zz^3\right)\boldsymbol{e}_z$$

$$\boldsymbol{B} = \nabla\times\boldsymbol{A} = -I*(\nabla\wedge\boldsymbol{A}) = -k\left(2a^xxy + a^yx^2 + 3a^yy^2 + a^yz^2 + 2a^zyz\right)\boldsymbol{e}_x + k\left(3a^xx^2 + a^xy^2 + a^xz^2 + 2a^yxy + 2a^zxz\right)\boldsymbol{e}_y$$

Cylindrical Coordinates

$$g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{R} = r\boldsymbol{e}_r + z\boldsymbol{e}_z$$

$$\boldsymbol{a} = \left(a_x\cos(\theta) + a_y\sin(\theta)\right)\boldsymbol{e}_r + \left(-a_x\sin(\theta) + a_y\cos(\theta)\right)\boldsymbol{e}_\theta + a_z\boldsymbol{e}_z$$

$$\boldsymbol{A} = -k(\boldsymbol{a}\cdot\boldsymbol{R})\boldsymbol{R}^2\boldsymbol{e}_z = -k\left(a_xr^3\cos(\theta) + a_xrz^2\cos(\theta) + a_yr^3\sin(\theta) + a_yrz^2\sin(\theta) + a_zr^2z + a_zz^3\right)\boldsymbol{e}_z$$

$$\boldsymbol{B} = \nabla\times\boldsymbol{A} = -I*(\nabla\wedge\boldsymbol{A}) = k\left(a_xr^2\sin(\theta) + a_xz^2\sin(\theta) - a_yr^2\cos(\theta) - a_yz^2\cos(\theta)\right)\boldsymbol{e}_r + k\left(3a_xr^2\cos(\theta) + a_xz^2\cos(\theta) + 3a_yr^2\sin(\theta) + a_yz^2\sin(\theta) + 2a_zrz\right)\boldsymbol{e}_\theta$$

Nonorthogonal Coordinates

$$g = \begin{bmatrix} 1 & 0 & \cos(\theta) \\ 0 & 1 & 0 \\ \cos(\theta) & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{R} = x_a\boldsymbol{e}_a + x_p\boldsymbol{e}_p + z\boldsymbol{e}_z$$

$$\boldsymbol{a} = a\boldsymbol{e}_a$$

$$\boldsymbol{A} = -k(\boldsymbol{a}\cdot\boldsymbol{R})\boldsymbol{R}^2\boldsymbol{e}_z = -ak\left((x_a)^3 + 3(x_a)^2z\cos(\theta) + x_a(x_p)^2 + 2x_az^2\cos(\theta)^2 + x_az^2 + (x_p)^2z\cos(\theta) + z^3\cos(\theta)\right)\boldsymbol{e}_z$$

$$\boldsymbol{B} = \nabla\times\boldsymbol{A} = -I*(\nabla\wedge\boldsymbol{A}) = -2akx_p\left(x_a + z\cos(\theta)\right)\boldsymbol{e}_a + ak\left(3(x_a)^2 + 4x_az\cos(\theta) + (x_p)^2 + z^2\right)\sin(\theta)^2\boldsymbol{e}_p + 2akx_p\left(x_a + z\cos(\theta)\right)\cos(\theta)\boldsymbol{e}_z$$