This is the matrix you have been looking at.

```
In[1]:= f[m_] := PadRight[#, m] & /@ Table[Floor[n / k], {n, m}, {k, n}]
```

```
In[2]:= f[15] // MatrixForm
```

Out[2]//MatrixForm=

1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	0	0	0	0	0	0	0	0	0	0	0	0
4	2	1	1	0	0	0	0	0	0	0	0	0	0	0
5	2	1	1	1	0	0	0	0	0	0	0	0	0	0
6	3	2	1	1	1	0	0	0	0	0	0	0	0	0
7	3	2	1	1	1	1	0	0	0	0	0	0	0	0
8	4	2	2	1	1	1	1	0	0	0	0	0	0	0
9	4	3	2	1	1	1	1	1	0	0	0	0	0	0
10	5	3	2	2	1	1	1	1	1	0	0	0	0	0
11	5	3	2	2	1	1	1	1	1	1	0	0	0	0
12	6	4	3	2	2	1	1	1	1	1	1	0	0	0
13	6	4	3	2	2	1	1	1	1	1	1	1	0	0
14	7	4	3	2	2	2	1	1	1	1	1	1	1	0
15	7	5	3	3	2	2	1	1	1	1	1	1	1	1

Whenever I see Floor I also want to see the fractional part. But fractions are complicated, so I rationalize.

```
In[3]:= ug[m_] := PadRight[#, m] & /@ Table[n - k Floor[n / k], {n, 3, m + 2}, {k, 2, n}]
```

```
In[4]:= ug[15] // MatrixForm
```

Out[4]//MatrixForm=

1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	2	1	0	0	0	0	0	0	0	0	0	0	0
1	1	3	2	1	0	0	0	0	0	0	0	0	0	0
0	2	0	3	2	1	0	0	0	0	0	0	0	0	0
1	0	1	4	3	2	1	0	0	0	0	0	0	0	0
0	1	2	0	4	3	2	1	0	0	0	0	0	0	0
1	2	3	1	5	4	3	2	1	0	0	0	0	0	0
0	0	0	2	0	5	4	3	2	1	0	0	0	0	0
1	1	1	3	1	6	5	4	3	2	1	0	0	0	0
0	2	2	4	2	0	6	5	4	3	2	1	0	0	0
1	0	3	0	3	1	7	6	5	4	3	2	1	0	0
0	1	0	1	4	2	0	7	6	5	4	3	2	1	0
1	2	1	2	5	3	1	8	7	6	5	4	3	2	1

It is rather obvious that the row sums of g are
A004125 Sum of remainders of n mod k, for k = 1,2,3,...,n.

```
In[5]:= Total /@ ug@15
```

```
Out[5]= {1, 1, 4, 3, 8, 8, 12, 13, 22, 17, 28, 31, 36, 36, 51}
```

These are the remainders again, but in the lower triangle 0 is replaced by k to get 1, 2, 3, ..., k, which looks better than 1, 2, 3, ..., k-1, 0.

```
In[6]:= g[m_] := PadRight[#, m] & /@ Table[Mod[n, k, 1], {n, 3, m + 2}, {k, 2, n - 1}]
```

In[7]:= `g[15] // MatrixForm`

Out[7]//MatrixForm=

1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	1	0	0	0	0	0	0	0	0	0	0	0	0
2	3	2	1	0	0	0	0	0	0	0	0	0	0	0
1	1	3	2	1	0	0	0	0	0	0	0	0	0	0
2	2	4	3	2	1	0	0	0	0	0	0	0	0	0
1	3	1	4	3	2	1	0	0	0	0	0	0	0	0
2	1	2	5	4	3	2	1	0	0	0	0	0	0	0
1	2	3	1	5	4	3	2	1	0	0	0	0	0	0
2	3	4	2	6	5	4	3	2	1	0	0	0	0	0
1	1	1	3	1	6	5	4	3	2	1	0	0	0	0
2	2	2	4	2	7	6	5	4	3	2	1	0	0	0
1	3	3	5	3	1	7	6	5	4	3	2	1	0	0
2	1	4	1	4	2	8	7	6	5	4	3	2	1	0
1	2	1	2	5	3	1	8	7	6	5	4	3	2	1

The inverse has a tridiagonal band. The rows underneath come in almost matching pairs.

In[8]:= `Inverse@g@18 // MatrixForm`

Out[8]//MatrixForm=

1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-2	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-4	3	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
4	-3	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0
14	-8	4	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0
-20	11	-4	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0
-2	2	-10	5	0	0	1	-2	1	0	0	0	0	0	0	0	0	0
8	-5	10	-5	0	0	0	1	-2	1	0	0	0	0	0	0	0	0
-16	13	10	-12	6	0	0	0	1	-2	1	0	0	0	0	0	0	0
16	-13	-10	12	-6	0	0	0	0	1	-2	1	0	0	0	0	0	0
30	-21	0	7	-14	7	0	0	0	0	1	-2	1	0	0	0	0	0
-46	29	-10	-2	14	-7	0	0	0	0	0	1	-2	1	0	0	0	0
142	-80	46	-5	8	-16	8	0	0	0	0	0	1	-2	1	0	0	0
-126	72	-36	0	-8	16	-8	0	0	0	0	0	0	1	-2	1	0	0
-208	120	-30	-12	6	9	-18	9	0	0	0	0	0	0	1	-2	1	0
208	-120	30	12	-6	-9	18	-9	0	0	0	0	0	0	0	1	-2	1

The positions of the -1 entries in the first column of the inverse of g seem to be related to the evil numbers.
A001969 Evil numbers: numbers with an even number of 1's in their binary expansion.

