

# Electrical Component Model for a Lithium Iron Phosphate Cell

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**Abstract** A lumped parameter electrical component model of a 26650 size LiFePO<sub>4</sub> cell was developed based on charge and discharge plots at various rates. Step responses of the cell from seconds to hours were considered. An impedance element model was developed with components associated with the current collection grid, the active plate materials, and the electrolyte. Model elements may be approximated as resistors and capacitors; however some of the values of capacitor and resistor elements vary as a function of state of charge, load current, and temperature. Voltage dependant voltage sources were included in series with the bulk capacitor to model the nonlinear characteristics of the open circuit voltage at high and low states of charge. Constant current charge and discharge data were used to develop components of the open circuit voltage model and the total value of cell impedance as a function of current. Resistance components and pulse capacitance were determined using data from higher rate pulse test cycles. Charge and discharge plots at various temperatures in addition to cell temperature plots were used to develop the thermal model.

## Introduction

Lithium Ion batteries continue to be developed because of their power, energy and cycling advantages over more conventional battery chemistries such as lead acid, NiCd and nickel metal hydride. Lithium Ion batteries are themselves really a family of different combinations anode and cathode materials which can both intercalate lithium ions. The charged and discharged state of the cell results from the transfer of Li ions from the anode to the cathode within the cell while transferring electrons through an external circuit doing useful electrical work.

There are many manufacturers of Li Ion cells and they generally provide data sheets with a limited number of discharge curves typically at one temperature. It is well known that Li-Ion cells can be damaged and do damage when operated outside limits of recommended voltages and temperatures. This becomes especially important near the high temperature, high discharge rate limits which are challenging to estimate and often require extensive and expensive experimental testing. It is therefore desirable to have good models of cell behavior especially when designing battery systems with large numbers of cells.

Modeling has been done on Li-Ion batteries ranging from fundamental physics based models to simple empirical correlations. The former are useful for cell design and optimization but can be cumbersome when designing banks of cells. The latter approach is less computationally intensive but requires extensive data. An intermediate approach involves developing a model of cell behavior around standard circuit elements. The thus derived circuit elements can then be ported to conventional circuit modeling software such as P-Spice to design an optimized multi-cell system.

The most mature lithium ion batteries are based on LiCoO<sub>2</sub> anodes and carbon cathodes which have open circuit voltages in the middle of their discharge curves of about 3.6 volts. LiCoO<sub>2</sub> type lithium ion cells as well as the polymer varieties provide very high energy density. More recent development has focused on LiFePO<sub>4</sub> cathodes. LiFePO<sub>4</sub> cells feature a moderate cell voltage of about 3.2 volts and fairly high specific energy with long cycle life and improved stability compared with LiCoO<sub>2</sub> and other lithium ion cells with metal oxide cathode materials. This translates to a higher degree of safety.

For the present study we have chosen to focus on LiFePO<sub>4</sub> chemistry, in particular the ANR26650M1A cell. This cell has a good compromise between power and energy characteristics and there is a significant body of published data to calibrate the model [1],[2],[3].

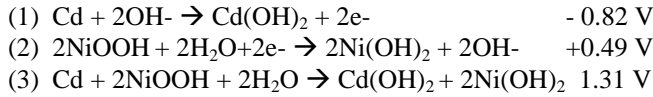
Many groups have looked at modeling LiFePO<sub>4</sub> batteries. Recently Prada [3] et.al discuss much of the full physics based models together with associated challenges and present in their paper a physics based model which features a current dependent radius pseudo particle. As reported the model produces good agreement with the A123 26650 experimental results for continuous discharge/charge but does not reproduce interval data although the authors report they are continuing to develop their model. Interval discharge data is important and has been the subject of recent experimental efforts developing repetitive pulsed power supplies using cells such as the 26650. [4],[5],[6]. Other authors have reported similar challenges modeling interval data. Chow et.al. report successfully reproducing continuous discharge data for the A123 26650 by fitting a multi parameter resistance expression but require using Matlab Simulink to reproduce the interval data [7]. Matlab can be used to estimate parameters in multi time constant LiFePO<sub>4</sub> cell model [8] for simulations with interval data. A prismatic lithium ion cell was modeled using PSCAD in [9] and included a 3 RC circuit impedance model and a polynomial fit for open circuit voltage (OCV) which was used for continuous cycle simulation, but not tested for interval data.

In the present study we chose to develop the circuit element based model in spread sheet form to make it easily understood, usable for a wide variety of load and charging profiles including pulsed interval currents, while being implemented in widely accessible, inexpensive software. The circuit parameters developed can then be ported to circuit element based software such as PSpice to continue to investigate the performance of multi cell designs with variations in component parameter values that can simulate real world imbalances.

