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BIOMECHANICAL CONSEQUENCES OF CALLUS DEVELOPMENT IN HOFFMANN, WAGNER, ORTHOFIX AND ILIZAROV EXTERNAL FIXATORS

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Abstract—A theoretical analysis by a finite elements model (FEM) of some external fixators (Hoffmann, Wagner, Orthofix and Ilizarov) was carried out. This study considered a logarithmic progress of callus elastic characteristics. A standard configuration of each fixator was defined where design and application characteristics were modified. A comparison among standard configurations and influence of every variation was made with regard to displacement and load transmission at the fracture site. An experimental evaluation of standard configurations was performed with a testing machine. After experimental validation of the theoretical model was achieved, an application of physiological loads which act on a fractured limb during normal gait was analysed.

A minimal contribution from an external fixator to the total rigidity of the bone–callus–fixator system was assessed when a callus showing minimum elastic characteristics had just been established. Insufficient rigidity from the fixation devices to assure an adequate immobilization during the early stages of fracture healing was verified. However, regardless of the external fixator, callus development was the overriding element for the rigidity of the fixator–bone system.

INTRODUCTION

A review about external fixation in Orthopaedics reveals a great number of reports based on the clinical evaluation of these systems. Data from mechanical behaviour, considering callus evolution in external fixation systems, are scarce (Beaupré *et al.*, 1983; Nishimura, 1984) although studies about the bone-frame structure are substantial (Adrey, 1970; Chao *et al.*, 1979, 1982; Chao and Malluege, 1981; Chao and An, 1982). Due to this lack of information, some accurate clinical application criteria about optimum fixator design and configuration characteristics have not yet been attained.

The first biomechanical investigations about external fixation systems and their structural components were achieved on cadaveric bones and bone models (Burny and Bourgois, 1972; Jorgensen, 1972; Vera *et al.*, 1986). At this first stage, the analysis of the Hoffmann fixator showed a wide variation in its mechanical performance, depending on the spatial frame configuration and its clinical mode of application. From these studies, the fixation rigidity analysis was considered to be an essential procedure to accurately compare different external fixators.

On the basis of this criterion, Chao *et al.* (1979) carried out their first theoretical study about external fixation systems using finite elements modelling (FEM). In this study a plane bone-frame system was modelled without defining the callus features.

With a similar methodology, although with important modifications regarding the model, the research group of 'Instituto de Biomecánica de Valencia' (Vera *et al.*, 1981, 1985, 1986) modelled an external fixator allowing progressive loading of the callus during fracture healing. Such modelling considers a callus with developing mechanical features where four loading hypotheses are assumed. Considering these hypotheses, different loading regimens to the immobilized limb were tested by means of FEM. Besides, these authors pointed out that a significant load transmission through the callus is presented even with a high rigidity of the external fixation system.

In relation to this modelling, the development of the present paper is established considering both callus presence during fracture healing and the influence of callus development on the total rigidity of the bone–callus–external-fixator system. This type of modelling is used as a tool to analyse the behaviour of different external fixation systems.

MATERIALS AND METHODS

Four external fixation systems with vast clinical acceptance and which follow main design and application theories on orthopaedic surgery and traumatology were selected. These four fixators were: Hoffmann, Wagner, Orthofix and Ilizarov.

Theoretical study

In this study, a structural linear FEM for bar structures, modelled by beam elements, has been used. The program algorithm required the definition of each component which constitutes the system according to its geometric and mechanical characteristics.

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Initially, a three-dimensional model representing the bone–callus–external-fixator system on the four fixators was achieved. Each fixator was modelled on some beams connected by grids (Fig. 1). Ilizarov rings were modelled on 12 identical beams. Longitudinal elastic modulus (E) and Poisson's ratio of the materials which constituted the system (stainless steel, titanium, bone and callus) were defined. Five different types of callus according to a logarithmic pattern were assumed in order to detect rigidity differences when small variations of callus mechanical characteristics appear in the first stages of bone repair (Table 1). Thus, Stage 1 corresponded to an early callus developed a few days after osteotomy and Stage 5 to intact cortical bone.

Finally, after the bone–callus–fixator system had been absolutely defined, six loading cases were considered. In each of them 1000 N forces (F_j) and 1000 N mm moments (M_j) along and around the Cartesian axes, associated with the proximal end system, were applied (Fig. 2).

After this process was achieved, four standard configurations were defined for each fixator. These configurations were the starting point for different variations aimed to study the effect on mechanical behaviour of fracture callus (Table 2). The variations considered were as follows:

(1) combinations of transfixing pins and side bars materials,
(2) number and diameter of pins,
(3) separation between groups of pins,
(4) separation between pins belonging to the same group,
(5) angle of insertion of pins,
(6) lateral separation between side bars,
(7) number and diameter of rings,
(8) number of side bars,
(9) telescoping of device, and
(10) callus length.

Linear and angular displacements (T_i and θ_i) as well as forces and moments (F'_i and M'_i) at bone callus were computed for every type of callus, as previously defined.

Forces and moments at the bone's proximal end were related to linear and angular displacements at the callus site by a compliance matrix (Fig. 3). The 36 coefficients of the compliance matrix for each callus stage were calculated by means of the expressions

$$A_{ij(c)} = \frac{T_i \text{ or } \theta_i}{F_j \text{ or } M_j}$$

$i, j = 1, 2, 3, 4, 5, 6,\ c = 1, 2, 3, 4, 5$ (callus stages).