In[9]:= `g300r1sign = Inverse@g@300 // Transpose // First // Sign`

Out[9]=
```
{1, -1, 1, -1, -1, 1, 1, -1, -1, 1, -1, 1, 1, -1, 1, -1, -1, 1, -1, 1, 1, -1, -1, 1, 1, -1,
 1, -1, -1, 1, 1, -1, -1, 1, -1, 1, 1, -1, -1, 1, 1, -1, 1, -1, -1, 1, -1, 1, 1, -1, 1,
 -1, -1, 1, 1, -1, -1, 1, -1, 1, 1, -1, 1, -1, -1, 1, -1, 1, 1, -1, -1, 1, 1, -1, 1, -1,
 -1, 1, -1, 1, 1, -1, 1, -1, -1, 1, 1, -1, -1, 1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1,
 1, 1, -1, -1, 1, -1, 1, 1, -1, 1, -1, -1, 1, -1, 1, 1, -1, -1, 1, 1, -1, 1, -1, -1, 1,
 1, -1, -1, 1, -1, 1, 1, -1, -1, 1, 1, -1, 1, -1, -1, 1, -1, 1, 1, -1, 1, -1, -1, 1, 1,
 -1, -1, 1, -1, 1, 1, -1, -1, 1, -1, 1, 1, -1, -1, 1, 1, -1, -1, 1, -1, 1, 1, -1, 1, -1,
 -1, 1, -1, 1, 1, -1, -1, 1, 1, -1, 1, -1, -1, 1, 1, -1, -1, 1, 1, -1, -1, 1, 1, -1, -1,
 1, -1, 1, 1, -1, 1, -1, -1, 1, -1, 1, 1, -1, -1, 1, 1, -1, 1, -1, -1, 1, 1, -1, -1, 1,
 -1, 1, 1, -1, -1, 1, 1, -1, 1, -1, -1, 1, -1, 1, 1, -1, 1, -1, -1, 1, 1, -1, -1, 1, -1,
 1, 1, -1, 1, -1, -1, 1, -1, 1, 1, -1, -1, 1, 1, -1, 1, -1, -1, 1, -1, 1, 1, -1, 1, -1,
 -1, 1, 1, -1, -1, 1, -1, 1, 1, -1, -1, 1, 1, -1, 1, -1, -1, 1, 1, -1, -1, 1, -1, 1}
```

```
In[10]:= almostevil = 1 + Position[g300r1sign, -1] // Flatten

Out[10]= {3, 5, 6, 9, 10, 12, 15, 17, 18, 20, 23, 24, 27, 29, 30, 33, 34, 36, 39, 40, 43, 45, 46, 48, 51,
  53, 54, 57, 58, 60, 63, 65, 66, 68, 71, 72, 75, 77, 78, 80, 83, 85, 86, 89, 90, 92, 95,
  97, 99, 101, 102, 105, 106, 108, 111, 113, 114, 116, 119, 120, 123, 125, 126, 129, 130,
  132, 135, 136, 139, 141, 142, 144, 147, 149, 150, 153, 154, 156, 159, 160, 162, 165,
  166, 169, 170, 172, 175, 177, 178, 180, 183, 184, 187, 189, 190, 193, 194, 197, 198,
  201, 202, 204, 207, 209, 210, 212, 215, 216, 219, 221, 222, 225, 226, 228, 231, 232,
  235, 237, 238, 240, 243, 245, 246, 249, 250, 252, 255, 257, 258, 260, 263, 264, 267,
  269, 270, 272, 275, 277, 278, 281, 282, 284, 287, 288, 291, 293, 294, 297, 298, 300}

In[11]:= Evil[n_] := Select[Range[0, n], EvenQ[DigitCount[#, 2][[1]]] &]

In[12]:= evil = Evil[300]

Out[12]= {0, 3, 5, 6, 9, 10, 12, 15, 17, 18, 20, 23, 24, 27, 29, 30, 33, 34, 36, 39, 40, 43, 45, 46, 48,
  51, 53, 54, 57, 58, 60, 63, 65, 66, 68, 71, 72, 75, 77, 78, 80, 83, 85, 86, 89, 90, 92,
  95, 96, 99, 101, 102, 105, 106, 108, 111, 113, 114, 116, 119, 120, 123, 125, 126, 129,
  130, 132, 135, 136, 139, 141, 142, 144, 147, 149, 150, 153, 154, 156, 159, 160, 163, 165,
  166, 169, 170, 172, 175, 177, 178, 180, 183, 184, 187, 189, 190, 192, 195, 197, 198,
  201, 202, 204, 207, 209, 210, 212, 215, 216, 219, 221, 222, 225, 226, 228, 231, 232,
  235, 237, 238, 240, 243, 245, 246, 249, 250, 252, 255, 257, 258, 260, 263, 264, 267,
  269, 270, 272, 275, 277, 278, 281, 282, 284, 287, 288, 291, 293, 294, 297, 298, 300}
```

The differences are off by 1 alternately up and down.

```
In[13]:= Complement[evil, almostevil]

Out[13]= {0, 96, 163, 192, 195}

In[14]:= Complement[almostevil, evil]

Out[14]= {97, 162, 193, 194}
```

There is a fair overlap.

```
In[21]:= Length /@ {almostevil, evil, Intersection[evil, almostevil]}

Out[21]= {150, 151, 146}
```

The positions of the 1 entries in the first column of the inverse of g seem to be related to the odius numbers.
A000069 Odious numbers: numbers with an odd number of 1's in their binary expansion.

```
In[26]:= almostodius = 1 + Position[g300r1sign, 1] // Flatten

Out[26]= {2, 4, 7, 8, 11, 13, 14, 16, 19, 21, 22, 25, 26, 28, 31, 32, 35, 37, 38, 41, 42, 44, 47, 49, 50,
  52, 55, 56, 59, 61, 62, 64, 67, 69, 70, 73, 74, 76, 79, 81, 82, 84, 87, 88, 91, 93, 94,
  96, 98, 100, 103, 104, 107, 109, 110, 112, 115, 117, 118, 121, 122, 124, 127, 128, 131,
  133, 134, 137, 138, 140, 143, 145, 146, 148, 151, 152, 155, 157, 158, 161, 163, 164,
  167, 168, 171, 173, 174, 176, 179, 181, 182, 185, 186, 188, 191, 192, 195, 196, 199,
  200, 203, 205, 206, 208, 211, 213, 214, 217, 218, 220, 223, 224, 227, 229, 230, 233,
  234, 236, 239, 241, 242, 244, 247, 248, 251, 253, 254, 256, 259, 261, 262, 265, 266,
  268, 271, 273, 274, 276, 279, 280, 283, 285, 286, 289, 290, 292, 295, 296, 299, 301}

In[24]:= Odius[n_] := Select[Range[300], OddQ[DigitCount[#, 2][[1]]] &]

In[25]:= odius = Odius[300]

Out[25]= {1, 2, 4, 7, 8, 11, 13, 14, 16, 19, 21, 22, 25, 26, 28, 31, 32, 35, 37, 38, 41, 42, 44, 47, 49,
  50, 52, 55, 56, 59, 61, 62, 64, 67, 69, 70, 73, 74, 76, 79, 81, 82, 84, 87, 88, 91, 93,
  94, 97, 98, 100, 103, 104, 107, 109, 110, 112, 115, 117, 118, 121, 122, 124, 127, 128,
  131, 133, 134, 137, 138, 140, 143, 145, 146, 148, 151, 152, 155, 157, 158, 161, 162,
  164, 167, 168, 171, 173, 174, 176, 179, 181, 182, 185, 186, 188, 191, 193, 194, 196,
  199, 200, 203, 205, 206, 208, 211, 213, 214, 217, 218, 220, 223, 224, 227, 229, 230,
  233, 234, 236, 239, 241, 242, 244, 247, 248, 251, 253, 254, 256, 259, 261, 262, 265,
  266, 268, 271, 273, 274, 276, 279, 280, 283, 285, 286, 289, 290, 292, 295, 296, 299}
```

Again the differences are off by 1 alternately up and down (never mind the solo 1).

```
In[27]:= Complement[odius, almostodius]

Out[27]= {1, 97, 162, 193, 194}

In[28]:= Complement[almostodius, odius]

Out[28]= {96, 163, 192, 195, 301}
```

Again there is a fair overlap.

```
In[29]:= Length /@ {almostodius, odius, Intersection[odius, almostodius]}

Out[29]= {150, 150, 145}
```

The row sums of the inverse of g show pairs x, -x.

```
In[19]:= t100 = Total /@ Inverse@g@100

Out[19]= {1, -1, 2, -2, -1, 1, 10, -13, -5, 8, 1, -1, 9, -22, 103, -90, -124, 124, -50, 54, 86,
  -90, 93, -103, -1, -109, 263, -143, -397, 397, 1738, -1653, -1613, 1525, -2333,
  2336, 2358, -2374, -954, 970, 1260, -1260, 1990, -2450, -1608, 2068, 3973, -3966,
  -2640, 1100, 1504, 29, -5299, 5377, 7509, -5234, -6498, 4145, -13 287, 13 287,
  12 309, -11 288, 56 333, -57 377, -52 553, 52 576, -56 362, 54 289, 55 523, -53 450,
  -84 012, 84 012, 86 434, -89 352, 94 888, -91 875, -92 727, 92 632, -37 482, 34 419,
  42 835, -39 772, 61 564, -63 104, -52 638, 50 028, 93 780, -89 630, -124 906, 124 900,
  -76 022, 88 332, 84 894, -94 852, 247 947, -250 293, -194 199, 139 618, -81 044, 135 625}
```

The position of these pairs seem to be related to twin primes.

A144834 Numbers n such that the two numbers n+1 and n+3 are both prime.

```
In[20]:= Flatten[Position[Most[t100] + Rest[t100], 0]]

Out[20]= {1, 3, 5, 11, 17, 29, 41, 59, 71}
```