In [10]:
```
from sympy import symbols, sin, cos
from ga import Ga
from printer import Format, Fmt
from IPython.display import Latex
Format()
```

In [11]:
```
xyz_coords = (x, y, z) = symbols('x y z', real=True)
(o3d, ex, ey, ez) = Ga.build('e', g=[1, 1, 1], coords=xyz_coord
s, norm=True)
o3d.g
```

Out[11]:
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In [12]:
```
f = o3d.mv('f', 'scalar', f=True)
f
```

Out[12]:
$$f = f$$

In [13]:
```
F = o3d.mv('F', 'vector', f=True)
lap = o3d.grad*o3d.grad
lap.Fmt(1,r'\nabla^{2}')
```

Out[13]:
$$\nabla^{2} = \frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial y^{2}} + \frac{\partial^{2}}{\partial z^{2}}$$

In [ ]:
```
```

In [14]:
```
lap.Fmt(1,r'\nabla^{2}')
```

Out[14]:
$$\nabla^{2} = \frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial y^{2}} + \frac{\partial^{2}}{\partial z^{2}}$$

In [15]:
```
lapf = lap*f
lapf
```

Out[15]:
$$\partial^{2}_{x} f + \partial^{2}_{y} f + \partial^{2}_{z} f$$

In [16]:
```
lapf = o3d.grad | (o3d.grad * f)
lapf.Fmt(1,r'\nabla \cdot (\nabla f)')
```

Out[16]:
```
'\\nabla \\cdot (\\nabla f) = \\partial^{2}_{x} f  + \\partia
l^{2}_{y} f  + \\partial^{2}_{z} f '
```

In [17]:
```
divF = o3d.grad|F
divF.Fmt(1,'x =')
```

Out[17]:
```
'x = = \\partial_{x} F^{x}  + \\partial_{y} F^{y}  + \\partia
l_{z} F^{z} '
```

In [18]:
```
gradF = o3d.grad * F
gradF.Fmt(1,r'\nabla F')
```

Out[18]:
```
'\\nabla F = \\left ( \\partial_{x} F^{x}  + \\partial_{y} F^{y}
+ \\partial_{z} F^{z} \\right )  + \\left ( - \\partial_{y}
F^{x}  + \\partial_{x} F^{y} \\right ) \\boldsymbol{e}_{x}\\wedg
e \\boldsymbol{e}_{y} + \\left ( - \\partial_{z} F^{x}  + \\part
ial_{x} F^{z} \\right ) \\boldsymbol{e}_{x}\\wedge \\boldsymbo
l{e}_{z} + \\left ( - \\partial_{z} F^{y}  + \\partial_{y} F^{z}
\\right ) \\boldsymbol{e}_{y}\\wedge \\boldsymbol{e}_{z}'
```

In [19]:
```
sph_coords = (r, th, phi) = symbols('r theta phi', real=True)
(sp3d, er, eth, ephi) = Ga.build('e', g=[1, r**2, r**2 * sin(t
h)**2], coords=sph_coords, norm=True)
sp3d.g_raw
```

Out[19]:
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & r^{2} & 0 \\ 0 & 0 & r^{2} \sin^{2}\left(\theta\right) \end{bmatrix}$$

In [20]:
```
sp3d.grad.Fmt(1,r'\nabla')
```

Out[20]:
$$\nabla = \boldsymbol{e}_{r} \frac{\partial}{\partial r} + \boldsymbol{e}_{\theta} \frac{1}{r} \frac{\partial}{\partial \theta} + \boldsymbol{e}_{\phi} \frac{1}{r \sin\left(\theta\right)} \frac{\partial}{\partial \phi}$$

In [21]:
```
f = sp3d.mv('f', 'scalar', f=True)
F = sp3d.mv('F', 'vector', f=True)
B = sp3d.mv('B', 'bivector', f=True)
sp3d.grad.Fmt(1,r'\nabla')
lap = sp3d.grad*sp3d.grad
lap.Fmt(1,r'\nabla^{2} ')
```

Out[21]:
$$\nabla^{2} = \frac{2}{r} \frac{\partial}{\partial r} + \frac{1}{r^{2} \tan\left(\theta\right)} \frac{\partial}{\partial \theta} + \frac{\partial^{2}}{\partial r^{2}} + r^{-2} \frac{\partial^{2}}{\partial \theta^{2}} + \frac{1}{r^{2} \sin^{2}\left(\theta\right)} \frac{\partial^{2}}{\partial \phi^{2}}$$

