Examples by topic

Differentiation

The operator `df` is used to represent partial differentiation with respect to one or more variables.

```
syntax: df exprn [var <num>]+.
```

Differentiation of the function $x^2 y^3 z^4$ with respect to x, y, z , two, three and four times respectively, i.e $\frac{\partial^9 x^2 y^3 z^4}{\partial x^2 \partial y^3 \partial z^4}$:

```
> simplify $ df (x^2*y^3*z^4) x 2 y 3 z 4 ;
288
```

The derivative of $\log \sin(x)^2$:

```
> simplify $ df (log(sin x)^2) x;
2*cos x*log (sin x)/sin x

Note the parentheses.
```

Suppose $z(\cos(x), y)$: Let's calculate $\frac{\partial \sin(z)}{\partial \cos(x)}$ and $\frac{\partial z^2}{\partial x}$:

```
> declare depend [z,cos x,y];
[]
> simplify (df (sin z) (cos x));
cos z*df z (cos x)
> simplify (df (z^2) x);
2*df z x*z

Note how to declare dependencies.
```

The results are $\cos(z)\,\frac{\partial z}{\partial\cos(x)}$ and $2z\,\frac{\partial z}{\partial x})$ respectively, as expected.

Integration

INT is an operator in REDUCE for indefinite integration using a combination of the Risch-Norman algorithm and pattern matching.

```
syntax: intg exprn var.
```

Note that in Pure the operator is called intg in order not to clash with the integer type int.

Example 1:

$$\int \frac{1}{ax+b}\,dx$$

```
> simplify $ intg (1/(a*x+b)) x;
log (a*x+b)/a
```

Example 2:

$$I(a,b,n) = \int x^2\,(ax+b)^n\,dx$$

```
> I a b n = simplify $ intg (x^2*(a*x+b)^n) x;
> I a b n;
((a*x+b)^n*a^3*n^2*x^3+3*(a*x+b)^n*a^3*n*x^3+2*(a*x+b)^n*a^3*x^3+
 (a*x+b)^n*a^2*b*n^2*x^2+(a*x+b)^n*a^2*b*n*x^2-2*(a*x+b)^n*a*b^2*
  n*x+2*(a*x+b)^n*b^3)/(a^3*n^3+6*a^3*n^2+11*a^3*n+6*a^3)
> I a b 0 ;
x^3/3
> I 0 b n;
b^n*x^3/3
> I a 0 k;
x^k*a^k*x^3/(k+3)
```

Example 3:

$$\int \frac{\sqrt{x+\sqrt{x^2+1}}}{x}$$

```
> simplify $ intg (sqrt(x+sqrt(x^2+1))/x) x ;
intg (sqrt (sqrt (x^2+1)+x)/x) x
```

Apparently no solution was found. There is a package `ALGINT` in REDUCE, that is specialized to deal with algebraic functions. The [UserGuide] says

> ... *will analytically integrate a wide range of expressions involving square roots where the answer exists in that class of functions. It is an implementation of the work described in J.H. Davenport* [LNCS102]

```
> reduce::load "algint" ;
0
> simplify $ intg (sqrt(x+sqrt(x^2+1))/x) x ;
atan ((sqrt (sqrt (x^2+1)+x)*sqrt (x^2+1)-sqrt (sqrt (x^2+1)+x)*x-sqrt
(sqrt (x^2+1)+x))/2)+2*sqrt (sqrt (x^2+1)+x)+log (sqrt (sqrt
(x^2+1)+x)-1)-log (sqrt (sqrt (x^2+1)+x)+1)

Note how to load packages.
```

[UserGuide]	REDUCE User Guide ref.
[LNCS102]	On the Integration of Algebraic Functions, LNCS 102, Springer Verlag, 1981.