## Model Development

Both deep cycle type lead acid [10] and Nickel-Cadmium [11] batteries are mature technologies used in a wide range of applications and have been previously modeled with similar techniques described herein.

The net electrochemical equation in a NiCd cell for example is the sum of the anode and cathode equations and the nominal cell voltage is the potential difference. In a NiCd cell OH<sup>-</sup> ions travel through the electrolyte, but there is no net change in the electrolyte. There is however a transient component of the model because at high rates the hydroxide can be depleted or possibly over concentrated near one of the electrode surfaces and in pores of the cell electrodes. The voltages at the electrodes of this cell are close to the edge of the aqueous electrolyte window, so when the cell is near a full state of charge (SOC) hydrogen and oxygen gasses can be evolved and these recombine inside the cell releasing heat. This is modeled as a non-linear self discharge component in parallel with the terminal voltage which can shunt excess charge current. A battery model can be applied in simulations such as [12] used for electric vehicle applications.



The net electrochemical equation in a lithium ion cell such as the LiCoO<sub>2</sub> cell is also a sum of the two equations and the nominal cell voltage is the potential difference. Aprotic solutions are required to support the high operating voltage of lithium ion cells. Similarly in the case of lithium iron phosphate the negative anode voltage requires an aprotic electrolyte, however the lower voltage cathode provides a wider voltage safety margin. Unlike NiCd cells and other aqueous electrolyte types, lithium ion cells cannot accept any overcharge because reversible breakdown of the electrolyte cannot be used as an electron shuttle mechanism. In addition the self discharge rate of the lithium iron phosphate cell is very low so this can be neglected except when modeling cycles with very long time duration therefore the model currently includes no shunt impedance component.

The present LiFePO<sub>4</sub> model includes two RC circuits, one series resistor alone  $R_s$  and one parallel RC circuit  $R_p$  and  $C_p$  and a larger capacitor  $C_b$  that is in series with both resistors. The resistance components are functions of current, SOC, and temperature. The OCV model is associated with the larger capacitor. This includes multiple Voltage Controlled Voltage Source VCVS components that are used to develop an OCV model to fit test data which can be either symmetric or asymmetric.

### Impedance Model Element Values

Model element values are determined from experimental battery cycle test data on the ANR26650M1A published by Reid [1] and Jeevarajan [2] et.al. The values of impedance components  $R_s$  and  $R_p$  are functions of current, temperature and  $V_b$  which is a linear function of SOC.  $R_s$  is equivalent to AC impedance in some datasheets and the sum of  $R_s$  and  $R_p$  is equivalent to DC impedance in some datasheets.

$$(4) \quad R = F(I, T, V_b) = F(I) * F(T) * F(V_b)$$

The resistance value is the product of a function of current and factors that compensate for temperature and SOC. The current factor is 1 at a reference temperature of 20 deg C and the SOC factor is close to 1 at mid SOC which corresponds to 0 volts across capacitor  $V_b$ .

The battery model shown in Figure 1 includes two series impedance elements and a non-linear bulk capacitance and voltage source section that can be modeled using a constant bulk capacitor  $C_b$  that is the maximum cell capacitance value in series with three voltage dependent voltage sources all of which are functions of  $V_b$ . These include symmetric source  $V_{c0}$  and asymmetric sources  $V_{c1}$  and  $V_{c2}$ . The shunt element included in aqueous cell models such as NiCd [11] is a relatively large resistance and is not included in this model because lithium ion cells have low self discharge rates and cannot accept overcharge. The terminal voltage,  $V_t$  is the load or battery charger voltage.  $I_L$  is the load current which is negative when the battery is being charged.  $R_s$  is the ac impedance of the cell. It can be measured automatically by some battery evaluation systems which generate a ripple current and determine the impedance by measuring the magnitude of the resulting ripple voltage. It is also specified on some manufacturer's datasheets and can be found from the instantaneous voltage drop in a transient step response plot. The RC time constant associated with impedance elements consisting of  $C_p$  and  $R_p$  and is presumed to result from a charge depletion layer on the plates and current flow constriction that may be caused by low Li<sup>+</sup> ion concentration in electrolyte in pores of electrode active materials or excess Li<sup>+</sup> in pores of electrode active materials. The open circuit float voltage  $V_f$  is the sum of nominal open circuit voltage  $V_n$ , the voltage across the bulk capacitance  $C_b$  and the voltages across the three VCVS elements that make the net effective capacitance become nonlinear.

