Moebius

Extensions of the Möbius function

Enrique Pérez Herrero
Sun, 12 Feb 2012 14:13:42 -0800 (PST)
Re: Is the A123706 triangle an extension of the Moebius function?

Peter:

I think that we must use the "true" meaning of mu, not the formal definition that cames in all books:

mu(n)=((-1)*[omega(n)==Omega(n)])^omega(n), where [] means Iverson bracket, an amazing notation that you teach me that it existed.

Best Regards, Enrique.

```
def omega(n) : return QQ(sloane.A001221(n))
def Omega(n) : return QQ(sloane.A001222(n))

print [omega(i) for i in (1..24)]
print [Omega(i) for i in (1..24)]
```

```
[0, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 2, 1, 2, 2, 1, 1, 2, 1, 2, 2, 2, 1, 2]
[0, 1, 1, 2, 1, 2, 1, 3, 2, 2, 1, 3, 1, 2, 2, 4, 1, 3, 1, 3, 2, 2, 1, 4]
```

Enrique, this is a splendid idea!

This idea can be easily used to fill all these zeros in the Möbius function ;-) Indeed this leads to several continuous functions by linear interpolation as you can see below. Enjoy!

```
def Moebius(n) : return ((-1)*(omega(n)//Omega(n)))^omega(n)

print [moebius(i) for i in (2..20)]
print [Moebius(i) for i in (2..20)]
```

```
[-1, -1, 0, -1, 1, -1, 0, 0, 1, -1, 0, -1, 1, 1, 0, -1, 0, -1, 0]
[-1, -1, -1/2, -1, 1, -1, -1/3, -1/2, 1, -1, 4/9, -1, 1, 1, -1/4, -1,
4/9, -1, 4/9]
```

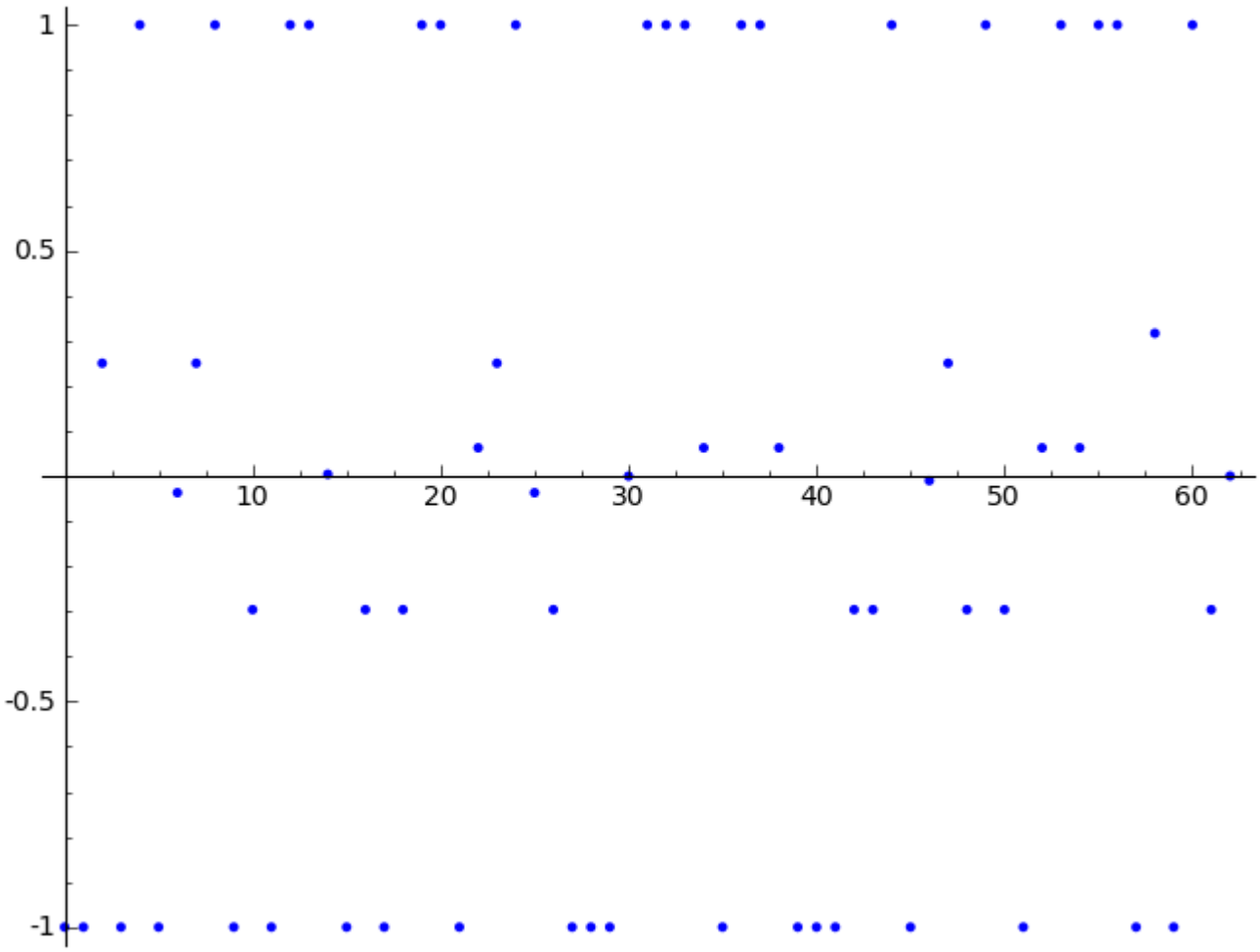
Now more systematically:

```
def muOBB(n) : return ((-1)*(omega(n)/Omega(n)))^Omega(n)
def muOBO(n) : return ((-1)*(omega(n)/Omega(n)))^omega(n)
def muBOO(n) : return ((-1)*(Omega(n)/omega(n)))^omega(n)
def muBOB(n) : return ((-1)*(Omega(n)/omega(n)))^Omega(n)
```

```
L = 64
MUOBB = [muOBB(i) for i in (2..L)]
MUOBO = [muOBO(i) for i in (2..L)]
MUBOO = [muBOO(i) for i in (2..L)]
MUBOB = [muBOB(i) for i in (2..L)]
```

```
"((-1)*(omega(n)/Omega(n)))^Omega(n)"
print MUOBB; print
list_plot(MUOBB)
```

