In [134]:
```
from sympy import symbols, sin, cos
from ga import Ga
from printer import Format, Fmt
from IPython.display import Latex
Format()
Latex(r'$\boldsymbol{E}$')
```

Out[134]: $\boldsymbol{E}$

In [135]:
```
xyz_coords = (x, y, z) = symbols('x y z', real=True)
(o3d, ex, ey, ez) = Ga.build('e', g=[1, 1, 1], coords=xyz_coords
o3d.g
```

Out[135]:
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In [136]:
```
f = o3d.mv('f', 'scalar', f=True)
f
```

Out[136]: $f = f$

In [137]:
```
F = o3d.mv('F', 'vector', f=True)
lap = o3d.grad*o3d.grad
lap.Fmt(1,r'\nabla^{2}')
```

Out[137]:
$$\nabla^{2} = \frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial y^{2}} + \frac{\partial^{2}}{\partial z^{2}}$$

In [ ]:

In [138]:
```

lap.Fmt(1,r'\nabla^{2}')
```

Out[138]:
$$\nabla^{2} = \frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial y^{2}} + \frac{\partial^{2}}{\partial z^{2}}$$

In [139]:
```
lapf = lap*f
lapf
```

Out[139]: $\partial^{2}_{x} f + \partial^{2}_{y} f + \partial^{2}_{z} f$

In [140]:
```
lapf = o3d.grad | (o3d.grad * f)
#lapf.Fmt(1,r'\nabla \cdot (\nabla f)')
lapf
```

Out[140]: $\partial^{2}_{x} f + \partial^{2}_{y} f + \partial^{2}_{z} f$

In [141]:
```
divF = o3d.grad|F
#divF.Fmt(1,'x =')
divF
```
Out[141]:
$$\partial_{x} F^{x} + \partial_{y} F^{y} + \partial_{z} F^{z}$$

In [142]:
```
gradF = o3d.grad * F
#gradF.Fmt(1,r'\nabla F')
gradF
```
Out[142]:
$$\begin{aligned} & \left (\partial_{x} F^{x} + \partial_{y} F^{y} + \partial_{z} F^{z}\right) \\ & + \left (- \partial_{y} F^{x} + \partial_{x} F^{y}\right) \boldsymbol{e}_{x}\wedge \boldsymbol{e}_{y} \\ & + \left (- \partial_{z} F^{x} + \partial_{x} F^{z}\right) \boldsymbol{e}_{x}\wedge \boldsymbol{e}_{z} \\ & + \left (- \partial_{z} F^{y} + \partial_{y} F^{z}\right) \boldsymbol{e}_{y}\wedge \boldsymbol{e}_{z} \end{aligned}$$

In [143]:
```
sph_coords = (r, th, phi) = symbols('r theta phi', real=True)
(sp3d, er, eth, ephi) = Ga.build('e', g=[1, r**2, r**2 * sin(th)
sp3d.g_raw
```
Out[143]:
$$\left[\begin{matrix}1 & 0 & 0\\0 & r^{2} & 0\\0 & 0 & r^{2} \sin^{2}{\left (\theta \right)}\end{matrix}\right]$$

In [144]:
```
sp3d.grad.Fmt(1,r'\nabla')
```
Out[144]:
$$\nabla = \boldsymbol{e}_{r} \frac{\partial}{\partial r} + \boldsymbol{e}_{\theta} \frac{1}{r} \frac{\partial}{\partial \theta} + \boldsymbol{e}_{\phi} \frac{1}{r \sin{\left (\theta \right)}} \frac{\partial}{\partial \phi}$$

In [145]:
```
f = sp3d.mv('f', 'scalar', f=True)
F = sp3d.mv('F', 'vector', f=True)
B = sp3d.mv('B', 'bivector', f=True)
sp3d.grad.Fmt(1,r'\nabla')
lap = sp3d.grad*sp3d.grad
lap.Fmt(1,r'\nabla^{2} ')
```
Out[145]:
$$\nabla^{2} = \frac{2}{r} \frac{\partial}{\partial r} + \frac{1}{r^{2} \tan{\left (\theta \right)}} \frac{\partial}{\partial \theta} + \frac{\partial^{2}}{\partial r^{2}} + r^{-2} \frac{\partial^{2}}{\partial \theta^{2}} + \frac{1}{r^{2} \sin^{2}{\left (\theta \right)}} \frac{\partial^{2}}{\partial \phi^{2}}$$