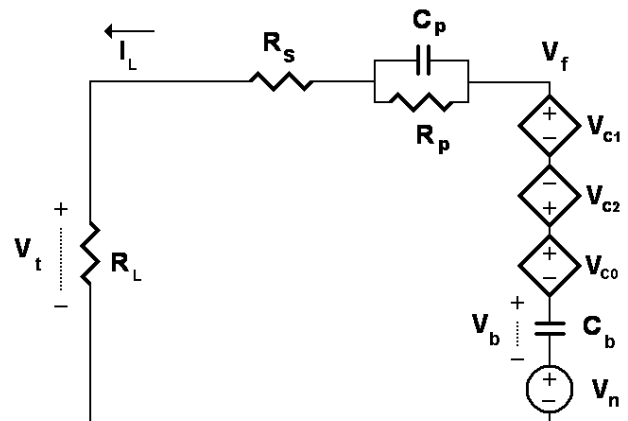


Figure 1 Component Lithium Ion Cell Model

Projections of the impedance at various test current levels based on [1] are plotted in Figure 2. This value is a function of current, however at high rates it is fairly close to 25 milli-ohms near 50% SOC and 25 deg C. At low current rates it has a non-linear characteristic similar to two anti-parallel diodes.

(5)  $F(I)=R_b*(1+K_g*(Abs(I_L)+K_y)^{K_z}+R_{bd}* \text{Discharge State}$   
 (6)  $F_s(I)=R_{bs}*(1+K_{gs}*(Abs(IL)+K_{ys})^{K_{zs}}+R_{bd}*SIGN(-I_L)+1)/2$   
 (7)  $F_p(I)=R_{bp}*(1+K_{gp}*(Abs(IL)+K_{yp})^{K_{zp}}$   
 where  $R_{bs}=5, R_{bp}=16, R_{bd}=8, K_{gs}=8, K_{ys}=1, K_{zs}=-2, K_{gp}=1, K_{yp}=1, K_{zp}=-1$

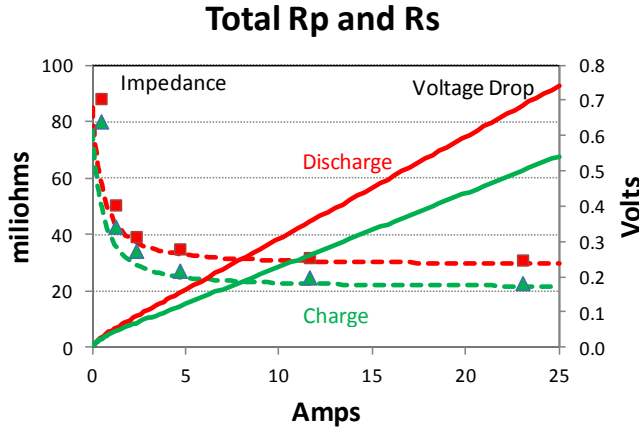


Figure 2 Total Impedance as a Function of Current at mid SOC using data scaled to 20 deg C to compensate for self-heating

The pulse test depicted in Figure 3 and 4 at different temperatures is a charge and discharge cycle with periodic high rate and low rate discharge. Results of the pulse test were used to determine parameter values associated with  $R_s$  and  $R_p$ , as well as the value of  $C_p$  which is currently modeled as a constant value 3 kFarads by matching step values and discharge voltage slopes from test data and model outputs. High rate test data superimposed on a plot of the model output as shown in Figure 5 was used to calibrate thermal model heat capacity and thermal impedance of the cell in the test fixture. The corresponding values are a thermal resistance of 8.5 degC/Watt and a heat capacity of 120 J/degC.

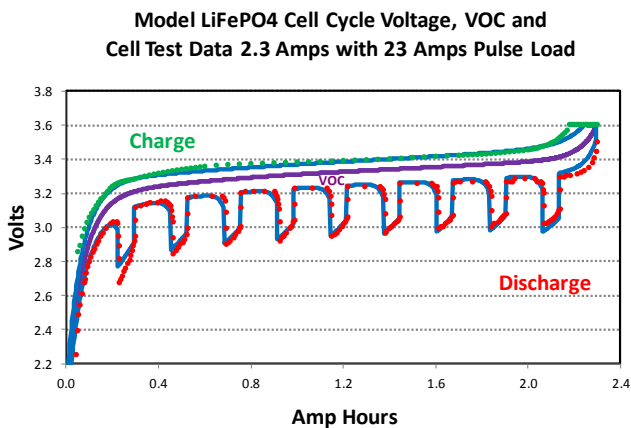


Figure 3 10 C Pulse Test Cycle at 25 deg C Data points derived from [2] and solid curves from model

Model LiFePO4 Cell Cycle Voltage, VOC and Cell Test Data 2.3 Amps with 23 Amps Pulse Load

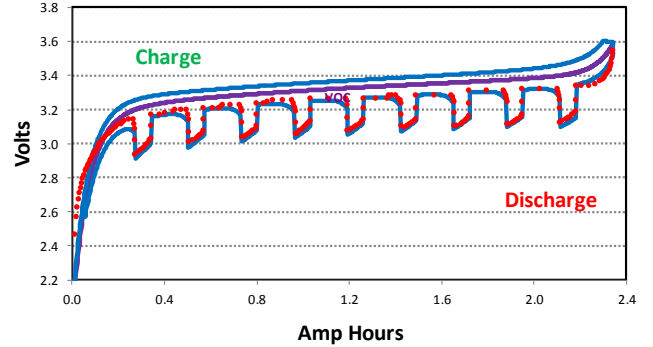


Figure 4 10 C Pulse Test Cycle at 60 deg C. Data points derived from [2] and solid curves from model

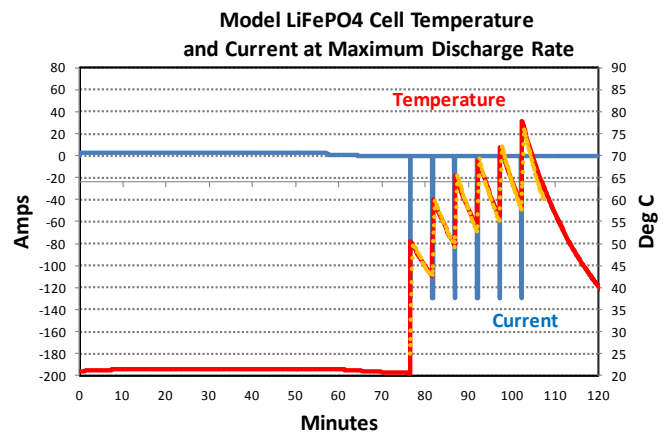


Figure 5 130 Amp Test Cycle with Temperature Data points derived from [2] and solid curves from model

The measured voltage drop across  $V_p$  is across a parallel RC circuit with a time constant on the order of 1 minute and a corresponding value of  $C_p$  of approximately 3000F. The resulting component characteristics are depicted in Figures 6a, 6b, and 6c. Compared with the reference unity value at 20 deg C the temperature coefficient causes an eight fold increase in the impedance of the cell at -20C and a decrease to about 60% at +60 deg C as indicated in [13] as well as increasing voltage drops shown in low temperature data from other sources. Some data such as in [2] also indicates reduced cell capacity at very low temperatures however Figure 12 in [14] indicates that the nominal open circuit voltage  $V_n$  changes only slightly as a function of temperature.

The model does not specifically include hysteresis as a separate component or two different OCV curves, however energy stored in the smaller capacitor can reproduce what other researchers refer to as hysteresis. Since the impedance components are non-linear and diode like at low current levels, the hysteresis like energy storage can persist for a fairly long time in the model output.

The impedance elements are also functions of SOC with an SOC factor of close to 1 at  $V_b$  close to 0 therefore mid SOC and increasing values at high and low SOC. This value is also a function of current direction and is therefore different when charging and discharging the cell with different parameters for  $R_s$  and  $R_p$ .

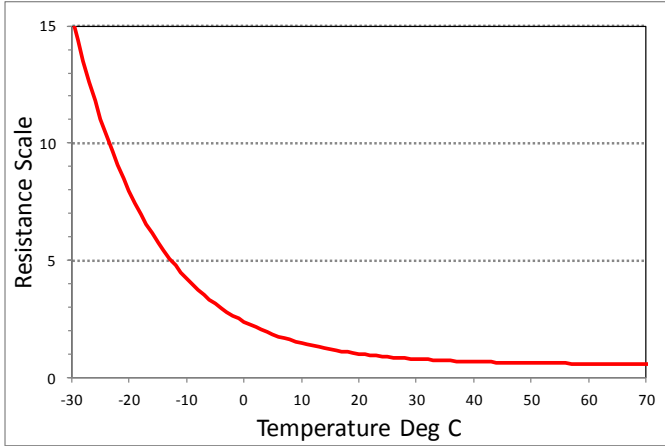


Figure 6a Impedance Temperature Scale Factor with 20 deg C Base

$$(8) F(T) = K_n * \text{Exp}(K_t * T) + K_m$$

$$(9) F(T) = 1.82 * \text{Exp}(-0.07 * T) + 0.56$$

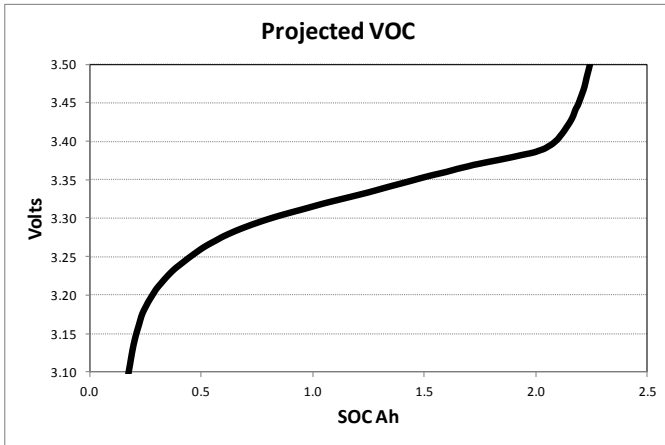


Figure 6b Open Circuit Voltage from Average of Projections

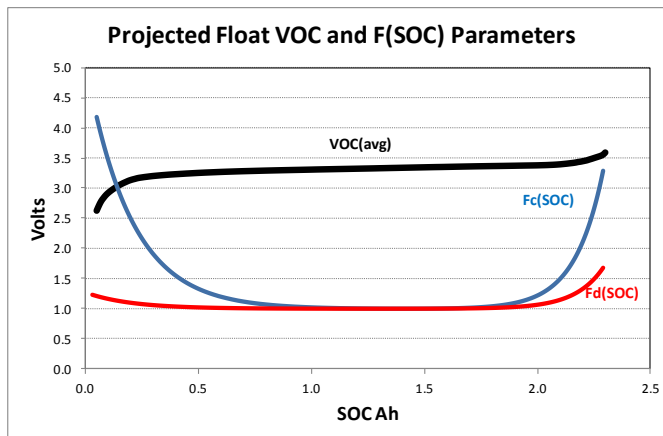


Figure 6c Average Pulse Impedance Scale Factors and  $V_f$

$$(10) F(V_b) = (1 + (K_{c0} \text{ or } K_{d0}) * \text{Exp}((K_{x0} - V_b) * K_{e0})) * (1 + (K_{c1} \text{ or } K_{d1}) * \text{Exp}((V_b - K_{x1}) * K_{e1}))) \text{ where}$$

$$K_{c0s} = 1.5, K_{d0s} = 0.1, K_{x0s} = -0.097, K_{e0s} = 75, K_{c1s} = 0.5, K_{d1s} = 0.15,$$

$$K_{x1s} = 0.035, K_{e1s} = 120, K_{c0p} = 0, K_{d0p} = -0.4, K_{x0p} = -0.104, K_{e0p} = 30,$$

$$K_{c1p} = 0.5, K_{d1p} = 0, K_{x1p} = 0.035, K_{e1p} = 45$$

### Open Circuit Float Voltage Component Parameters

The OCV related components can be projected by running an impedance sub-model over a range of current levels and averaging the resulting OCV estimates. The impedance parameters are corrected by finding a set of OCV solutions that converge towards a common solution. The slope of the curve near mid SOC corresponds to the maximum capacitance as shown in Figure 7a. The sum of  $V_n$  and  $V_b$  is a linear plot and the voltage controlled voltage source components VCVS0 VCVS1 and VCVS2 add the nonlinear parts of the plot at high and low SOC. These are a polynomial sum and two exponential functions in equation 12. The capacitance is found from the projections based on test data with voltage drops from the impedance sub-model subtracted. The equivalent capacitance of  $C_b$  and the three VCVS components is the slope found from numerical differentiation.

Capacitance plots based on data points and the model are plotted as a function of  $V_f$  in Figure 7b and as a function of SOC in Figure 7c. The capacitance peak of  $C_b = 52$  kFarads corresponding to the minimum slope of the projected  $V_f$  plot is at SOC of 1.6 amp hours and  $V_n = 3.355$  volts float voltage. Figure 8a and 8b show the model output for one continuous 10C rate charge and discharge cycle including temperature while Figure 9 shows the model output compared to NASA data[1]. The model run starts with charge so the SOC and voltage conditions transition around the loop clockwise starting in the lower left corner of the SOC vs voltage plot.

$$(11) V_f = V_n + V_b + V_{\text{poly}} + \text{Exp1} + \text{Exp2}$$

$$(12) V_f = 3.355 + V_b + k_3 * V_b^3 + k_5 * V_b^5 + k_{g0} * \text{Exp}(-V_b * k_{m0}) + k_{g1} * \text{Exp}(V_b * k_{m1})$$

where

$$k_3 = 35, k_5 = 2000, k_{g0} = 9.4E-11, k_{m0} = 210, k_{g1} = 1.2E-5, k_{m1} = 200$$

$$(13) V_b = 3600 * (\text{SOC} - 1.6) / C_b$$

$$(14) C_{eq} = I \Delta t / \Delta V \text{ Farads}$$

**Open Circuit Voltage Model and Data**

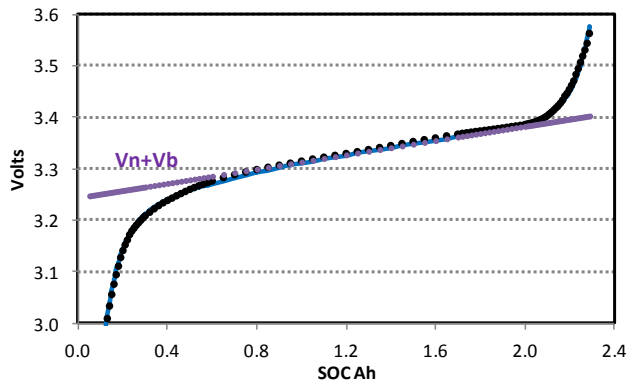


Figure 7a Float Voltage using sum of Nominal Voltage  $V_n$ , Bulk Capacitor Voltage  $V_b$  and VCVS components as a function of SOC.

**VCVS Equivalent Capacitance 2.3 Ah**

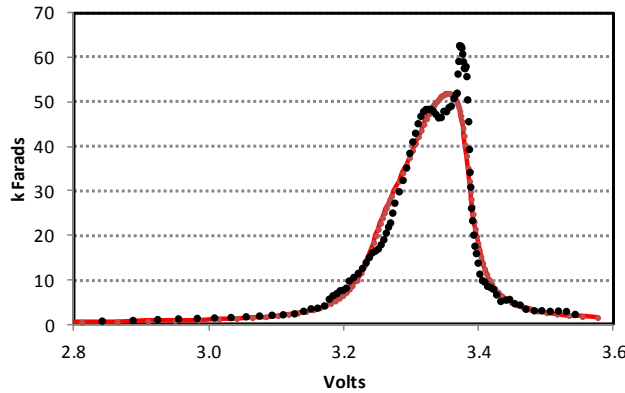


Figure 7b Net Equivalent OCV Capacitance as a function of  $V_f$  with points from average projected voc using equation 15 and solid curves from model

**SOC Dependent Equivalent Capacitance**

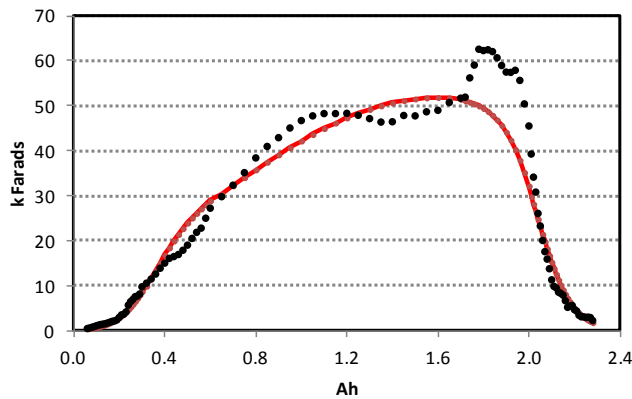


Figure 7c Net Equivalent OCV Capacitance as a Function of SOC with points from average projected voc using equation 15 and solid curves from model

**Model LiFePO4 Cell Voltage and Current 23 Amps**

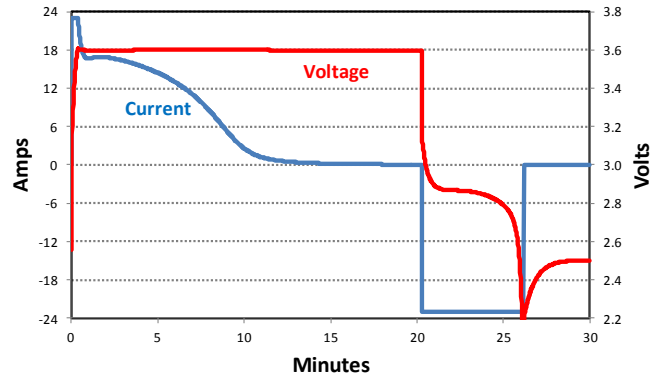


Figure 8a 10C Model Cycle

**Model LiFePO4 Cell Temperature and Current at 10C Discharge Rate**

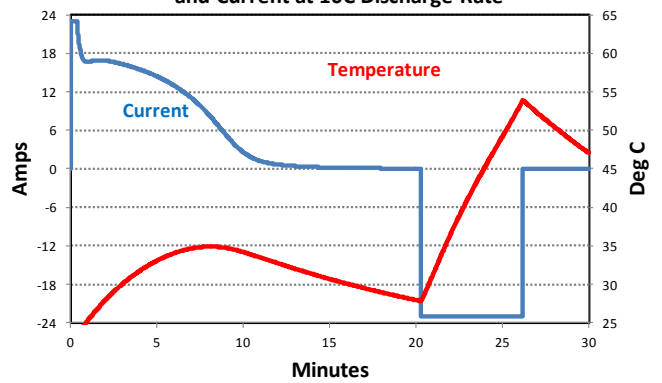


Figure 8b 10C Test Cycle Temperature Projection

**Model LiFePO4 Cell Cycle Voltage, VOC and Cell Test Data 23 Amps**

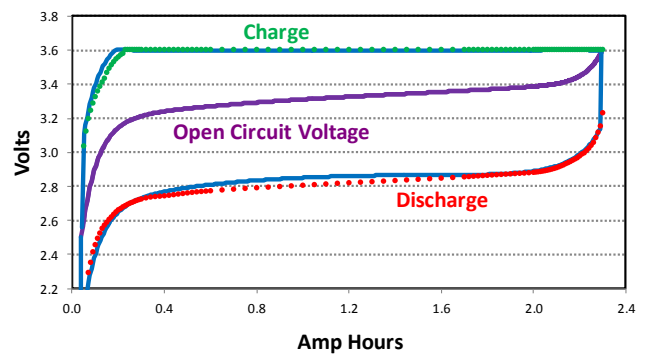


Figure 9 Model and Test Data Cycle as a Function of SOC Data points derived from [1] and solid curves from model

## List of Model Variables and Parameters

$V_t$  : Terminal voltage  
 $I_L$  : Load current  
 $R_L$  : Cell load resistance  
 $R_s$  : Series resistor  $R_s$  or AC impedance of cell  
 $R_p$  : Resistor in parallel RC circuit.  
 $C_p$  : Capacitor in parallel RC circuit  
 VOC or OCV : open circuit voltage  
 SOC : State of Charge in amp hours.  
 $V_f$  : Float open circuit voltage = OCV after  $V(C_p) \rightarrow 0$   
 $C_{eq}$  : Capacitance equivalent to circuit associated with  $V_f$   
 $V_{c0}$  : Polynomial VCVS term of  $V_f$   
 $V_{c1}$  : Exponential VCVS term of  $V_f$  at high SOC  
 $V_{c2}$  : Exponential VCVS term of  $V_f$  at high SOC  
 $V_n$  : Nominal cell voltage  
 $C_b$  : Bulk capacitance of cell at mid SOC and  $V_f = V_n$   
 $V_b$  : Voltage across  $C_b$   
 $T$  : Temperature in degrees C  
 $t$  : Time in seconds  
 $F(I), F_s(I), F_p(I)$  : Component of  $R_s$  or  $R_p$  Function of  $I_L$   
 $F(I), F_s(I), F_p(I)$  : Component of  $R_s$  or  $R_p$  Function of  $T$   
 $F(I), F_s(I), F_p(I)$  : Component of  $R_s$  or  $R_p$  Function of  $V_b$   
 $R_{bs}$  : Base resistance of series resistor  $R_s$   
 $R_{bd}$  : Additional base resistance of  $R_s$  when discharging  
 $R_{bp}$  : Base resistance of parallel resistor  $R_p$   
 $K_{gs}$  : Gain coefficient of  $R_s(I)$   
 $K_{gp}$  : Gain coefficient of  $R_p(I)$   
 $K_{ys}$  : Offset coefficient of  $R_s(I)$   
 $K_{yp}$  : Offset coefficient of  $R_p(I)$   
 $K_{zs}$  : Power coefficient of  $R_s(I)$   
 $K_{zp}$  : Power coefficient of  $R_p(I)$   
 $K_m$  : Minimum gain at high temperature of  $R(T)$   
 $K_n$  : Gain coefficient of resistance temperature compensation  
 $K_r$  : Exponent coefficient of resistance temp compensation  
 $K_{c0s}$  : Gain coefficient of  $R_s(V_b)$  at low SOC when charging  
 $K_{c1s}$  : Gain coefficient of  $R_s(V_b)$  at high SOC when charging  
 $K_{c0p}$  : Gain coefficient of  $R_p(V_b)$  at low SOC when charging  
 $K_{c1p}$  : Gain coefficient of  $R_p(V_b)$  at high SOC when charging  
 $K_{d0s}$  : Gain coefficient of  $R_s(V_b)$  at low SOC when discharging  
 $K_{d1s}$  : Gain coefficient of  $R_s(V_b)$  at high SOC when discharging  
 $K_{d0p}$  : Gain coefficient of  $R_p(V_b)$  at low SOC when discharging  
 $K_{d1p}$  : Gain coefficient of  $R_p(V_b)$  at high SOC when discharging  
 $K_{e0s}$  : Exponent coefficient of  $R_s(V_b)$  at low SOC  
 $K_{e1s}$  : Exponent coefficient of  $R_s(V_b)$  at high SOC  
 $K_{e0p}$  : Exponent coefficient of  $R_p(V_b)$  at low SOC  
 $K_{e1p}$  : Exponent coefficient of  $R_p(V_b)$  at high SOC  
 $K_{x0s}$  : Offset coefficient of  $R_s(V_b)$  at low SOC  
 $K_{x1s}$  : Offset coefficient of  $R_s(V_b)$  at high SOC  
 $K_{x0p}$  : Offset coefficient of  $R_p(V_b)$  at low SOC  
 $K_{x1p}$  : Offset coefficient of  $R_p(V_b)$  at high SOC  
 $k_3$  : Cubic gain of polynomial  $V_f$  term  
 $k_5$  : Fifth order gain of polynomial  $V_f$  term  
 $k_{g0}$  : Gain coefficient of low SOC exponential term  
 $k_{m0}$  : Multiplier coefficient of low SOC exponential term  
 $k_{g1}$  : Gain coefficient of high SOC exponential term  
 $k_{m0}$  : Multiplier coefficient of high SOC exponential term

## Conclusions and Future Work

A spreadsheet based electrical component cell model with typical component values indicated in Figure 10 was developed for the ANR26650M1A lithium iron phosphate cell using published NASA test data. The model reproduces the NASA continuous charge and discharge, interval and pulsed data over a range of temperatures. This model could be converted into a model for use in a simulation tool such as PSpice in order to model a system using many cells and other electrical components such as charge control circuitry and loads. Such a model could project temperatures and voltages in a battery system that may include cell parameter variations and heat transfer variations because of different cell locations in a battery system. Such models may help support system designs or be used as battery fuel gauge applications to estimate remaining state of charge using voltage current and temperature measurements. Models for other types of cells and possibly other energy storage and conversion devices such as ultracapacitors and fuel cells could also be developed.

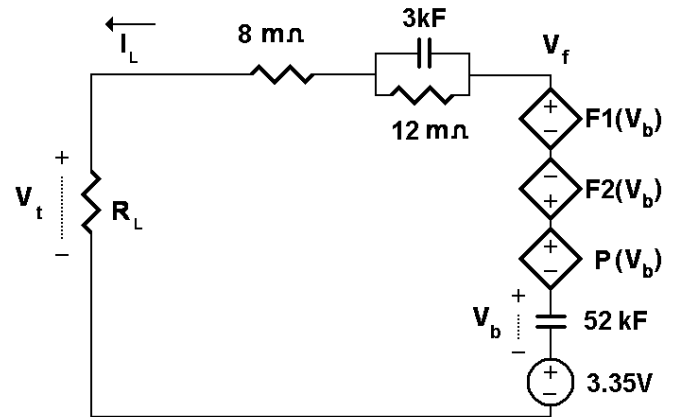


Figure 10 LiFePO<sub>4</sub> Cell Model with Typical Element Values

## References

- [1] C. M. Reid, "Lithium Iron Phosphate Cell Performance Evaluations for Lunar Extravehicular Activities", 10th Electrochemical Power Sources R+D Symposium, Williamsburg, VA, 20-23 Aug, 2007
- [2] J. Jeevarajan, B. Strangways and T. Nelson, "Performance and Safety Evaluation of High Rate 18650 Lithium Phosphate Cells", NASA Battery Workshop, November, 2009
- [3] E. Prada, D. Di Domenico, Y. Creff, J. Bernard, V. Sauvaut-Moynot, F. Huet, "Simplified Electrochemical and Thermal Model of LiFePO<sub>4</sub>-Graphite Li-Ion Batteries for Fast Charge Applications", Journal of The Electrochemical Society 159, 9 (2012) A1508-A1519