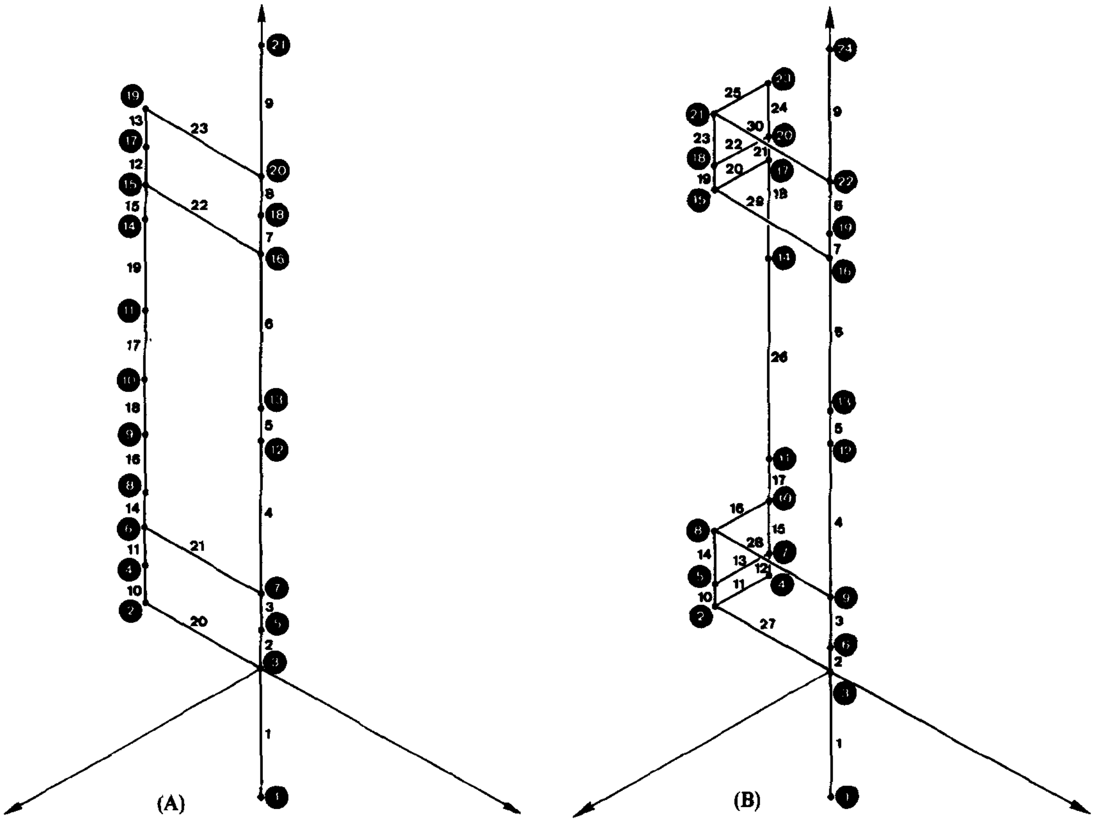

Fig. 1(A, B).

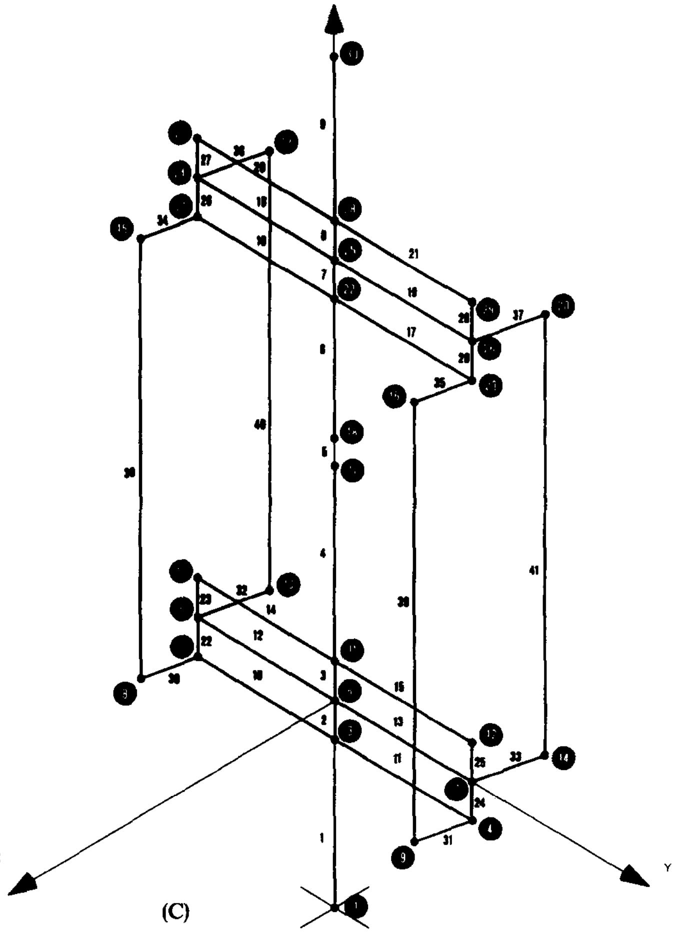

Fig. 1(C).

In an analogous way, a load transfer matrix which relates the loads applied on this proximal end with loads on callus was calculated (Fig. 4):

$$A_{ij(c)} = \frac{F'_i \text{ or } M'_i}{F_j \text{ or } M_j}$$

$i, j = 1, 2, 3, 4, 5, 6,\ c = 1, 2, 3, 4, 5$ (callus stages).

Both matrices together described the system's mechanical behaviour, and the variation of their coefficients during consolidation (Stages 1–5) allowed a determination of the callus relative contribution to the total system rigidity.

Both matrix coefficients were then compared to evaluate the effect of design and parameter variation on the system rigidity for each fixator. This comparison was the criterion used to determine theoretical performance differences between the four systems tested.

Experimental study

The performance of the four standard configurations in the theoretical study was analysed using compression tests on a static testing machine (INSTRON 1185).

Skeletal elements were replaced by polymethylmethacrylate (PMMA) bars whose geometry and mechanical properties were known (McCrum *et al.*, 1988; Tsai, 1987). Fracture calluses were replaced by rubber conglomerate pieces whose elastic non-linear characteristics were previously determined through compression tests, and Poisson's ratio was considered constant (Table 3).

PMMA bars, rubber conglomerate pieces and external fixators were arranged to perform compression tests (Fig. 5). The tests were carried out under load control. Load application speed was set to $10\ \mathrm{N\,s^{-1}}$ and the tests were stopped at a load of 600 N. Sixteen tests were carried out in order to characterize the rigidity of the four fixators considered in the four callus stages, simulated with the rubber conglomerate pieces and the gap.

A theoretical model including PMMA bars and rubber conglomerate pieces, geometrical and elastic characteristics, was developed to compare experimental and theoretical results. Once matching be-

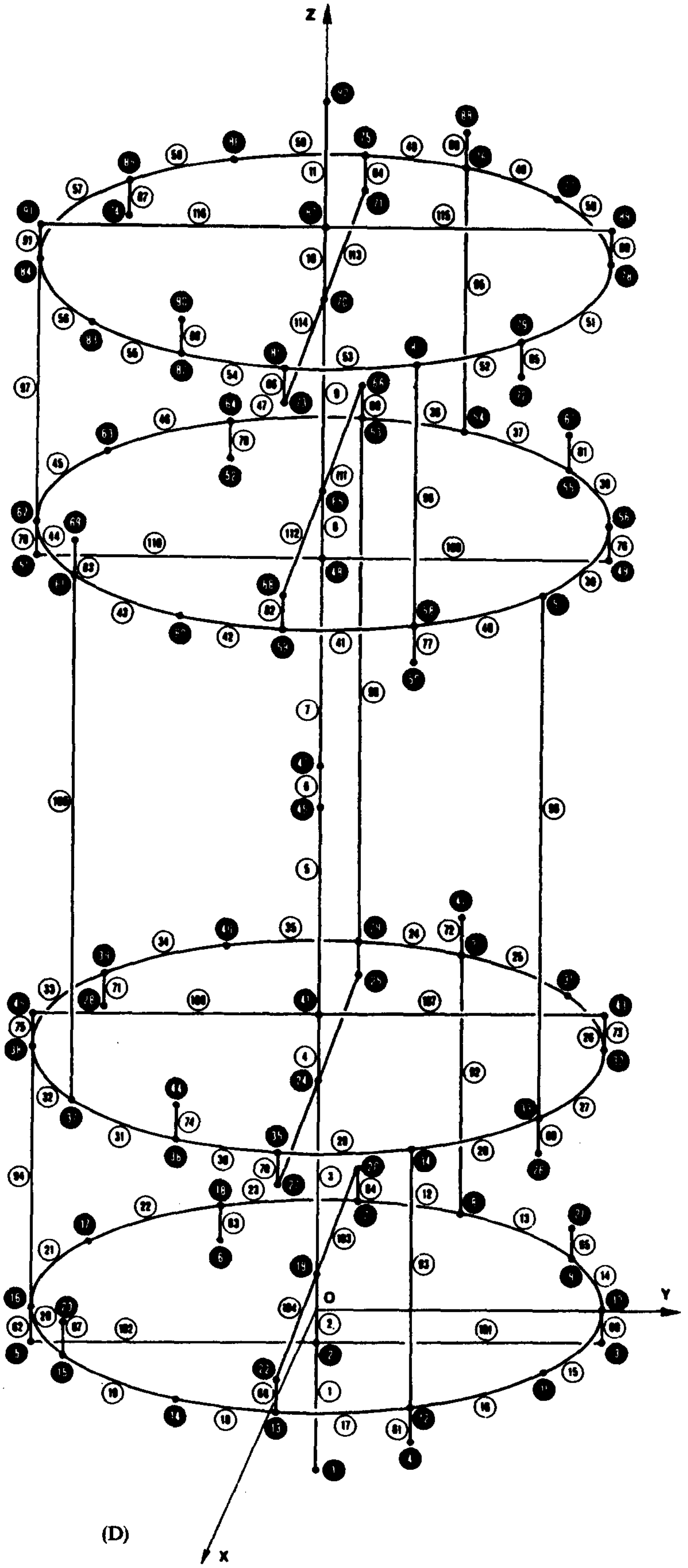

Fig. 1. Finite element model (FEM) of external fixators: (A) Hoffmann; (B) Wagner; (C) Orthofix; (D) Ilizarov.

Table 1. Elastic features of materials in the theoretical model

Material	E(N mm^{-2})	Poisson's ratio
Stainless steel	210000	0.36
Titanium	106000	0.23
Callus 1	1	0.39
Callus 2	10	0.39
Callus 3	100	0.39
Callus 4	1000	0.39
Callus 5	14240	0.39

tween them was achieved, a validated theoretical model was applied to compare the four standard fixator configurations under physiological loads. In order to reproduce the standard conditions during normal human gait, the maximum loads acting on the upper end of the tibia have been considered. These values were obtained from Morrison (1970) (Table 4).

RESULTS

A comparison among the theoretical results from FEM application was accomplished by studying the main diagonal coefficient patterns of both compliance and load transfer matrices. These compliance matrix coefficients relate forces and moments applied on the proximal element of fractured bone to linear and angular displacements, calculated at the callus site, in the same direction of load application. In the load transfer matrix these coefficients relate applied forces and calculated forces at the callus, as well as applied moments and calculated moments at the callus, that presented the same direction. This comparison was carried out in each of the five healing stages considered, except for the Ilizarov fixator, where a completely different behaviour was found in the first healing stage (Callus 1).

Compliance matrix coefficients development during the consolidation process, for the four standard external fixators studied, is shown in Fig. 6. A decrease of such coefficients' values was observed as bone repair evolves.

On the other hand, there is a noticeable quick increase of the load transfer matrix coefficients (Fig. 7). This increment points out an important increase of load transmission through the callus. These patterns are repeated irrespective of the fixator considered. So then, an increase of callus elastic characteristics causes an evident increase of load transmission at the callus site and, for this reason, callus assumes the most significant part of the total rigidity.

In an analogous way, the effect of design and application characteristics variation on the total rigidity of the system was analysed. On the basis of this comparison, a sensitivity analysis of the rigidity of the four external fixators showed the influence of each parameter on the variations of the matrix coefficients.

In relation to the Hoffmann fixator, Table 5 points out these described parameters:

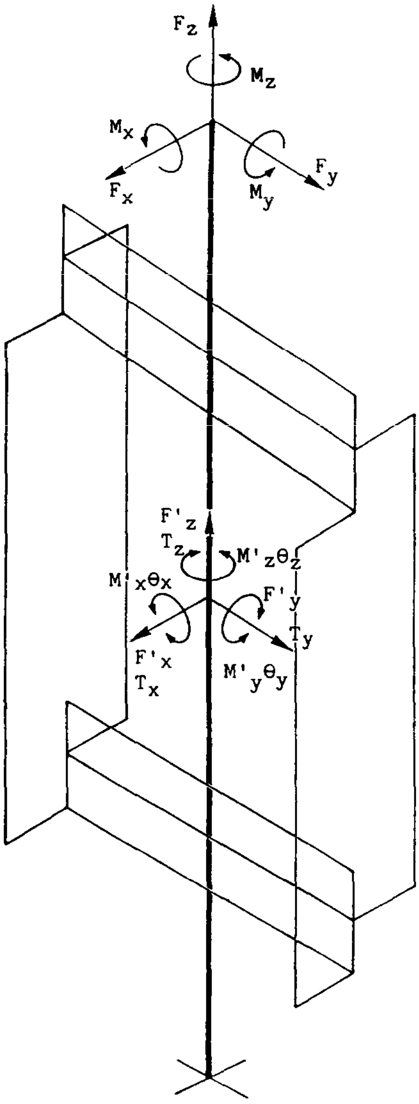

Fig. 2. Forces and moments applied on the proximal end system. Displacements and loads calculated in the callus element.

- — material and pin diameter,
- — number of pins and bars,
- — lateral side bars separation,
- — bars telescoping, and
- — callus size.

The Wagner fixator (Table 6), due to its simpler design, presented less possibilities of change and the most striking guidelines on its rigidity are

- — material,
- — side bar separation, and
- — callus size.

The Orthofix fixator rigidity (Table 7) was sensitive to

- — material,
- — side bar separation,
- — pins angle,
- — bar telescoping, and
- — callus size.

Table 2. List of variations analysed in the theoretical study

Parameter	Hoffmann	Wagner	Orthofix	Ilizarov
Pins and bars material	Steel, titanium	Steel, titanium	Steel, titanium in pins	
Pins separation	Clamp range	Clamp range	Clamp range	
Pin diameter (mm)	3, 4, 5, 6			1.5, 1.8
Number of pins (per group)	2, 3		2, 3	
Pin groups separation (cm)	−3, 0, 3	−2, 0, 2	−1, 0, 1	
Frontal plane pins angle (deg)		−30, 0, 30	−30, 0, 30	
Number of rings				2–4
Ring diameter (mm)				72, 90, 97
Number of pins (per ring)				2, 3, 4
Rings separation (cm)				−2.5, 0, 2.5
Ring groups separation (cm)				−3, 0, 3
Pins angle (in the rings) (deg)				30, 60, 90
Number of side bars	1, 2, 4		2, 3, 4	
Lateral side bars separation (cm)	−2, 0, 2	−2, 0, 2	−2, 0, 2	
AP side bars separation (cm)	−1, 0, 1			
Angle between side bars (deg)				30–120
Telescoping	Yes, no		Yes, no	
Callus size (mm)	2, 3, 5	2, 3, 5	2, 3, 5	2, 3, 5

Table 3. Elastic features of materials in the experimental model

Material	E (N mm^{-2}) as a function of the strain	Poisson's ratio
PMMA bars	3.200	0.36
Rubber conglomerate 1	$5.58+3.4\varepsilon+123\varepsilon^2$	0.40
Rubber conglomerate 2	$8.51+16.6\varepsilon-200\varepsilon^2$	0.42
Rubber conglomerate 3	$7.79+132\varepsilon+1670\varepsilon^2$	0.43

$$\begin{bmatrix} Tx \\ Ty \\ Tz \\ \theta x \\ \theta y \\ \theta z \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\ A_{21} & A_{22} & A_{21} & A_{24} & A_{25} & A_{26} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{bmatrix} \ast \begin{bmatrix} Fx \\ Fy \\ Fz \\ Mx \\ My \\ My \end{bmatrix}$$

Fig. 3. Compliance matrix.

$$\begin{bmatrix} F'x \\ F'y \\ F'z \\ M'x \\ M'y \\ M'z \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} & B_{15} & B_{16} \\ B_{21} & B_{22} & B_{21} & B_{24} & B_{25} & B_{26} \\ B_{33} & B_{32} & B_{33} & B_{34} & B_{35} & B_{36} \\ B_{41} & B_{42} & B_{43} & B_{44} & B_{45} & B_{46} \\ B_{51} & B_{52} & B_{53} & B_{54} & B_{55} & B_{56} \\ B_{61} & B_{62} & B_{63} & B_{64} & B_{65} & B_{66} \end{bmatrix} \ast \begin{bmatrix} Fx \\ Fy \\ Fz \\ Mx \\ My \\ Mz \end{bmatrix}$$

Fig. 4. Load transfer matrix.

Finally, the parameters concerning the Ilizarov fixator (Table 8) were

— pins group separation,
— pin separation,
— number of rings and bars, and
— callus size.

In spite of the influence of the configuration parameters on external fixator rigidity, these differences among configurations disappear as soon as the presence of Callus 3 (theoretically modelled) is considered.