In [146]:
```
Lapf = lap*f
#Lapf.Fmt(1,r'\nabla^{2} f')
Lapf
```
Out[146]:
$$\frac{1}{r^{2}} \left(r^{2} \partial^{2}_{r} f + 2 r \partial_{r} f + \partial^{2}_{\theta} f + \frac{\partial_{\theta} f}{\tan{\left (\theta \right)}} + \frac{\partial^{2}_{\phi} f}{\sin^{2}{\left (\theta \right)}}\right)$$

In [147]:
```
lapf = sp3d.grad | (sp3d.grad * f)
#lapf.Fmt(1,r'\nabla \cdot (\nabla f)')
lapf
```

Out[147]:
$$\frac{1}{r^{2}}\left(r^{2}\partial^{2}_{r}f + 2r\partial_{r}f + \partial^{2}_{\theta}f + \frac{\partial_{\theta}f}{\tan{(\theta)}} + \frac{\partial^{2}_{\phi}f}{\sin^{2}{(\theta)}}\right)$$

In [148]:
```
gradF = sp3d.grad | F
#gradF.Fmt(1,r'\nabla F')
gradF
```

Out[148]:
$$\frac{1}{r}\left(r\partial_{r}F^{r} + 2F^{r} + \frac{F^{\theta}}{\tan{(\theta)}} + \partial_{\theta}F^{\theta} + \frac{\partial_{\phi}F^{\phi}}{\sin{(\theta)}}\right)$$

In [149]:
```
curlF = sp3d.grad ^ F
#curlF.Fmt(1,r'\nabla \wedge F')
curlF
```

Out[149]:
$$\begin{aligned}&\frac{1}{r}\left(r\partial_{r}F^{\theta} + F^{\theta} - \partial_{\theta}F^{r}\right)\boldsymbol{e}_{r}\wedge\boldsymbol{e}_{\theta}\\&+\frac{1}{r}\left(r\partial_{r}F^{\phi} + F^{\phi} - \frac{\partial_{\phi}F^{r}}{\sin{(\theta)}}\right)\boldsymbol{e}_{r}\wedge\boldsymbol{e}_{\phi}\\&+\frac{1}{r}\left(\frac{F^{\phi}}{\tan{(\theta)}} + \partial_{\theta}F^{\phi} - \frac{\partial_{\phi}F^{\theta}}{\sin{(\theta)}}\right)\boldsymbol{e}_{\theta}\wedge\boldsymbol{e}_{\phi}\end{aligned}$$

In [150]:
```
divB = sp3d.grad | B
#divB.Fmt(1,r'\nabla \cdot B')
divB
```

Out[150]:
$$\begin{aligned}&-\frac{1}{r}\left(\frac{B^{r\theta}}{\tan{(\theta)}} + \partial_{\theta}B^{r\theta} + \frac{\partial_{\phi}B^{r\phi}}{\sin{(\theta)}}\right)\boldsymbol{e}_{r}\\&+\frac{1}{r}\left(r\partial_{r}B^{r\theta} + B^{r\theta} - \frac{\partial_{\phi}B^{\phi\phi}}{\sin{(\theta)}}\right)\boldsymbol{e}_{\theta}\\&+\frac{1}{r}\left(r\partial_{r}B^{r\phi} + B^{r\phi} + \partial_{\theta}B^{\phi\phi}\right)\boldsymbol{e}_{\phi}\end{aligned}$$

In [151]:
```
F
```

Out[151]:
$$\begin{aligned}F = &F^{r}\boldsymbol{e}_{r}\\&+F^{\theta}\boldsymbol{e}_{\theta}\\&+F^{\phi}\boldsymbol{e}_{\phi}\end{aligned}$$

In [152]:
```
F.Fmt(3,'F')
```

Out[152]:
```
'F =  \\begin{align*}  & F^{r}  \\boldsymbol{e}_{r} \\\\  &  +
F^{\\theta }  \\boldsymbol{e}_{\\theta } \\\\  &  + F^{\\phi }
 \\boldsymbol{e}_{\\phi }  \\end{align*} \n'
```

In [ ]: