1. Match the APs given in column A with suitable common differences given in column B.

Column A		Column B	
(A_1)	$2, -2, -6, -10, \ldots$	(B_1)	$\frac{2}{3}$
(A_2)	$a = -18, n = 10, a_n = 0$	(B_2)	-5
(A_3)	$a = 0, a_{10} = 6$	(B_3)	4
(A_4)	$a_2 = 13, a_4 = 3$	(B_4)	-4
		(B_5)	2
		(B_6)	$\frac{1}{2}$
		(B_7)	5

2. Verify that each of the following is an AP, and then write its next three terms.

(i) $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$

(ii) $5, \frac{14}{3}, \frac{13}{3}, 4, \ldots$

(iii) $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots$

(iv) $a + b, (a + 1) + b, (a + 1) + (b + 1), \ldots$

(v) $a, 2a + 1, 3a + 2, 4a + 3, \ldots$

3. Write the first three terms of the APs when a and d are as given below:

(i) $a = \frac{1}{2}$, $d = -\frac{1}{6}$

(ii) $a = -5$, $d = -3$

(iii) $a = \sqrt{2}$, $d = \frac{1}{\sqrt{2}}$

4. Find a, b and c such that the following numbers are in AP: a, 7, b, 23, c.

5. Determine the AP whose fifth term is 19 and the difference of the eighth term from the thirteenth term is 20.

6. The 26th, 11th and the last term of an AP are 0, 3 and $-\frac{1}{5}$, respectively. Find the common difference and the number of terms.

7. The sum of the 5th and the 7th terms of an AP is 52 and the 10th term is 46. Find the AP.

8. Find the 20th term of the AP whose 7th term is 24 less than the 11th term, first term being 12.

9. If the 9th term of an AP is zero, prove that its 29th term is twice its 19th term.

10. Find whether 55 is a term of the AP: 7, 10, 13,--- or not. If yes, find which term it is.

11. Determine k so that $k^2 + 4k + 8$, $2k^2 + 3k + 6$, $3k^2 + 4k + 4$ are three consecutive terms of an AP.

12. Split 207 into three parts such that these are in AP and the product of the two smaller parts is 4623.

13. The angles of a triangle are in AP. The greatest angle is twice the least. Find all the angles of the triangle.

14. If the nth terms of the two APs: 9, 7, 5, ... and 24, 21, 18,... are the same, find the value of n. Also find that term.

15. If sum of the 3rd and the 8th terms of an AP is 7 and the sum of the 7th and the 14th

terms is –3, find the 10^{th} term.

16. Find the 12^{th} term from the end of the AP: –2, –4, –6,..., –100.

17. Which term of the AP: 53, 48, 43,... is the first negative term?

18. How many numbers lie between 10 and 300, which when divided by 4 leave a remainder 3?

19. Find the sum of the two middle most terms of the AP: $-\frac{4}{3}, -1, -\frac{2}{3}, \ldots, 4\frac{1}{3}$.

20. The first term of an AP is –5 and the last term is 45. If the sum of the terms of the AP is 120, then find the number of terms and the common difference.

21. Find the sum:

(i) $1 + (-2) + (-5) + (-8) + \ldots + (-236)$

(ii) $4 - \frac{1}{n} + 4 - \frac{2}{n} + 4 - \frac{3}{n} + \ldots$ upto n terms

(iii) $\frac{a-b}{a+b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} + \cdots$ to 11 terms.

22. Which term of the AP: –2, –7, –12,... will be –77? Find the sum of this AP upto the term –77.

23. If $a_n = 3 - 4n$, show that $a_1, a_2, a_3, \ldots$ form an AP. Also find S_{20}.

24. In an AP, if $S_n = n\ (4n + 1)$, find the AP.

25. In an AP, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k.

26. If S_n denotes the sum of first n terms of an AP, prove that

$$S_{12} = 3(S_8 - S_4)$$

27. Find the sum of first 17 terms of an AP whose 4^{th} and 9^{th} terms are –15 and –30 respectively.

28. If sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.

29. Find the sum of all the 11 terms of an AP whose middle most term is 30.

30. Find the sum of last ten terms of the AP: 8, 10, 12,---, 126.

31. Find the sum of first seven numbers which are multiples of 2 as well as of 9.

[**Hint:** Take the LCM of 2 and 9]

32. How many terms of the AP: –15, –13, –11,--- are needed to make the sum –55? Explain the reason for double answer.

33. The sum of the first n terms of an AP whose first term is 8 and the common difference is 20 is equal to the sum of first $2n$ terms of another AP whose first term is – 30 and the common difference is 8. Find n.

34. Kanika was given her pocket money on Jan 1^{st}, 2008. She puts Re 1 on Day 1, Rs 2 on Day 2, Rs 3 on Day 3, and continued doing so till the end of the month, from this money into her piggy bank. She also spent Rs 204 of her pocket money, and found that at the end of the month she still had Rs 100 with her. How much was her pocket money for the month?

35. Yasmeen saves Rs 32 during the first month, Rs 36 in the second month and Rs 40 in the third month. If she continues to save in this manner, in how many months will she save Rs 2000?

36. The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the sum of the first ten terms of this AP is 235, find the sum of its first twenty terms.

37. Find the

(i) sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.

(ii) sum of those integers from 1 to 500 which are multiples of 2 as well as of 5 .

(iii) sum of those integers from 1 to 500 which are multiples of 2 or 5.

[Hint (iii) : These numbers will be : multiples of 2 + multiples of 5 – multiples of 2 as well as of 5]

38. The eighth term of an AP is half its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15^{th} term.

39. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the AP.

40. Find the sum of the integers between 100 and 200 that are

(i) divisible by 9

(ii) not divisible by 9

[Hint (ii) : These numbers will be : Total numbers – Total numbers divisible by 9]

41. The ratio of the 11^{th} term to the 18^{th} term of an AP is 2 : 3. Find the ratio of the 5^{th} term to the 21^{st} term, and also the ratio of the sum of the first five terms to the sum of the first 21 terms.

42. Show that the sum of an AP whose first term is a, the second term b and the last term c, is equal to

$$\frac{a \quad c \quad b \quad c-2a}{2 \quad b-a}$$

Solve the equation

$$-4+(-1)+2+\ldots+x=437$$

43. Jaspal Singh repays his total loan of Rs 118000 by paying every month starting with the first instalment of Rs 1000. If he increases the instalment by Rs 100 every month, what amount will be paid by him in the 30^{th} instalment? What amount of loan does he still have to pay after the 30^{th} instalment?

44. The students of a school decided to beautify the school on the Annual Day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 m. The flags are stored at the position of the middle most flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time. How much distance did she cover in completing this job and returning back to collect her books? What is the maximum distance she travelled carrying a flag?

Question-10

Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers.

Solution:

Let the three numbers be a – d, a, a+ d.

$a - d + a + a + d = 27$

$3a = 27$

$a = 9$

$(a - d)(a)(a + d) = 648$

$a(a^2 - d^2) = 648$

$9(9^2 - d^2) = 648$

$9^2 - d^2 = 72$

$d^2 = 81 - 72$

$d^2 = 9$

$d = 3$

The numbers are 6, 9, 12.

Question-8

An A.P consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32^{nd} term.

Solution:

Given, $n = 60$, $a_1 = 7$,

and $a_{60} = 125$

$\Rightarrow a_1 + 59d = 125$

$7 + 59d = 125$

$59d = 118$

$d = 118/59 = 2$

$a_{32} = a_1 + 31d = 7 + 31(2) = 7 + 62$

$\therefore a_{32} = 69$.

Question-7

The sum of the first six terms of an A.P is zero and the fourth term is 2. Find the sum of its first 30 terms.

Solution:

Let the sum of first 30 terms be S_{30}, first term be a, fourth term be a_4 and the sum of first six terms be S_6.

Given that $S_6 = 0$ and fourth term $a_4 = 2$

$\Rightarrow a + 3d = 2$ ....(i)

$S_6 = 0$

$\frac{n}{2}(2a + 5d) = 0$

$\Rightarrow 2a + 5d = 0$ ....(ii)

(i) × 2,

$2a + 6d = 4$ ....(iii)

(iii) – (ii),

$\therefore d = 4$

Substituting the value of d = 4 in (i),

$a + 3 \times (4) = 2$

$\Rightarrow a = 2 - 12 = -10$

$\therefore a_{30} = a + 29d$

$= -10 + 29 \times (4)$

$= -10 + 116$

$= 106$

$\therefore$ Sum to first 30 terms $= S_{30} = \frac{n}{2}(a + l)$

$= \frac{30}{2}(-10 + 106)$

$= 15 \times 96$

$= 1140.$

Question-9

Find the sum of the series 51 + 50 + 49 + ... + 21.

Solution:

51 + 50 + 49 + ... + 21

$a = 51, d = -1, a_n = 21$

$\therefore a + (n - 1)d = a_n$

$51 + (n - 1)(-1) = 21$

$(n - 1)(-1) = 21 - 51$

$n - 1 = 30$

$\therefore n = 31$

$\therefore$ Sum of the series $= \frac{31}{2}(51 + 21)$

$= \frac{31}{2} \times 72$

$= 1116$

$\therefore$ The sum of the series 51 + 50 + 49 + ... + 21 = 1116.

Question-3

In a certain A.P the 24th term is twice the 10th term. Prove that the 72nd term is twice the 34th term.

Solution:

Given, $a_{24} = 2a_{10}$

$a_{24} = a + 23d$ and $a_{10} = a + 9d$

To prove: $a_{72} = 2a_{34}$

$a_{72} = a + 71d$

$a_{34} = a + 33d$

$a_{24} = 2a_{10}$ (Given)

$a + 23d = 2(a + 9d)$

$a + 23d = 2a + 18d$

$a - 5d = 0$

$a = 5d$..........................(i)

$a_{72} = 2a_{34}$

$a + 71d = 2(a + 33d)$

$a + 71d = 2a + 66d$

$a - 5d = 0$

$a = 5d$..........................(ii)

from, (1) and (2) $a_{72} = 2a_{34}$

Hence proved.

Question-2

Find the A.P. whose 10th term is 5 and 18th term is 77.

Solution:

given, 10th term of an A.P= 5

Þ $a + (10 - 1)d = 5$

$\Rightarrow a + 9d = 5$ ....(i)

and 18th term = 77

Þ$a + (18 - 1)d = 77$

$\Rightarrow a + 17d = 77$ ....(ii)

(ii) – (i), $8d = 72$

$\therefore d = 9$

Substituting the value of $d = 9$ in (i),

$a + 81 = 5$

$a = 5 - 81 = -76$

$\therefore$ The A.P. is $-76, -67,$ ....

Question-5

If 9^{th} term of an A.P. is zero, prove that its 29^{th} term is double the 19^{th} term.

Solution:

9^{th} term = 0

$a_1 + 8d = 0$

$a_{29} = a_1 + 28d = a_1 + 8d + 20d = 0 + 20d = 20d$

$a_{19} = a_1 + 18d = a_1 + 8d + 10d = 0 + 10d = 10d$

$a_{29} = 2a_{19}$.

Question-11

How many terms of A.P -10, -7, -4, -1, ... must be added to get the sum -104?

Solution:

-10, -7, -4, -1, ...

a = - 10, d = 3

$S_n = \frac{1}{2}n\{2a + (n - 1) d\}$

$-104 = \frac{1}{2}n \{2(-10) + (n - 1) 3\}$

$= \frac{1}{2}n (-20 + 3n - 3)$

$-208 = n (3n - 23)$

$3n^2 - 23n + 208 = 0$

$3n^2 - 39n + 16n + 208 = 0$

$3n (n - 13) + 16 (n - 13) = 0$

$(n - 13) (3n + 16) = 0$

$\therefore n = 13$

$\therefore$ 13 terms must be added to get the sum of the A.P – 104.

Question-12

If the sum of p terms of an A.P is $3p^2 + 4p$, find its n^{th} term.

Solution:

$S_p = 3p^2 + 4p$

$t_n = S_n - S_{n-1}$

$= (3n^2 + 4n) - [3(n - 1)^2 + 4(n - 1)]$

$= (3n^2 + 4n) - [3(n^2 - 2n + 1) + 4(n - 1)]$

$= (3n^2 + 4n) - [3n^2 - 6n + 3 + 4n - 4]$

$= (3n^2 + 4n) - [3n^2 - 2n - 1]$

$= 3n^2 + 4n - 3n^2 + 2n + 1$

$= 6n + 1$

Therefore the n^{th} term is $6n + 1$.

Question-6

Determine the A.P whose third term is 16 and the difference of 5th from 7th term is 12.

Solution:

Let the A.P. be a, a + d, a + 2d, ……..

⇒ The third term = a_3 = a + 2d = 16 ………………(i)

and seventh term = a_7 = a + 6d

Given that $a_7 - a_5 = 12$

⇒ (a + 6d) – (a + 4d) = 12

⇒ a + 6d – a – 4d = 12

⇒ 2d = 12

⇒ d = 6

Substituting the value of d = 6 in (i),

a + 12 = 16

a = 4

∴ The first term of the A.P. is 4 and the common difference is 6.

∴ The A.P. is 4, 10, 16, 22, 28, 34, …

∴ The fifth term = a_5 = a + 4d.

Question-1

Find the sum of the following A.P. 1, 3, 5, 7, ...,199.

Solution:

Given, a = 1, d = 2 , a_n = l = 199,

a + (n – 1) d = 199

1 + (n – 1) 2 = 199

⇒ 1 + 2n – 2 = 199

⇒ 2n = 200

∴ $n = \frac{200}{2}$

n = 100.

S_n = n/2 (a + l)

= 50(1 + 199)

= 50(200)

= 10000

Question-4

a, b and c are in A.P. Prove that b + c, c + a and a + b are in A.P.

Solution:

Given, a, b and c are in A.P.

$\therefore$ b – a = c – b

To prove : **b + c, c + a and a + b are in A.P.**

c + a – (b + c) = a + b – (c + a)

$\Rightarrow$ c + a – b – c = a + b – c – a

a – b = b – c

$\Rightarrow$ b – a = c – b

$\therefore$ a, b, c are in A.P.

$\therefore$ b + c, c + a and a + b are in A.P.