In order to validate the theoretical model, a comparison between theoretical and experimental results was carried out (Fig. 8). The comparison showed some discrepancies as follows.

(1) The experimental values for Wagner and Orthofix fixators showed less-rigid fixator performance than that obtained from the theoretical model.

(2) On the other hand, Hoffmann and Ilizarov fixators, where transfixing pins are used, showed an opposite behaviour; that is to say, experimental models presented higher rigidity levels than theoretical models.

These dissimilarities between both groups of results can be explained as effects of clamp rigidity and pin span. Absolutely rigid clamps were considered for unilateral Wagner and Orthofix fixators in the theoretical model but, in fact, the experimental model performed in a different way. Also, stress levels at these clamps were tenfold higher than those reached by the Hoffmann and Ilizarov fixator clamps. On the other hand, in the theoretical model, the pin zones embedded in the clamp and bone segment were not considered. Besides, Hoffmann and Ilizarov fixators had a higher number of pins than unilateral fixators did. For these reasons, when experimental study was achieved, pin-span values were smaller than those of the theoretical model and because of this, the experimental model presented higher rigidity values than those of the theoretical model.

Matching between theoretical and experimental models was achieved by means of a theoretical decrease of the pin span in the Hoffmann and Ilizarov fixators and a theoretical modification of the inertial characteristics of the clamps—reducing cross-sectional areas of the clamps—in the Orthofix and Wagner fixators.

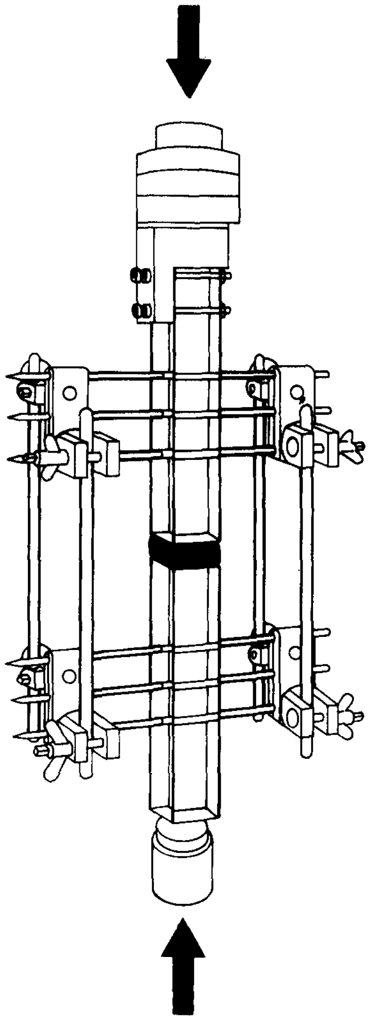

Fig. 5. Compression tests on the Hoffmann fixator.

Table 4. Maximum loads acting on the knee during walking

Direction	Forces (N)	Moments (N mm)
X	353.5	47472.0
Y	182.0	75286.0
Z	212.0	11868.1

Table 5. Effects of different variations on matrix coefficients. Hoffmann fixator: (− no influence, ± slight influence, + remarkable influence)

Compliance matrix						Variations	Load transfer matrix					
A_{11}	A_{22}	A_{33}	A_{44}	A_{55}	A_{66}		B_{11}	B_{22}	B_{33}	B_{44}	B_{55}	B_{66}
+	−	+	−	+	+	Material	−	−	±	+	+	+
+	−	+	−	+	+	Pin diameter	−	−	+	±	+	+
−	−	−	−	−	±	Pins groups separation	+	±	−	−	−	−
−	−	−	−	−	−	Pins separation	±	−	−	−	−	−
+	−	−	−	+	±	Pins number	+	−	−	−	±	−
+	−	+	−	+	+	Side bars lateral separation	−	−	+	−	+	−
−	−	−	−	−	−	Side bars posterior separation	−	−	−	−	−	−
+	−	−	−	+	+	Bars number	−	+	+	+	+	+
−	−	−	−	−	−	Clamps position	−	±	−	−	−	−
+	+	+	+	+	+	Telescoping	−	+	+	+	−	−
+	+	+	−	−	−	Callus size	−	−	−	−	+	+

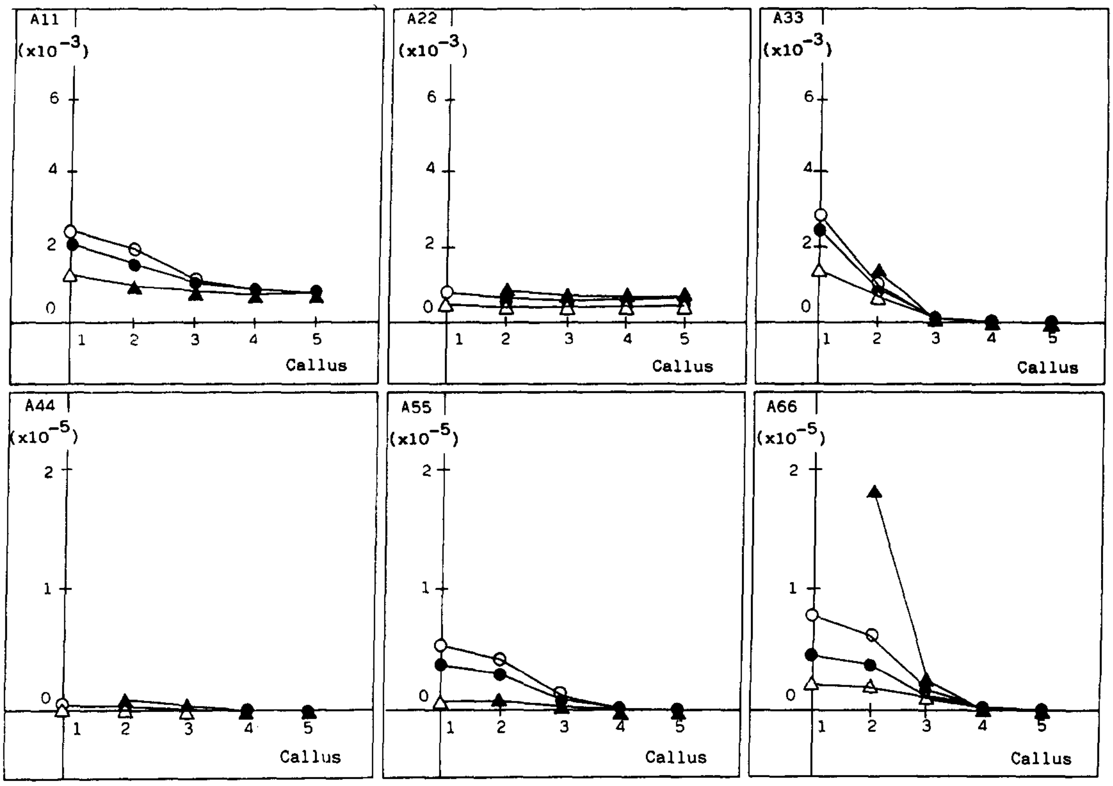

Fig. 6. Evolution of compliance matrix coefficients. (○) Hoffmann; (●) Wagner; (△) Orthofix; (▲) Ilizarov.

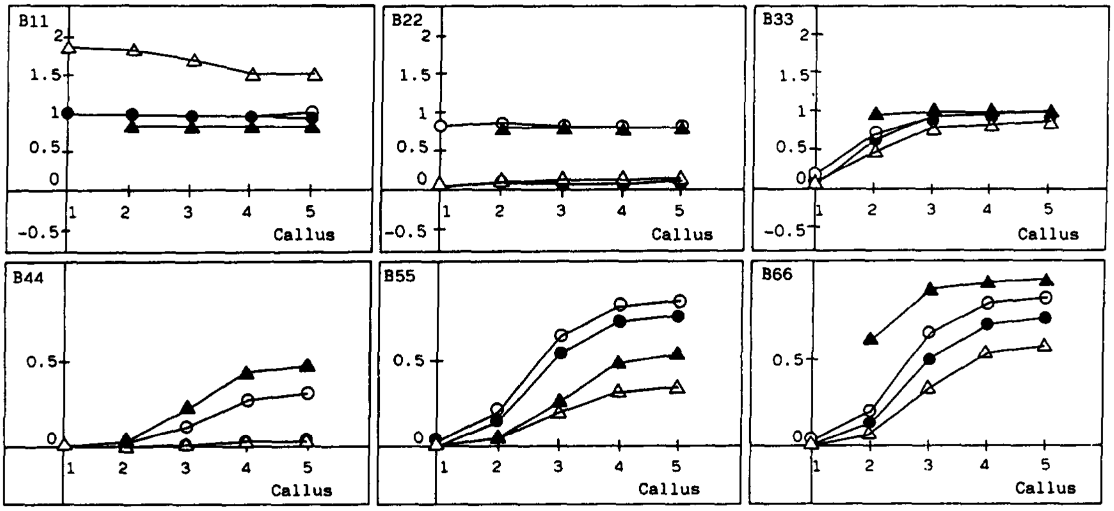

Fig. 7. Evolution of load transfer matrix coefficients. (○) Hoffmann; (●) Wagner; (△) Orthofix; (▲) Ilizarov.

When a coincidence between theoretical and experimental data was attained, we were able to use FEM as a method to compare the mechanical performance among the four fixators with different loads during human walking. This comparison showed the following.

(1) A negligible rigidity of each fixator during early consolidation stages to control the linear displacements in the X (anteroposterior) and Z (longitudinal) axis directions and the angular displacements around the transversal axis (Y) (Fig. 9).

(2) A higher load transmission through the callus in the anteroposterior (X) and longitudinal (Z) axis directions and higher moments around the transversal axis (Y) (Fig. 10). These facts showed the parallelism between the results obtained in both matrices.

Table 6. Effects of different variations on matrix coefficients. Wagner fixator (– no influence, ± slight influence, + remarkable influence)

Compliance matrix						Variations	Load transfer matrix					
A_{11}	A_{22}	A_{33}	A_{44}	A_{55}	A_{66}		B_{11}	B_{22}	B_{33}	B_{44}	B_{55}	B_{66}
+	–	+	–	+	+	Material	–	–	–	–	+	+
–	–	–	–	–	–	Pins groups separation	–	–	–	–	–	–
–	–	–	–	–	–	Pins separation	–	–	–	–	–	–
+	–	+	–	+	+	Side bar separation	–	–	+	–	+	+
–	+	–	–	–	–	Pins angle	–	+	–	–	–	–
+	+	–	–	–	–	Callus size	–	–	–	–	±	–

Table 7. Effects of different variations on matrix coefficients. Orthofix fixator (– no influence, ± slight influence, + remarkable influence)

Compliance matrix						Variations	Load transfer matrix					
A_{11}	A_{22}	A_{33}	A_{44}	A_{55}	A_{66}		B_{11}	B_{22}	B_{33}	B_{44}	B_{55}	B_{66}
–	–	+	–	–	+	Material	±	±	±	–	±	±
–	–	–	–	–	–	Pins groups separation	–	–	–	–	–	–
–	–		–	–	±	Pins number	–	–	–	–	–	+
–	–	–	–	–	–	Pins separation	+	+	–	–	±	–
±	–	+	–	±	±	Side bar separation	±	–	±	–	±	±
–	–	+	–	–	–	Pins angle (plane)	+	+	–	–	–	–
–	–	–	–	–	–	Pins angle (space)	–	–	–	–	–	–
–	–	+	–	–	+	Telescoping	–	–	+	–	–	–
+	+	–	–	–	–	Callus size	–	–	–	–	–	–

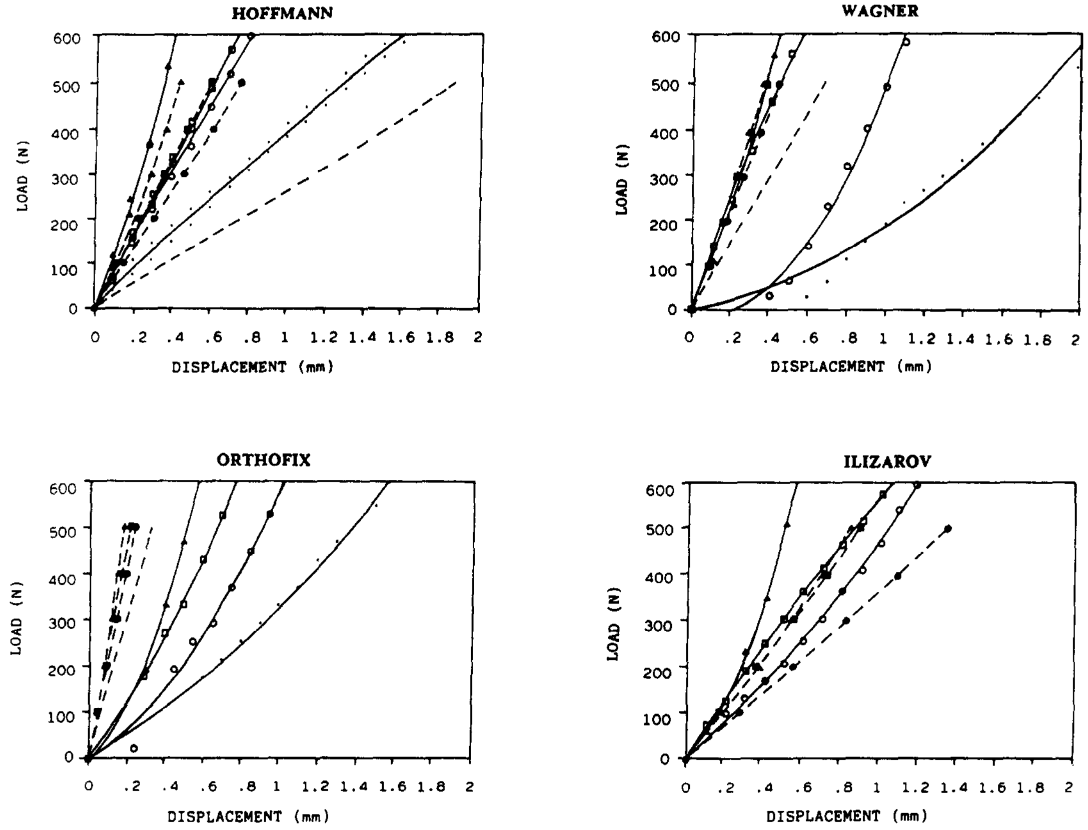

Fig. 8. Comparison between experimental and theoretical data. *Experimental:* (·) without callus; (○) Callus 1; (□) Callus 2; (△) Callus 3. *Theoretical:* (—) without callus; (●) Callus 1; (■) Callus 2; (▲) Callus 3.

Table 8. Effects of different variations on matrix coefficients. Ilizarov fixator (− no influence, ± slight influence, + remarkable influence)

Compliance matrix						Variations	Load transfer matrix					
A_{11}	A_{22}	A_{33}	A_{44}	A_{55}	A_{66}		B_{11}	B_{22}	B_{33}	B_{44}	B_{55}	B_{66}
−	−	−	−	−	±	Pin diameter	−	−	−	−	−	±
−	−	−	−	−	−	Pins groups separation	+	+	−	±	±	−
−	−	−	−	−	−	Pins separation	+	+	±	±	±	±
−	−	−	−	−	±	Pins number	−	−	−	−	−	±
+	+	±	+	+	±	Rings number	+	+	±	+	+	±
−	−	−	−	−	−	Ring diameter	−	−	−	−	−	−
+	±	−	±	+	−	Bars number	±	±	−	+	+	−
−	−	−	−	−	−	Pins angle	−	−	−	−	±	−
±	−	−	−	±	−	Bars separation	−	−	−	±	+	−
±	±	±	−	−	+	Callus size	−	−	−	−	−	−

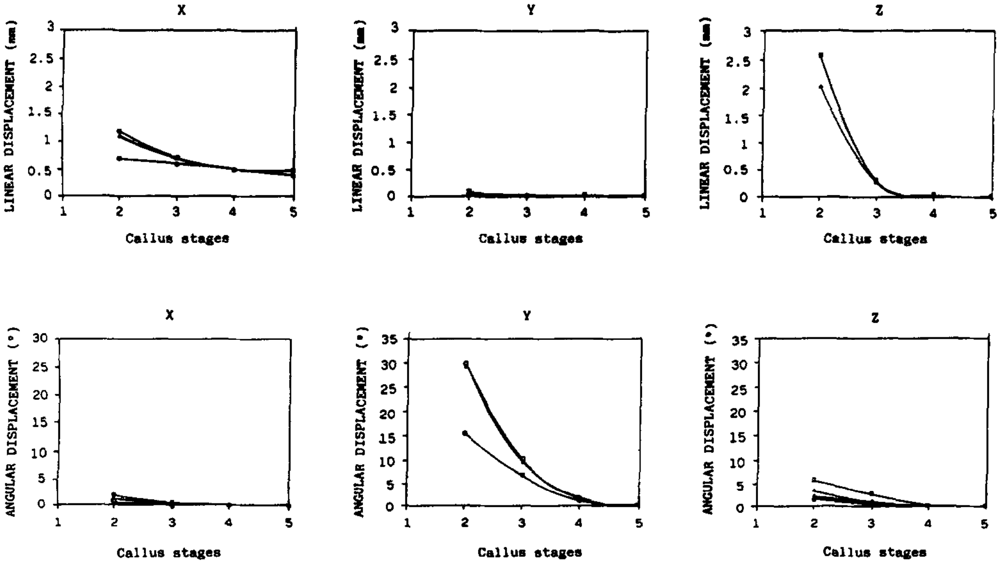

Fig. 9. Displacements calculated in the callus element during human gait. (△) Hoffmann; (▲) Wagner; (□) Orthofix; (■) Ilizarov.

