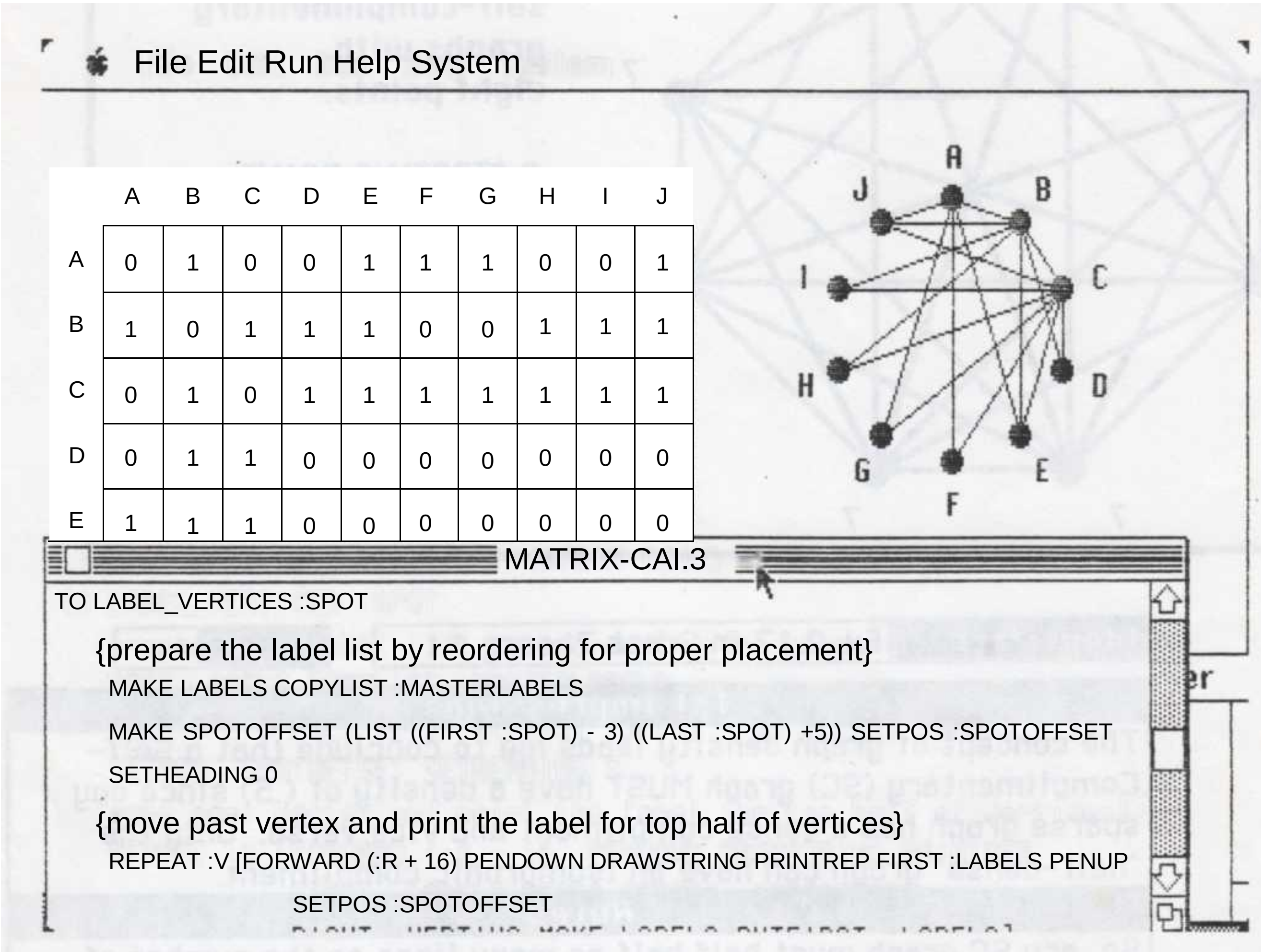
Graph Theory Instructional Courseware - Work in Progress

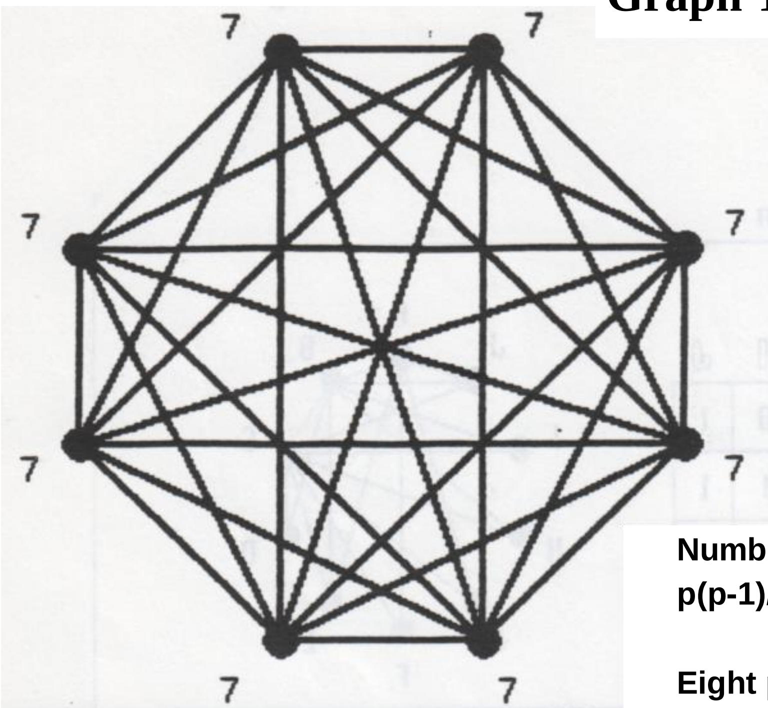


The above screen dump shows the exciting potential of highly graphical instructional material in graph theory which I am working on as an independent research project under the direction of Professors Boyd and Batchelder.

In addition to simple drill & practice presentation of the definitional basics of graph theory, an underpinning of intelligent graph algorithms will provide a "construction set" environment for student exploration of graphs, their various representations and mathematical manipulations associated with their adjacency matrices. This goal of a user-driven instructional environment guided selection of ExperLISP™ as the development environment.

Initial motivation to develop graph theoretical courseware came as a result of using Filevision™ for a graph theory class homework assignment. My professors were most

Graph Theory Homework #1.



Ex. 2.17 Draw the four self-complementary graphs with eight points.

A STARTING POINT:

The Complete Graph of Eight Points, K_8.

Number of Lines = p(p-1)/2 = 8*7/2 = 28.

Eight points of degree 7.

Identifier	Ex. 2.17 in Graph Theory # 1	Page	1

1st Impressions

The concept of graph density leads me to conclude that a Self-Complementary (SC) graph MUST have a density of (.5) since any sparse graph has a dense compliment and vice versa. Only the "half-dense" graph can have an isomorphic complement.

Notes

So, any SC graph must half half as many lines as the number of lines in the graph's associated Complete Graph, Ksub8 in this case.

Focus on the action of forming a graph's complement at the individual point level. Since a SC graph must have a density of .5, half of the lines will be incident to any given point in one version, to be replaced by the other half of the lines in its isomorphic reflection.

This half and half aspect is obvious and simple when the Complete Graph has an even point degree, as in K(sub)5 with ten lines. The first SC is easy, spread the point degrees evenly over the five points by degree two and the "Pentagon/Star" SC is the result. Additional SCs of Complete Graphs can be formed around mirror- image "stair-step" functions around the longest path [as in the degree path 1,3,2,3,1 on K(sub)5],

(See page 2 for SCs of Complete Graphs with odd point degrees.)

Identifier	Ex 2.17, Graph Theory # 1	Page	2

1st Impressions

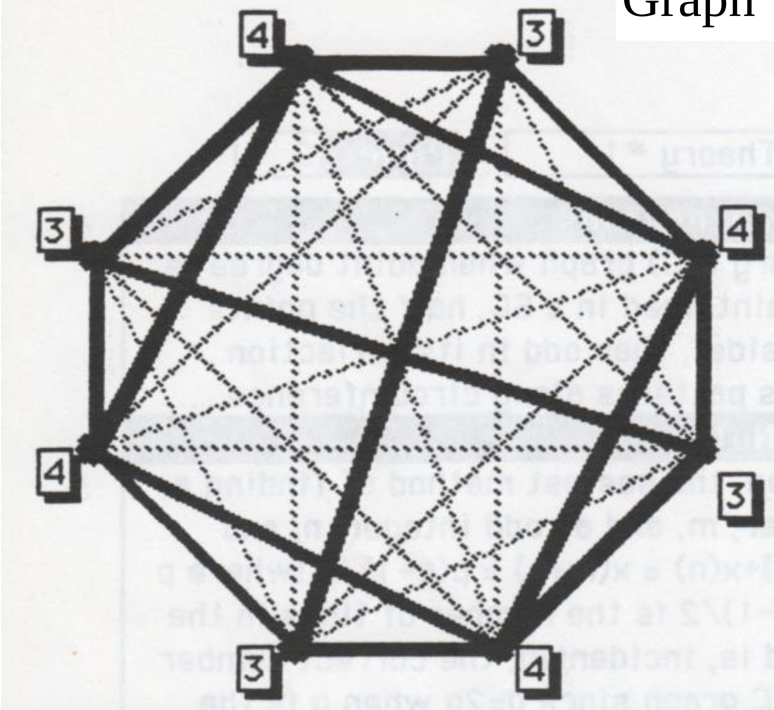
... forming a Self-Complementary (SC) graph when point degree is odd... since density must be maintained in a SC, half the points will have even degree on one "side", then odd in its reflection... variations are formed in series patterns along circumference ...

Notes

When facing an odd point degree, the easiest method of finding a SC is to look for on even integer, m, and an odd integer, n, and some number, x, such that $x(m)+x(n) = x(m+n) = p(p-1)/2$ [where p = the number of points and $p(p-1)/2$ is the number of lines in the associated Complete Graph and is, incidently, the correct number of point degrees desired in a SC graph since d=2q when q is the number of lines. Multiply 2q by the desired density of .5 and you get q].

In the case of K(sub)8, solving the equation gives; x=4, m=4 and n=3. Therefore, in any SC of K(sub)8, four points of three degrees will reflect into four of four degrees while four of four reflect on four of three. The four variations of this "3d/4d" balance are found by by developing "patterns" along the circumference. They are:

1. 3d , 4d , 3d , 4d , 3d , 4d ,3d , 4d (by one's)
2. 3d , 3d , 4d , 4d , 3d , 3d , 4d , 4d (by two's)
3. 3d , 3d , 3d , 4d , 4d , 4d , 3d , 4d (by three's & by one's)
4. 3d , 3d , 3d , 3d , 4d , 4d , 4d , 4d (by four's)



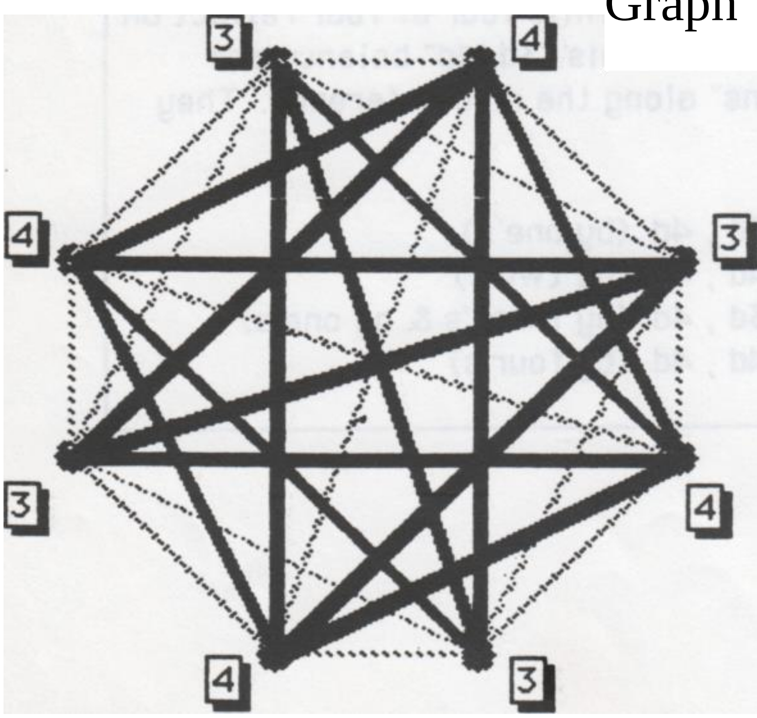
Graph Theory Homework #1.

Ex. 2.17 - Draw the four self-complementary graphs with eight points.

First Variation - Side 1:

Point degrees along the circumference are:

3 , 4 , 3 , 4 , 3 , 4 , 3 , 4



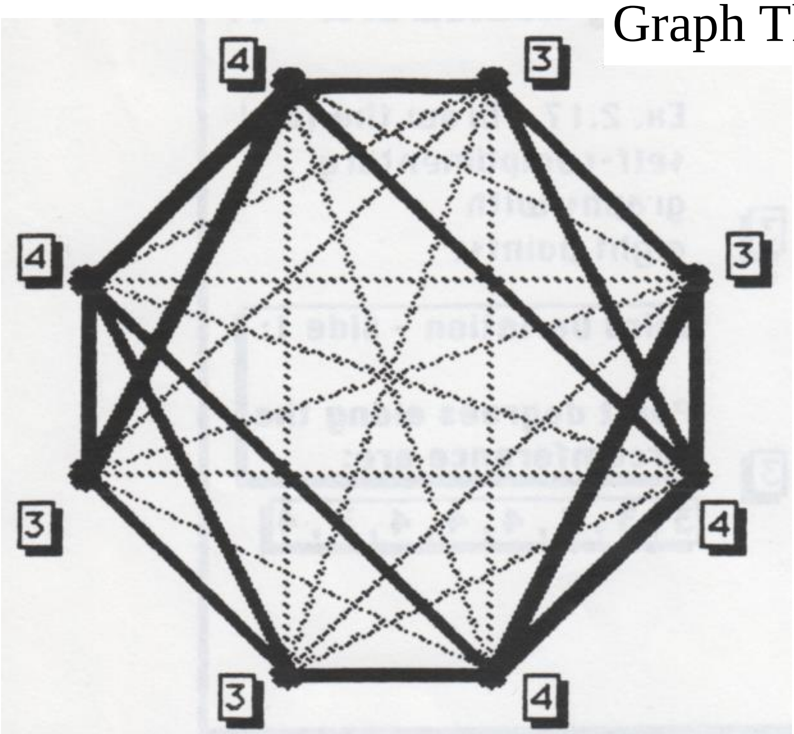
Graph Theory Homework #1.

Ex. 2.17 - Draw the four self-complementary graphs with eight points.

First Variation - Side 2:

Point degrees along the circumference are:

3 , 4 , 3 , 4 , 3 , 4 , 3 , 4



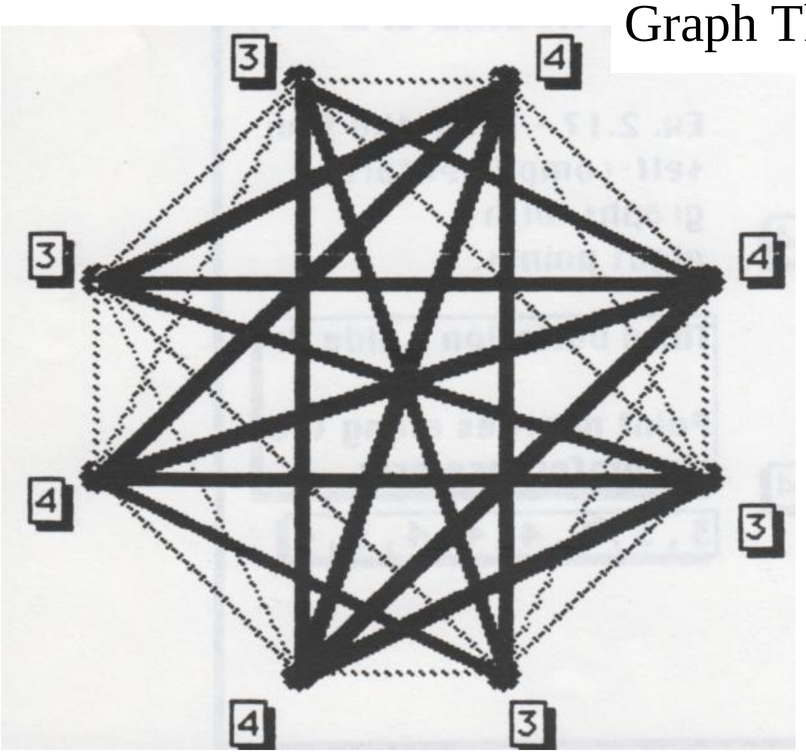
Graph Theory Homework #1.

Ex. 2.17 - Draw the four self-complementary graphs with eight points.

Second Variation - Side 1:

Point degrees along the circumference are:

3 , 3 , 4 , 4 , 3 , 3 , 4 , 4



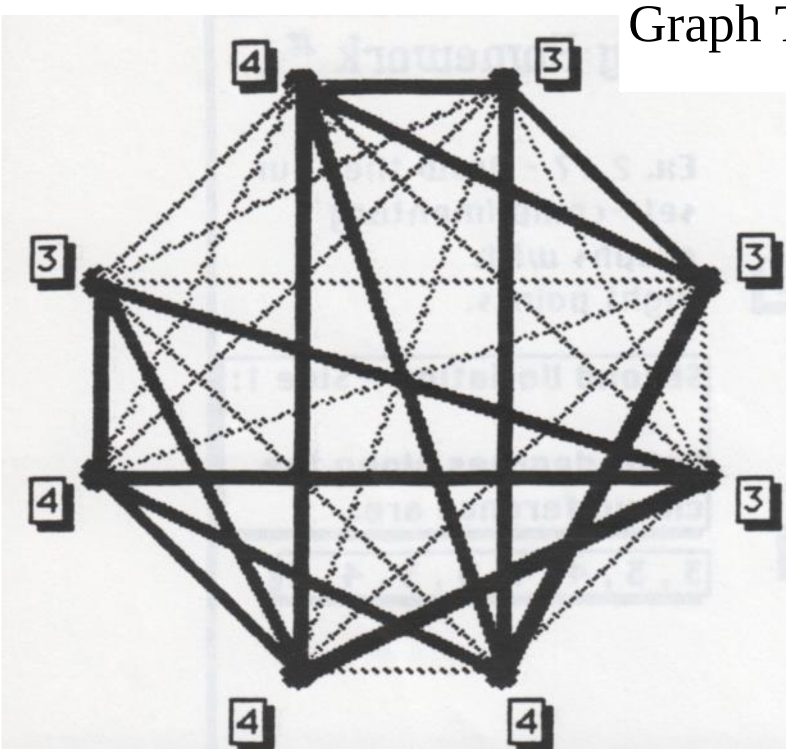
Graph Theory Homework #1.

Ex. 2.17 - Draw the four self-complementary graphs with eight points.

Second Variation - Side 2:

Point degrees along the circumference are:

3 , 3 , 4 , 4 , 3 , 3 , 4 , 4



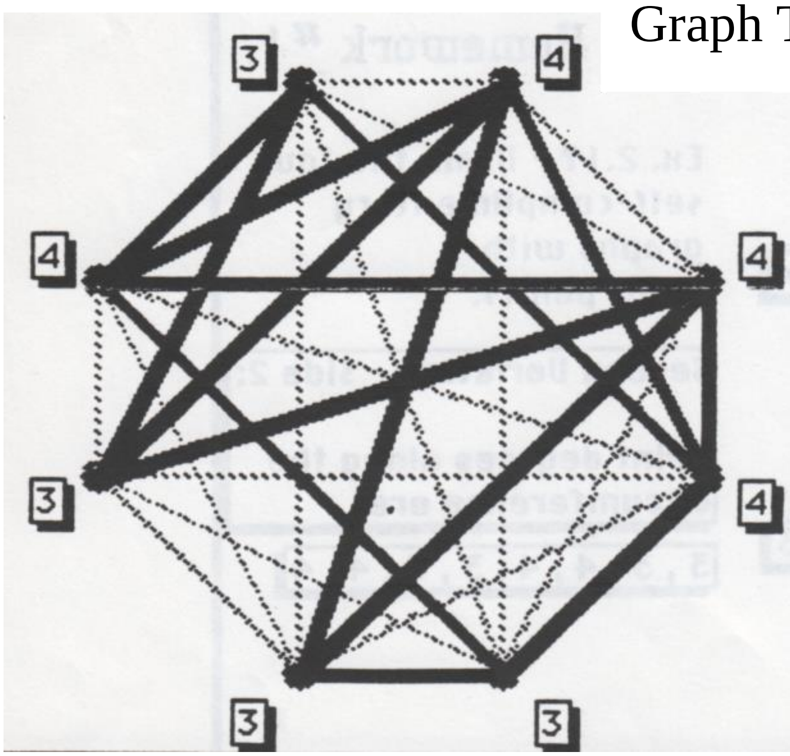
Graph Theory Homework #1.

Ex. 2.17 - Draw the four self-complementary graphs with eight points.

Third Variation - Side 1:

Point degrees along the circumference are:

3 , 3 , 3 , 4 , 4 , 4 , 3 , 4



Graph Theory Homework #1.

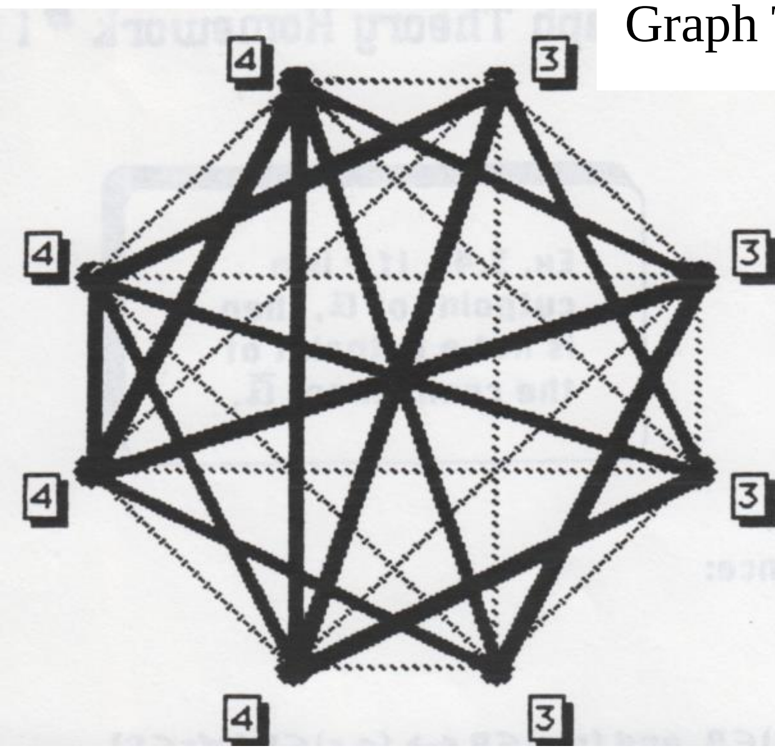
Ex. 2.17 - Draw the four self-complementary graphs with eight points.

Third Variation - Side 2:

Point degrees along the circumference are:

3 , 3 , 3 , 4 , 4 , 4 , 3 , 4

Graph Theory Homework #1.



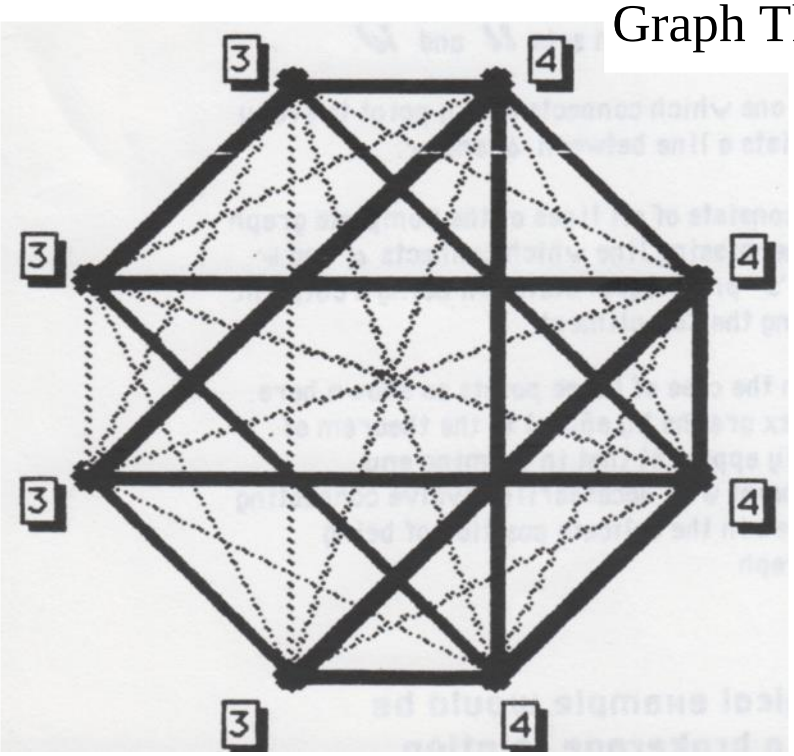
Ex. 2.17 - Draw the four self-complementary graphs with eight points.

Fourth Variation - Side 1:

Point degrees along the circumference are:

3 , 3 , 3 , 3 , 4 , 4 , 4 , 4

Graph Theory Homework #1.



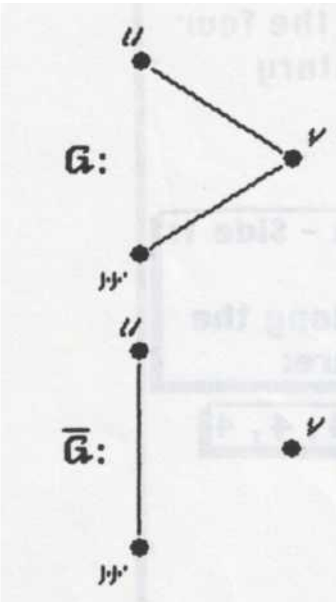
Ex. 2.17 - Draw the four self-complementary graphs with eight points.

Fourth Variation - Side 2:

Point degrees along the circumference are:

3 , 3 , 3 , 3 , 4 , 4 , 4 , 4

Graph Theory Homework #1



Ex. 3.4 - If *v* is a cutpoint of *G*, then *v* is not a cutpoint of the complement *Ḡ*.

Graph Equivalence:

For <G,R>

$p \equiv_G q \leftrightarrow (p,q) \epsilon R$, and $(p,r) \epsilon R \leftrightarrow (q,r) \epsilon R$ [ɄrϵG]

From the statement of graph theoretic equivalence, $\equiv_G$, the proof can be reduced to the simplest case of three points. In other words, *u* and *w* represent the collective cases of all elements in sets *U* and *W*

A Complete Graph on *V(G)* is one which connects every point to every other. This implies that there exists a line between *u* and *w*

Since the Complement of a graph consists of all lines of the Complete graph on the same point set, *V(G),* the missing line which connects *u* and *w* must be in the Complement. So *v*'s "privileged" status in being a cutpoint is necessarily destroyed in forming the Complement.

This relationship is easily seen in the case of three points as shown here. "Popping" back up to more complex graphs by appeal to the theorem of graph equivalence, it is intuitively apparent that in forming any complement of a graph with a cutpoint will necessarily involve connecting all points to all others that had been in the delicate position of being separable by *v* in the original graph.

A relevant sociological example would be the dissolution of a brokerage relation.