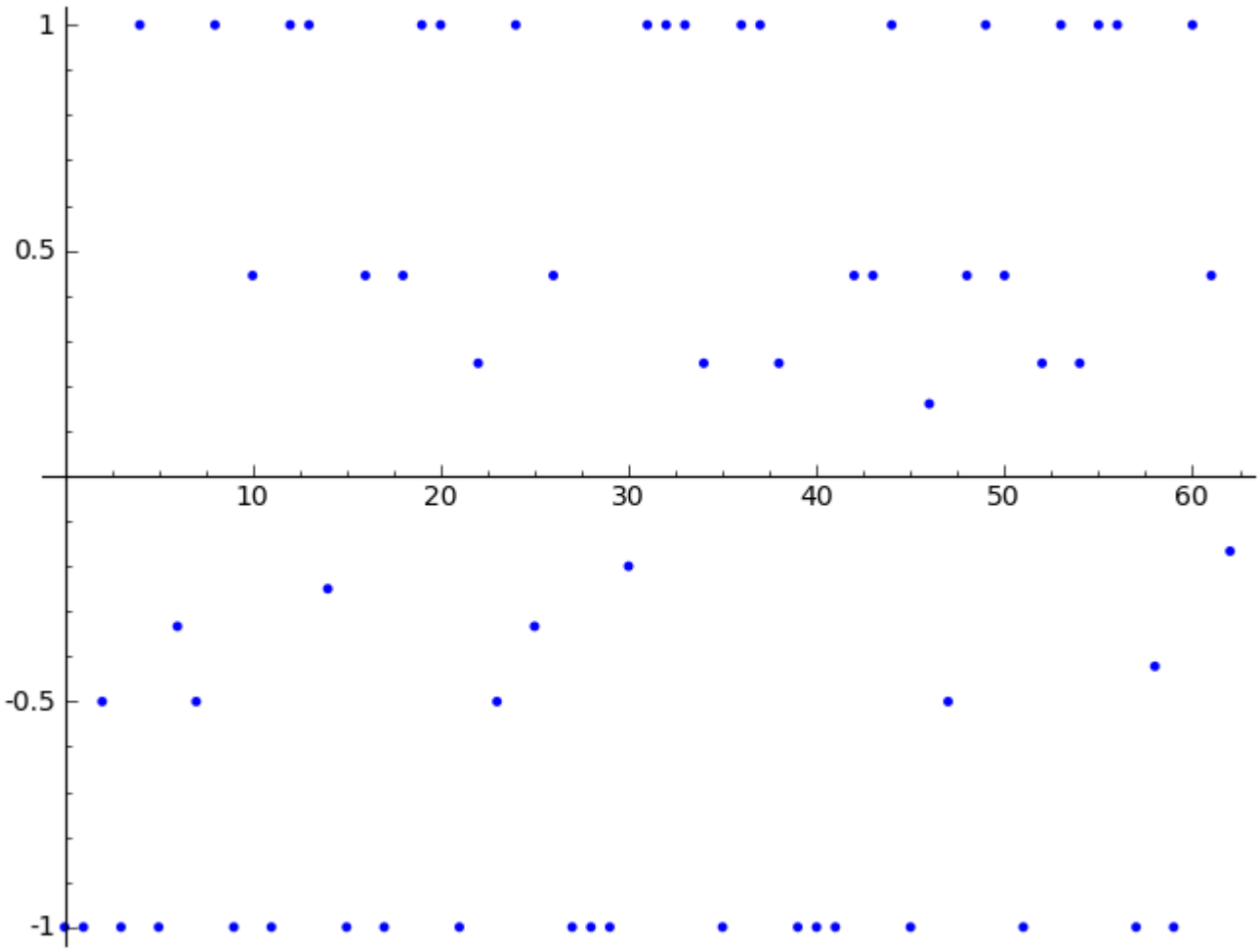
```
[-1, -1, 1/4, -1, 1, -1, -1/27, 1/4, 1, -1, -8/27, -1, 1, 1, 1/256, -1,
-8/27, -1, -8/27, 1, 1, -1, 1/16, 1/4, 1, -1/27, -8/27, -1, -1, -1,
-1/3125, 1, 1, 1, 1/16, -1, 1, 1, 1/16, -1, -1, -1, -8/27, -8/27, 1, -1,
-32/3125, 1/4, -8/27, 1, -8/27, -1, 1/16, 1, 1/16, 1, 1, -1, 81/256, -1,
1, -8/27, 1/46656]
```




```
"((-1)*(omega(n)/Omega(n)))^omega(n)"
print MUOBO; print
list_plot(MUOBO)
```

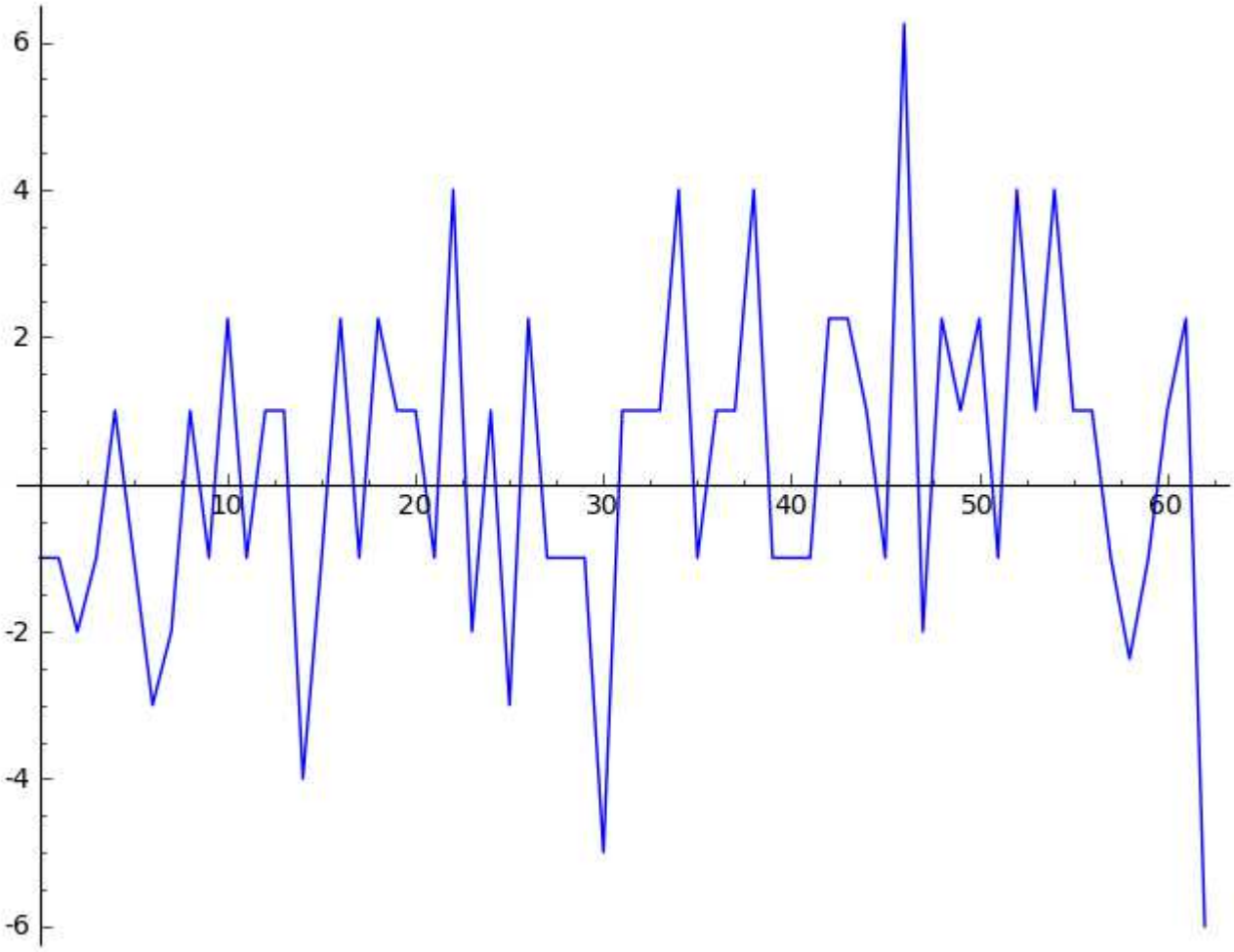
```
[-1, -1, -1/2, -1, 1, -1, -1/3, -1/2, 1, -1, 4/9, -1, 1, 1, -1/4, -1,
```

```
4/9, -1, 4/9, 1, 1, -1, 1/4, -1/2, 1, -1/3, 4/9, -1, -1, -1, -1/5, 1, 1,
1, 1/4, -1, 1, 1, 1/4, -1, -1, -1, 4/9, 4/9, 1, -1, 4/25, -1/2, 4/9, 1,
4/9, -1, 1/4, 1, 1/4, 1, 1, -1, -27/64, -1, 1, 4/9, -1/6]
```




```
"((-1)*(Omega(n)/omega(n)))^omega(n)"
print MUBOO; print
list_plot(MUBOO, plotjoined=True)
```

```
[-1, -1, -2, -1, 1, -1, -3, -2, 1, -1, 9/4, -1, 1, 1, -4, -1, 9/4, -1,
9/4, 1, 1, -1, 4, -2, 1, -3, 9/4, -1, -1, -1, -5, 1, 1, 1, 4, -1, 1, 1,
4, -1, -1, -1, 9/4, 9/4, 1, -1, 25/4, -2, 9/4, 1, 9/4, -1, 4, 1, 4, 1,
1, -1, -64/27, -1, 1, 9/4, -6]
```



evaluate