In [22]:
```
Lapf = lap*f
Lapf.Fmt(1,r'\nabla^{2} f')
```

Out[22]:
```
'\\nabla^{2} f = \\frac{1}{r^{2}} \\left(r^{2} \\partial^{2}_{r}
f  + 2 r \\partial_{r} f  + \\partial^{2}_{\\theta } f  + \\fra
c{\\partial_{\\theta } f }{\\tan{\\left (\\theta  \\right )}} +
\\frac{\\partial^{2}_{\\phi } f }{\\sin^{2}{\\left (\\theta  \\r
ight )}}\\right)'
```

In [23]:
```
lapf = sp3d.grad | (sp3d.grad * f)
lapf.Fmt(1,r'\nabla \cdot (\nabla f)')
```

Out[23]:
```
'\\nabla \\cdot (\\nabla f) = \\frac{1}{r^{2}} \\left(r^{2} \\pa
rtial^{2}_{r} f  + 2 r \\partial_{r} f  + \\partial^{2}_{\\theta
} f  + \\frac{\\partial_{\\theta } f }{\\tan{\\left (\\theta
\\right )}} + \\frac{\\partial^{2}_{\\phi } f }{\\sin^{2}{\\left
(\\theta  \\right )}}\\right)'
```

In [24]:
```
gradF = sp3d.grad | F
gradF.Fmt(1,r'\nabla F')
```

Out[24]:
```
'\\nabla F = \\frac{1}{r} \\left(r \\partial_{r} F^{r}  + 2
F^{r}  + \\frac{F^{\\theta } }{\\tan{\\left (\\theta  \\right
)}} + \\partial_{\\theta } F^{\\theta }  + \\frac{\\partial_{\\p
hi } F^{\\phi } }{\\sin{\\left (\\theta  \\right )}}\\right)'
```

In [25]:
```
curlF = sp3d.grad ^ F
curlF.Fmt(1,r'\nabla \wedge F')
```

Out[25]:
```
'\\nabla \\wedge F = \\frac{1}{r} \\left(r \\partial_{r} F^{\\th
eta }  + F^{\\theta }  - \\partial_{\\theta } F^{r} \\right) \\b
oldsymbol{e}_{r}\\wedge \\boldsymbol{e}_{\\theta } + \\frac{1}
{r} \\left(r \\partial_{r} F^{\\phi }  + F^{\\phi }  - \\fra
c{\\partial_{\\phi } F^{r} }{\\sin{\\left (\\theta  \\right
)}}\\right) \\boldsymbol{e}_{r}\\wedge \\boldsymbol{e}_{\\phi }
+ \\frac{1}{r} \\left(\\frac{F^{\\phi } }{\\tan{\\left (\\theta
\\right )}} + \\partial_{\\theta } F^{\\phi }  - \\frac{\\partia
l_{\\phi } F^{\\theta } }{\\sin{\\left (\\theta  \\right )}}\\ri
ght) \\boldsymbol{e}_{\\theta }\\wedge \\boldsymbol{e}_{\\phi }'
```

In [26]:
```
divB = sp3d.grad | B
divB.Fmt(1,r'\nabla \cdot B')
```

Out[26]:
```
'\\nabla \\cdot B = - \\frac{1}{r} \\left(\\frac{B^{r\\theta } }
{\\tan{\\left (\\theta  \\right )}} + \\partial_{\\theta }
B^{r\\theta }  + \\frac{\\partial_{\\phi } B^{r\\phi } }{\\si
n{\\left (\\theta  \\right )}}\\right) \\boldsymbol{e}_{r} + \\f
rac{1}{r} \\left(r \\partial_{r} B^{r\\theta }  + B^{r\\theta }
- \\frac{\\partial_{\\phi } B^{\\phi \\phi } }{\\sin{\\left (\\t
heta  \\right )}}\\right) \\boldsymbol{e}_{\\theta } + \\frac{1}
{r} \\left(r \\partial_{r} B^{r\\phi }  + B^{r\\phi }  + \\parti
al_{\\theta } B^{\\phi \\phi } \\right) \\boldsymbol{e}_{\\phi
}'
```

In [27]:
```
F
```

Out[27]:
$$F = F^{r} \boldsymbol{e}_{r} + F^{\theta} \boldsymbol{e}_{\theta} + F^{\phi} \boldsymbol{e}_{\phi}$$

In [28]:
```
F.Fmt(3,'F')
```

Out[28]:
```
'F =  \\begin{align*}  & F^{r}  \\boldsymbol{e}_{r} \\\\  &  +
F^{\\theta }  \\boldsymbol{e}_{\\theta } \\\\  &  + F^{\\phi }
\\boldsymbol{e}_{\\phi }  \\end{align*} \n'
```

In [ ]:
```
```