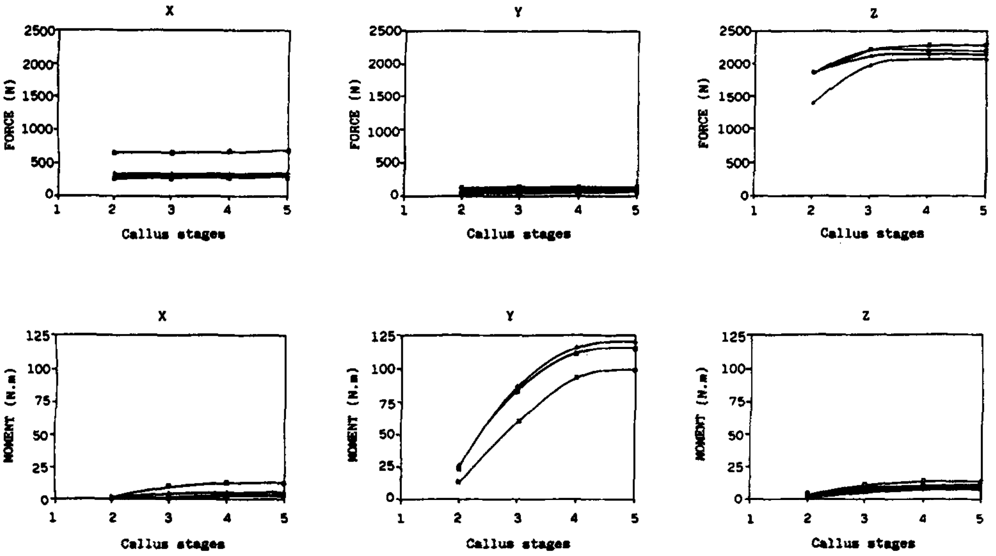

Fig. 10. Loads calculated in the callus element during human gait. (△) Hoffmann; (▲) Wagner; (□) Orthofix; (■) Ilizarov.

(3) A similar behaviour between the four fixators analysed, with some exceptions: it shows more load transmission along the X (anteroposterior) axis in the Orthofix fixator than in the others, and less levels of angular displacement around the Y (transversal) axis and less moment transmission around the same axis in the Ilizarov fixator than in the others.

DISCUSSION

A comparative theoretical study among the four external fixation devices shows two interesting and original aspects. First, the compliance and load transfer matrices characterize completely, by themselves, the mechanical performance of the bone–callus–external-fixator system. Secondly, fracture callus development and its consequences on the total system rigidity are considered.

After a validation of the theoretical model is carried out, variations of the four fixators are classified into three groups. Group I consists of modifications that obviously change the system rigidity. In Group III, variations with minimal effects on total rigidity are presented. In Group II, variations with an intermediate role are shown. Anyway, performance differences in Groups I and II are related to early fracture consolidation stages but, as the process evolves, these differences are minimum or absent.

Variables would be distributed as follows.

Group I
— pin material,
— pin diameter,
— number of pins on each bone fragment,
— lateral separation of side bars, and
— callus size.

Group II
— number of side bars,
— separation between pin groups,
— separation between pins, and
— angle of insertion of pins.

Group III
— side bar material,
— anterior and posterior separation of side bars, and
— side bar sliding.

From all this, it follows that Group I parameters present a major influence on system rigidity. Thus, variations of these parameters cause important changes on fixator performance.

The comparative analysis of the four fixators studied, bearing physiological loads, shows a scarce relationship between the defined parameters and the analysed fixator, and otherwise shows a great correlation between the same parameters and the callus presence and evolution. This comparison allows to distinguish between two types of variables. First, those which remain more or less constant during bone repair (Type I):

— X-linear displacements,
— X- and Y-forces, and
— X- and Z-moments.

Secondly, those which change with callus development (Type II):

— Z-linear displacements,
— Y- and Z-angular displacements,
— Z-forces, and
— Y-moments.

The possibility of modifying fixator performance acting on fixator design parameters affecting Type I variables is the main difference between these two groups. In Type II it is the callus development which regulates system rigidity.

Most of the researchers who have studied this problem (Currey, 1970; Bordás *et al.*, 1980; Briggs and Chao, 1982; Lortat-Jacob *et al.*, 1982; De Bastiani *et al.*, 1984; Finlay *et al.*, 1987; Cunningham *et al.*, 1987) have not considered the presence and development of the callus. They have compared several devices in an unvarying way. This kind of research only allows us to conclude about the mechanical performance of the device at the time of surgical application, but fixator performance varies as the healing process progresses.

The study of external fixation devices, taking into account callus presence, raises more interest than does frame rigidity analysis alone, because Calluses 3, 4 and 5 are present during 70–80% of the total healing time (Prat, 1990).

We conclude that the development of the callus plays an important role in total fixation system rigidity. Callus with minimal elastic characteristics causes some important variations in the load transmission pattern at the bone–callus–external-fixator structure. From an initial situation, when the fixator supports the total applied load, there follows another situation when callus assumes this function. We also observed some failures of the analysed fixators during early consolidation stages, when a callus immobilization is considered. A highly rigid external fixator would avoid some micromovements at early consolidation stages, but would not prevent load transmission through the callus when this callus appears.

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