In [1]:
```
from sympy import symbols, sin, cos
from ga import Ga
from printer import Format, Fmt
from IPython.display import Latex
Format()
```

In [2]:
```
xyz_coords = (x, y, z) = symbols('x y z', real=True)
(o3d, ex, ey, ez) = Ga.build('e', g=[1, 1, 1], coords=xyz_coo
rds, norm=True)
o3d.g
```

Out[2]:

$$\left[\begin{matrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{matrix}\right]$$

In [3]:
```
f = o3d.mv('f', 'scalar', f=True)
f
```

Out[3]: $f = f$

In [4]:
```
F = o3d.mv('F', 'vector', f=True)
lap = o3d.grad*o3d.grad
lap.Fmt(1,r'\nabla^{2}')
```

Out[4]:

$$\nabla^{2} = \frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial y^{2}} + \frac{\partial^{2}}{\partial z^{2}}$$

In [ ]:

In [5]:
```
lap.Fmt(1,r'\nabla^{2}')
```

Out[5]:

$$\nabla^{2} = \frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial y^{2}} + \frac{\partial^{2}}{\partial z^{2}}$$

In [6]:
```
lapf = lap*f
lapf
```

Out[6]: $\partial^{2}_{x} f + \partial^{2}_{y} f + \partial^{2}_{z} f$

In [7]:
```
lapf = o3d.grad | (o3d.grad * f)
lapf.Fmt(1,r'\nabla \cdot (\nabla f)')
```

Out[7]: $\nabla \cdot (\nabla f) = \partial^{2}_{x} f + \partial^{2}_{y} f + \partial^{2}_{z} f$

In [8]:
```
divF = o3d.grad|F
divF.Fmt(1,'x =')
```

Out[8]:
$$x == \partial_{x} F^{x} + \partial_{y} F^{y} + \partial_{z} F^{z}$$

In [9]:
```
gradF = o3d.grad * F
gradF.Fmt(1,r'\nabla F')
```

Out[9]:
$$\nabla F = \left (\partial_{x} F^{x} + \partial_{y} F^{y} + \partial_{z} F^{z}\right) + \left (- \partial_{y} F^{x} + \partial_{x} F^{y}\right) \boldsymbol{e}_{x}\wedge \boldsymbol{e}_{y} + \left (- \partial_{z} F^{x} +$$

In [10]:
```
sph_coords = (r, th, phi) = symbols('r theta phi', real=True)
(sp3d, er, eth, ephi) = Ga.build('e', g=[1, r**2, r**2 * sin(
th)**2], coords=sph_coords, norm=True)
sp3d.g_raw
```

Out[10]:
$$\left[\begin{matrix}1 & 0 & 0\\0 & r^{2} & 0\\0 & 0 & r^{2} \sin^{2}{\left (\theta \right)}\end{matrix}\right]$$

In [11]:
```
sp3d.grad.Fmt(1,r'\nabla')
```

Out[11]:
$$\nabla = \boldsymbol{e}_{r} \frac{\partial}{\partial r} + \boldsymbol{e}_{\theta} \frac{1}{r} \frac{\partial}{\partial \theta} + \boldsymbol{e}_{\phi} \frac{1}{r \sin{\left (\theta \right)}} \frac{\partial}{\partial \phi}$$

In [12]:
```
f = sp3d.mv('f', 'scalar', f=True)
F = sp3d.mv('F', 'vector', f=True)
B = sp3d.mv('B', 'bivector', f=True)
sp3d.grad.Fmt(1,r'\nabla')
lap = sp3d.grad*sp3d.grad
lap.Fmt(1,r'\nabla^{2} ')
```

Out[12]:
$$\nabla^{2} = \frac{2}{r} \frac{\partial}{\partial r} + \frac{1}{r^{2} \tan{\left (\theta \right)}} \frac{\partial}{\partial \theta} + \frac{\partial^{2}}{\partial r^{2}} + r^{-2} \frac{\partial^{2}}{\partial \theta^{2}} + \frac{1}{r^{2} \sin^{2}{\left (\theta \right)}} \frac{\partial^{2}}{\partial \phi^{2}}$$

In [13]:
```
Lapf = lap*f
Lapf.Fmt(1,r'\nabla^{2} f')
```

Out[13]:
$$\nabla^{2} f = \frac{1}{r^{2}} \left(r^{2} \partial^{2}_{r} f + 2 r \partial_{r} f + \partial^{2}_{\theta} f + \frac{\partial_{\theta} f}{\tan{\left (\theta \right)}} + \frac{\partial^{2}_{\phi} f}{\sin^{2}{\left (\theta \right)}}\right)$$

In [14]:
```
lapf = sp3d.grad | (sp3d.grad * f)
lapf.Fmt(1,r'\nabla \cdot (\nabla f)')
```

Out[14]:
$$\nabla \cdot (\nabla f) = \frac{1}{r^{2}} \left(r^{2} \partial^{2}_{r} f + 2 r \partial_{r} f + \partial^{2}_{\theta} f + \frac{\partial_{\theta} f}{\tan{\left (\theta \right)}} + \frac{\partial^{2}_{\phi} f}{\sin^{2}{\left (\theta \right)}}\right)$$

In [15]:
```
dviF = sp3d.grad | F
divF.Fmt(1,r'\nabla F')
```

Out[15]: $\nabla F = \partial_{x} F^{x} + \partial_{y} F^{y} + \partial_{z} F^{z}$

In [16]:
```
curlF = sp3d.grad ^ F
curlF.Fmt(1,r'\nabla \wedge F')
```

Out[16]:

$$\nabla \wedge F = \frac{1}{r}\left(r\partial_{r} F^{\theta} + F^{\theta} - \partial_{\theta} F^{r}\right) \boldsymbol{e}_{r}\wedge \boldsymbol{e}_{\theta} + \frac{1}{r}\left(r\partial_{r} F^{\phi} + F^{\phi} - \frac{\partial_{\phi} F^{r}}{\sin{\left(\theta \right)}}\right)$$

In [17]:
```
divB = sp3d.grad | B
divB.Fmt(1,r'\nabla \cdot B')
```

Out[17]:

$$\nabla \cdot B = -\frac{1}{r}\left(\frac{B^{r\theta}}{\tan{\left(\theta \right)}} + \partial_{\theta} B^{r\theta} + \frac{\partial_{\phi} B^{r\phi}}{\sin{\left(\theta \right)}}\right) \boldsymbol{e}_{r} + \frac{1}{r}\left(r\partial_{r} B^{r\theta} + B^{r\theta} - \frac{\partial_{\phi} I}{\sin}\right.$$

In [18]:
```
F
```

Out[18]: $F = F^{r} \boldsymbol{e}_{r} + F^{\theta} \boldsymbol{e}_{\theta} + F^{\phi} \boldsymbol{e}_{\phi}$

In [19]:
```
F.Fmt(3,'F')
```

Out[19]:

$$\begin{aligned} F = & F^{r} \boldsymbol{e}_{r} \\ & + F^{\theta} \boldsymbol{e}_{\theta} \\ & + F^{\phi} \boldsymbol{e}_{\phi} \end{aligned}$$

In [ ]: