CBSE Class 10 Mathematics

Important Questions

Chapter - 5

Arithmetic progressions

One of the endlessly alluring aspects of mathematics is that its thorniest paradoxes have a way of blooming into beautiful theories

1. **The fourth term of an AP is 0. Prove that its 25^{th} term is triple its 11^{th} term.**

   Ans: $a_4 = 0$

   $\Rightarrow a + 3d = 0$

   T.P a25= 3 (a11)

   $\Rightarrow a + 24d = 3(a + 10d)$

   $\Rightarrow a + 24d = 3a + 30d$

   RHS sub $a = -3d$

   $-3d + 24d = 21d$

   LHS $3a + 30d$

   $-9d + 30d = 21d$

   LHS = RHS. Hence proved

2. **Find the 20^{th} term from the end of the AP 3, 8, 13........253.**

   Ans: 3, 8, 13 253

   Last term = 253

   a20 from end

   $= l - (n-1)d$

   $253 - (20-1)\,5$

   $253 - 95$

   $= 158$

3. **If the p^{th}, q^{th} & r^{th} term of an AP is x, y and z respectively, show that $x(q-r) + y(r-p) + z(p-q) = 0$**

   Ans: p^{th} term $\Rightarrow x = A + (p-1)D$

   q^{th} term $\Rightarrow y = A + (q-1)D$

   r^{th} term $\Rightarrow z = A + (r-1)D$

1. Match the APs given in column A with suitable common differences given in column B.

Column A		Column B	
(A_1)	$2, -2, -6, -10, \ldots$	(B_1)	$\frac{2}{3}$
(A_2)	$a = -18, n = 10, a_n = 0$	(B_2)	-5
(A_3)	$a = 0, a_{10} = 6$	(B_3)	4
(A_4)	$a_2 = 13, a_4 = 3$	(B_4)	-4
		(B_5)	2
		(B_6)	$\frac{1}{2}$
		(B_7)	5

2. Verify that each of the following is an AP, and then write its next three terms.

(i) $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$

(ii) $5, \frac{14}{3}, \frac{13}{3}, 4, \ldots$

(iii) $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots$

(iv) $a + b, (a + 1) + b, (a + 1) + (b + 1), \ldots$

(v) $a, 2a + 1, 3a + 2, 4a + 3, \ldots$

3. Write the first three terms of the APs when a and d are as given below:

T.P x(q-r) + y(r-p) + z(p-q) = 0

={A+(p-1)D}(q-r) + {A + (q-1)D} (r-p)

+ {A+(r-1)D} (p-q)

A {(q-r) + (r-p) + (p-q)} + D {(p-1)(q-r)

+ (r-1) (r-p) + (r-1) (p-q)}

⇒ A.0 + D{p(q-r) + q(r-p) + r (p-q)

- (q-r) – (r-p)-(p-q)}

= A.0 + D.0 = 0. Hence proved

4. **Find the sum of first 40 positive integers divisible by 6 also find the sum of first 20 positive integers divisible by 5 or 6.**

Ans: No's which are divisible by 6 are 6, 12 .... 240.

S40 = [240] 6+240

= 20 x 246

= 4920

No's div by 5 or 6

30, 60 600

[220] 30+600

= 10 x 630

= 6300

5. **A man arranges to pay a debt of Rs.3600 in 40 monthly instalments which are in a AP. When 30 instalments are paid he dies leaving one third of the debt unpaid. Find the value of the first instalment.**

Ans: Let the value of I instalment be x S_{40} = 3600.

$\Rightarrow \frac{40}{2}[2a+39d]=3600$

⇒2a + 39d = 180 1

$S_{30}=\frac{30}{2}[2a+29d]=2400$

⇒ 30a + 435d = 2400

⇒ 2a + 29d = 160 2

Solve 1 & 2 to get

d = 2 a = 51.

∴ I instalment = Rs.51.

6. **Find the sum of all 3 digit numbers which leave remainder 3 when divided by 5.**

Ans: 103, 108..........998

(i) $a = \frac{1}{2}$, $d = -\frac{1}{6}$

(ii) $a = -5$, $d = -3$

(iii) $a = \sqrt{2}$, $d = \frac{1}{\sqrt{2}}$

4. Find a, b and c such that the following numbers are in AP: a, 7, b, 23, c.

5. Determine the AP whose fifth term is 19 and the difference of the eighth term from the thirteenth term is 20.

6. The 26^{th}, 11^{th} and the last term of an AP are 0, 3 and $-\frac{1}{5}$, respectively. Find the common difference and the number of terms.

7. The sum of the 5^{th} and the 7^{th} terms of an AP is 52 and the 10^{th} term is 46. Find the AP.

8. Find the 20^{th} term of the AP whose 7^{th} term is 24 less than the 11^{th} term, first term being 12.

9. If the 9^{th} term of an AP is zero, prove that its 29^{th} term is twice its 19^{th} term.

10. Find whether 55 is a term of the AP: 7, 10, 13,--- or not. If yes, find which term it is.

11. Determine k so that $k^2 + 4k + 8$, $2k^2 + 3k + 6$, $3k^2 + 4k + 4$ are three consecutive terms of an AP.

12. Split 207 into three parts such that these are in AP and the product of the two smaller parts is 4623.

13. The angles of a triangle are in AP. The greatest angle is twice the least. Find all the angles of the triangle.

14. If the nth terms of the two APs: 9, 7, 5, ... and 24, 21, 18,... are the same, find the value of n. Also find that term.

15. If sum of the 3^{rd} and the 8^{th} terms of an AP is 7 and the sum of the 7^{th} and the 14^{th}

a + (n-1)d = 998

⇒ 103 + (n-1)5 = 998

⇒ n = 180

$S_{180} = \frac{180}{2}[103 + 998]$

S_{180} = 99090

7. **Find the value of x if 2x + 1, x2 + x +1, 3x2 - 3x +3 are consecutive terms of an AP.**

   Ans: $a_2 - a_1 = a_3 - a_2$

   ⇒ $x^2 + x + 1 - 2x - 1 = 3x^2 - 3x + 3 - x^2 - x - 1$

   $x^2 - x = 2x^2 - 4x + 2$

   ⇒ $x^2 - 3x + 2 = 0$

   ⇒ (x -1) (x – 2) = 0

   ⇒ x = 1 or x = 2

8. **Raghav buys a shop for Rs.1,20,000.He pays half the balance of the amount in cash and agrees to pay the balance in 12 annual instalments of Rs.5000 each. If the rate of interest is 12% and he pays with the instalment the interest due for the unpaid amount. Find the total cost of the shop.**

   Ans: Balance = Rs.60,000 in 12 instalment of Rs.5000 each.

   Amount of I instalment $= 5000 + \frac{12}{100}60,000$

   II instalment = 5000 + (Interest on unpaid amount)

   = 5000 + 6600 $\left[\frac{12}{100} \times 55000\right]$

   = 11600

   III instalment = 5000 + (Interest on unpaid amount of Rs.50,000)

   ∴ AP is 12200, 11600, 11000

   D = is 600

   Cost of shop = 60000 + [sum of 12 instalment]

   $= 60,000 + \frac{12}{2}[24,400 - 6600]$

   = 1,66,800

9. **Prove that $a_{m+n} + a_{m-n} = 2a_m$**

   Ans: $a_{m+n} = a_1 + (m + n - 1)\,d$

   $a_{m-n} = a_1 + (m - n - 1)\,d$

   $a_m = a_1 + (m-1)\,d$

terms is –3, find the 10^{th} term.

16. Find the 12^{th} term from the end of the AP: –2, –4, –6,..., –100.

17. Which term of the AP: 53, 48, 43,... is the first negative term?

18. How many numbers lie between 10 and 300, which when divided by 4 leave a remainder 3?

19. Find the sum of the two middle most terms of the AP: $-\frac{4}{3}, -1, -\frac{2}{3}, ..., 4\frac{1}{3}$.

20. The first term of an AP is –5 and the last term is 45. If the sum of the terms of the AP is 120, then find the number of terms and the common difference.

21. Find the sum:

(i) $1 + (-2) + (-5) + (-8) + ... + (-236)$

(ii) $4 - \frac{1}{n} + 4 - \frac{2}{n} + 4 - \frac{3}{n} + ...$ upto n terms

(iii) $\frac{a-b}{a+b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} + \cdots$ to 11 terms.

22. Which term of the AP: –2, –7, –12,... will be –77? Find the sum of this AP upto the term –77.

23. If $a_n = 3 - 4n$, show that $a_1, a_2, a_3, ...$ form an AP. Also find S_{20}.

24. In an AP, if $S_n = n(4n + 1)$, find the AP.

25. In an AP, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k.

26. If S_n denotes the sum of first n terms of an AP, prove that

$$S_{12} = 3(S_8 - S_4)$$

27. Find the sum of first 17 terms of an AP whose 4^{th} and 9^{th} terms are –15 and –30 respectively.

Add 1 & 2

$a_{m+n} + a_{m-n} = a_1 + (m+n-1)\,d + a_1 + (m-n-1)d$

$= 2a_1+(m+n+m-n-1-1)d$

$= 2a_1+ 2(m-1)d$

$= 2[a_1+ (m-1)d]$

$= 2[a_1+ (m-1)d]$

$= 2a_m$. Hence proved.

10. **If the roots of the equation (b-c)x2 +(c-a)x +(a-b) = 0 are equal show that a, b, c are in AP.**

Ans: Refer sum No.12 of Q.E.

If $(b-c)x^2 + (c-a)x + (a-b)x$ have equal root.

$B^2-4AC=0.$

Proceed as in sum No.13 of Q.E to get c + a = 2b

$\Rightarrow b - a = c - b$

⇒ a, b, c are in AP

11. **Balls are arranged in rows to form an equilateral triangle .The first row consists of one ball, the second two balls and so on. If 669 more balls are added, then all the balls can be arranged in the shape of a square and each of its sides then contains 8 balls less than each side of the triangle. find the initial number of balls.**

Ans: Let their be n balls in each side of the triangle

$\therefore$ No. of ball (in Δ) = 1 + 2+ 3.......... $= \frac{n(n+1)}{2}$

No. of balls in each side square = n-8

No. of balls in square = $(n-8)^2$

APQ $\frac{n(n+1)}{2} + 660 = (n-8)^2$

On solving

$n^2 + n + 1320 = 2(n^2 - 16n + 64)$

$n^2 - 33n - 1210 = 0$

$\Rightarrow (n-55)(n+22) = 0$

n=-22 (N.P)

n=55

$\therefore$No. of balls = $\frac{n(n+1)}{2} = \frac{55\times 56}{2}$

28. If sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.

29. Find the sum of all the 11 terms of an AP whose middle most term is 30.

30. Find the sum of last ten terms of the AP: 8, 10, 12,---, 126.

31. Find the sum of first seven numbers which are multiples of 2 as well as of 9.

[Hint: Take the LCM of 2 and 9]

32. How many terms of the AP: –15, –13, –11,--- are needed to make the sum –55? Explain the reason for double answer.

33. The sum of the first n terms of an AP whose first term is 8 and the common difference is 20 is equal to the sum of first $2n$ terms of another AP whose first term is – 30 and the common difference is 8. Find n.

34. Kanika was given her pocket money on Jan 1st, 2008. She puts Re 1 on Day 1, Rs 2 on Day 2, Rs 3 on Day 3, and continued doing so till the end of the month, from this money into her piggy bank. She also spent Rs 204 of her pocket money, and found that at the end of the month she still had Rs 100 with her. How much was her pocket money for the month?

35. Yasmeen saves Rs 32 during the first month, Rs 36 in the second month and Rs 40 in the third month. If she continues to save in this manner, in how many months will she save Rs 2000?

= 1540

12. **Find the sum of $(1-\frac{1}{n})+(1-\frac{2}{n})+(1-\frac{3}{n})$ .... upto n terms.**

Ans: $\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)$- upto n terms

⇒ [1+1+.......+n terms] –$[\frac{1}{n}+\frac{2}{n}+$....+ n terms]

n – [S_n up to n terms]

$S_n=\frac{n}{2}[2a+(n-1d]\ (d=\frac{1}{n},a=\frac{1}{n})$

$=\frac{n}{2}\left[\frac{2}{n}+(n-1)\frac{1}{n}\right]$

$=\frac{n+1}{2}$ (on simplifying)

$n-\frac{n+1}{2}$

$=\frac{n-1}{2}$Ans

13. **If the following terms form a AP. Find the common difference & write the next 3 terms3, 3+ √2, 3+2√2, 3+3√2..........**

Ans: d= $\sqrt{2}$ next three terms $3+4\sqrt{^2},3+5\sqrt{^2},3+6\sqrt{^2}\ldots\ldots$

14. **Find the sum of a+b, a-b, a-3b, ... to 22 terms.**

Ans: a + b, a – b, a – 3b, up to 22 terms

d = a – b – a – b = 2b

$S_{22}=\frac{22}{2}[2(a+b)+21(-2b)]$

11[2a + 2b – 42b]

= 22a – 440b Ans.

15. **Write the next two terms √12, √27, √48, √75...................**

Ans: next two terms $\sqrt{108},\sqrt{147}$ AP is $2\sqrt{3},3\sqrt{3},4\sqrt{3},5\sqrt{3},6\sqrt{3},7\sqrt{3}\ldots\ldots$

16. **If the pth term of an AP is q and the qth term is p. P.T its nth term is (p+q-n).**

Ans: APQ

a_p = q

a_q = p

a_n =?

a + (p-1) d = q

a + (q-1) d = p

d [p – q] = q – p Sub d = -1 to get ⇒ = -1 ⇒ a = q + p -1

a_n = a + (n – 1)d

= a + (n - 1)d

= (q + p – 1) + (n – 1) - 1

36. The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the sum of the first ten terms of this AP is 235, find the sum of its first twenty terms.

37. Find the

(i) sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.

(ii) sum of those integers from 1 to 500 which are multiples of 2 as well as of 5 .

(iii) sum of those integers from 1 to 500 which are multiples of 2 or 5.

[Hint (iii) : These numbers will be : multiples of 2 + multiples of 5 – multiples of 2 as well as of 5]

38. The eighth term of an AP is half its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15^{th} term.

39. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the AP.

40. Find the sum of the integers between 100 and 200 that are

(i) divisible by 9

(ii) not divisible by 9

[Hint (ii) : These numbers will be : Total numbers – Total numbers divisible by 9]

41. The ratio of the 11^{th} term to the 18^{th} term of an AP is 2 : 3. Find the ratio of the 5^{th} term to the 21^{st} term, and also the ratio of the sum of the first five terms to the sum of the first 21 terms.

42. Show that the sum of an AP whose first term is a, the second term b and the last term c, is equal to

$$\frac{a \quad c \quad b \quad c-2a}{2 \; b-a}$$

Solve the equation

$a_n = (q + p - n)$

17. **If $\frac{1}{x+2}, \frac{1}{x+3}, \frac{1}{x+5}$ are in AP find x.**

Ans: $\frac{1}{x+2}, \frac{1}{x+3}, \frac{1}{x+5}$ are in AP find x.

$\frac{1}{x+3} - \frac{1}{x+3}, \frac{1}{x+5}$

$\Rightarrow \frac{1}{x^2+5x+6} = \frac{2}{x^2+8x+15}$

On solving we get x = 1

18. **Find the middle term of the AP 1, 8, 15....505.**

Ans: Middle terms

a + (n-1)d = 505

a + (n-1)7 = 505

$n - 1 = \frac{504}{7}$

n = 73

$\therefore$ 37^{th} term is middle term

$a_{37} = a + 36d$

= 1 + 36(7)

= 1 + 252

= 253

19. **Find the common difference of an AP whose first term is 100 and sum of whose first 6 terms is 5 times the sum of next 6 terms.**

Ans: a = 100

APQ $a_1 + a_2 + ... a_6 = 5 (a_7 + ... + a_{12}) 6\left(\frac{a_1+a_6}{2}\right) = 5 \times 6\left(\frac{a_7+a_{12}}{2}\right)$

⇒ a + a + 5d = 5[a + 6d + a + 11d]

⇒ 8a + 80d = 0 (a = 100)

⇒ d = - 10.

20. **Find the sum of all natural no. between 101 & 304 which are divisible by 3 or 5. Find their sum.**

Ans: No let 101 and 304, which are divisible by 3.

102, 105.............303 (68 terms)

No. which are divisible by 5 are 105, 110......300 (40 terms)

No. which are divisible by 15 (3 & 5) 105, 120...... (14 terms)

$\therefore$ There are 94 terms between 101 & 304 divisible by 3 or 5. (68 + 40 – 14)

$\therefore S_{68} + S_{40} - S_{14} = 19035$

$$-4 + (-1) + 2 + ... + x = 437$$

43. Jaspal Singh repays his total loan of Rs 118000 by paying every month starting with the first instalment of Rs 1000. If he increases the instalment by Rs 100 every month, what amount will be paid by him in the 30^{th} instalment? What amount of loan does he still have to pay after the 30^{th} instalment?

44. The students of a school decided to beautify the school on the Annual Day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 m. The flags are stored at the position of the middle most flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time. How much distance did she cover in completing this job and returning back to collect her books? What is the maximum distance she travelled carrying a flag?

21. The ratio of the sum of first n terms of two AP's is 7n+1:4n+27. Find the ratio of their 11th terms.

Ans: Let a_1, a_2... and d_1, d_2 be the I terms are Cd's of two AP's.

$S_n\ of\ one\ AP = \frac{7n+1}{4n+27}$

Sn of II AP

$\frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]} = \frac{7n+1}{4n+27}$

$\Rightarrow \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{7n+1}{4n+27}$

We have sub. n = 21.

$\frac{2a_1+20d_1}{2a_2+20d_2} = \frac{7\times21+1}{4(21)+27}$

$\Rightarrow \frac{a_1+10d_1}{a_2+10d_2} = \frac{148}{111}$

$= \frac{4}{3}$

∴ ratio of their 11th terms = 4 :3.

22. **If there are (2n+1)terms in an AP ,prove that the ratio of the sum of odd terms and the sum of even terms is (n+1):n**

Ans: Let a, d be the I term & Cd of the AP.

∴ ak = a + (k – 1) d

s_1 = sum to odd terms

$s_1 = a_1 + a_3 + \ldots\ldots\ldots a_{2n+1}$

$s_1 = \frac{n+1}{2}[2a_1 + 2nd]$

$= \frac{n+1}{2}[2a_1 + 2nd]$

s_1 = (n + 1) (a + nd)

s_2 = sum to even terms

$s_2 = a_2 + a_4 + \ldots\ldots a_{2n}$

$s_2 = \frac{n}{2}[a_2 + a_{2n}]$

$= \frac{n}{2}[a + d + a + (2n-1)d]$

=n [a + nd]

$\therefore s_1 : s_2 == \frac{(n+1)(a+nd)}{n(a+nd)}$

$= \frac{n+1}{n}$

23. **Find the sum of all natural numbers amongst first one thousand numbers which are neither divisible 2 or by 5**

Ans: Sum of all natural numbers in first 1000 integers which are not divisible by 2 i.e. sum of odd integers.

1 + 3 + 5 + ... + 999

n = 500

$S_{500} = \frac{500}{2}[1 + 999]$

= 2,50,000

No's which are divisible by 5

5 + 15 + 25 .... + 995

n = 100

$S_n = \frac{100}{2}[5 + 995]$

= 50 × 1000 = 50000

∴ Required sum = 250000 – 50,000

= 200000

CBSE Class 10 Mathematics

Important Questions

Chapter 5

Arithmetic Progressions

1 Marks Questions

1. The next term of the AP in $1^2, 5^2, 7^2, 73$.......... **is**

(a) 97

(b) 92

(c) 99

(d) 95

Ans. (a) 97

2. The 10th term of the AP in 2, 7, 12, ... is

(a) 45

(b) 47

(c) 48

(d) 50

Ans. (b) 47

3. If the sum of the circumferences of two circles with radii R_1 **and** R_2 **is equal to the circumference of a circle of Radius** R_1**, then**

(a) $R_1 + R_2 = R$

(b) $R_1 + R_2 > R$

(c) $R_1 + R_2 < R$

(d) None of these

Ans. a) $R_1 + R_2 = R$

4. If the perimeter of a circle is equal to that of a square, then the ratio of their area is

(a) 22:7

(b) 14:11

(c) 7:22

(d) 11:14

Ans. (c) 7:22

5. Area of a sector of angle p^0 of a circle with radius R is

(a) $\frac{P}{180} \times 2\pi R$

(b) $\frac{P}{180} \times \pi R^2$

(c) $\frac{P}{360} \times 2\pi R$

(d) $\frac{P}{720} \times 2\pi R^2$

Ans.

(d) $\frac{P}{720} \times 2\pi R^2$

6. Area of the sector of angle 60 ° of a circle with radius 10cm is

(a) $52\frac{5}{21}cm^2$

(b) $52\frac{8}{21}cm^2$

(c) $52\frac{4}{21}cm^2$

(d) none of there

Ans. (b) $52\frac{8}{21}cm^2$

7. 11^{th} term of the AP $-3, -\frac{1}{2}, 2, \ldots$ is

(a) 28

(b) 22

(c) -38

(d) $-48\frac{1}{2}$

Ans. (b) 22

8. If 17^{th} term of an AP exceeds its 10^{th} term by 7. The common difference is

(a) 2

(b) -1

(c) 3

(d) 1

Ans. (d) 1

9. Which of the following list of no. form an AP?

(a) 2, 4, 8, 16 ...

(b) 2, $\frac{5}{2}$ **, 3,** $\frac{7}{2}$ **,...**

(c) 0.2, 0.22, 0.222...

(d) 1, 3, 9, 27...

Ans. (b) 2, $\frac{5}{2}$, 3, $\frac{7}{2}$,...

10. The n^{th} term of the AP in 2, 5, 8... is

(a) 3n – 1

(b) 2n – 1

(c) 3n – 2

(d) 2n – 3

Ans. (a) 3n – 1

11. If $a,(a-2)$ **and 3a are in AP, then value of a is**

(a) -3

(b) -2

(c) 3

(d) 2

Ans. (b) -2

12. The sum of first n positive integers is given by

(a) $\frac{n(n-1)}{2}$.

(b) $\frac{n(2n+1)}{2}$.

(c) $\frac{n(n+1)}{2}$

(d) none of these

Ans. (c) $\frac{n(n+1)}{2}$

CBSE Class 10 Mathematics
Important Questions
Chapter 5
Arithmetic Progressions

2 Marks Questions

1. Find the missing variable from a, d, n and a_n, where a is the first term, d is the common difference and $a_n$is the nth term of AP.

(i) a= 7, d = 3, n = 8

(ii) a = –18, n= 10, a_n=0

(iii) d = –3, n = 18, a_n=-5

(iv) a = –18.9, d = 2.5, a_n=3.6

(v) a = 3.5, d = 0, n = 105

Ans. (i) a= 7, d = 3, n = 8

We need to find a_n here.

Using formula a_n=a+(n−1)d

Putting values of a, d and n,

a_n=7+(8−1)3=7+(7)3=7+21=28

(ii) a = –18, n= 10, a_n=0

We need to find d here.

Using formula a_n=a+(n−1)d

Putting values of a, a_n and n,

$0 = -18 + (10 - 1)d$

$\Rightarrow 0 = -18 + 9d$

$\Rightarrow 18 = 9d$

$\Rightarrow d = 2$

(iii) $d = -3$, $n = 18$, $a_n = -5$

We need to find a here.

Using formula $a_n = a + (n-1)d$

Putting values of d, a_n and n,

$-5 = a + (18 - 1)(-3)$

$\Rightarrow -5 = a + (17)(-3)$

$\Rightarrow -5 = a - 51$

$\Rightarrow a = 46$

(iv) $a = -18.9$, $d = 2.5$, $a_n = 3.6$

We need to find n here.

Using formula $a_n = a + (n-1)d$

Putting values of d, a_n and a,

$3.6 = -18.9 + (n - 1)(2.5)$

$\Rightarrow 3.6 = -18.9 + 2.5n - 2.5$

$\Rightarrow 2.5n = 25$

$\Rightarrow n = 10$

(v) $a = 3.5$, $d = 0$, $n = 105$

We need to find a_n here.

Using formula a_n=a+(n−1)d

Putting values of d, n and a,

a_n=3.5+(105−1)(0)

⇒ a_n=3.5 + 104 × 0

⇒ a_n=3.5 + 0

⇒ a_n=3.5

2. Choose the correct choice in the following and justify:

(i) 30thterm of the AP: 10,7,4... is

(A) 97

(B) 77

(C) −77

(D) −87

(ii) 11thterm of the AP: −3,−12,2...is

(A) 28

(B) 22

(C) −38

(D) $-48\frac{1}{2}$

Ans. (i) 10,7,4...

First term = a =10, Common difference = d = 7 – 10= 4 – 7= –3

And n = 30 {Because, we need to find 30thterm}

$a_n=a+(n-1)d$

$\Rightarrow a_{30}=10+(30-1)(-3)=10-87=-77$

Therefore, the answer is (C).

(ii) –3,–½,2...

First term = a = –3, Common difference = d = –½ –(–3)=2–(–½)= $\frac{5}{2}$

And n = 11 (Because, we need to find 11thterm)

$a_n=-3+(11-1)\frac{5}{2}=-3+25=22$

3. Which term of the AP: 3, 8, 13, 18 ... is 78?

Ans. First term = a=3, Common difference = d = 8 – 3=13 – 8=5 and a_n=78

Using formula $a_n=a+(n-1)d$,to find nth term of arithmetic progression,

$a_n=3+(n-1)5$,

$\Rightarrow 78=3+(n-1)5$

$\Rightarrow 75=5n-5$

$\Rightarrow 80=5n$

$\Rightarrow n=16$

It means 16thterm of the given AP is equal to 78.

4. Find the number of terms in each of the following APs:

(i) 7,13,19….,205

(ii) 18, $15\frac{1}{2}$ **,13…, -47**

Ans. (i) 7,13,19…, 205

First term = a =7, Common difference = d = 13 – 7= 19 – 13= 6

And a_n=205

Using formula $a_n=a+(n-1)d$, to find nth term of arithmetic progression,

$205=7+(n-1)6=7+6n-6$

$\Rightarrow 205=6n+1$

$\Rightarrow 204=6n$

$\Rightarrow n=34$

Therefore, there are 34 terms in the given arithmetic progression.

(ii) 18, $15\frac{1}{2}$,13…, -47

First term = a =18, Common difference = d = $15\frac{1}{2}-18=\frac{31}{2}-18=\frac{31-36}{2}=\frac{-5}{2}$

And a_n=-47

Using formula $a_n=a+(n-1)d$, to find nth term of arithmetic progression,

$-47=18+(n-1)\left(-\frac{5}{2}\right)=36-\frac{5}{2}n+\frac{5}{2}$

$\Rightarrow -94=36-5n+5$

$\Rightarrow 5n=135$

$\Rightarrow n=27$

Therefore, there are 27 terms in the given arithmetic progression

5. Check whether –150 is a term of the AP: 11,8,5,2...

Ans. Let −150 is the nth of AP 11,8,5,2... which means that $a_n=-150$

Here, First term = a = 11, Common difference = d = 8 – 11= –3

Using formula $a_n=a+(n-1)d$, to find nth term of arithmetic progression,

$-150=11+(n-1)(-3)$

$\Rightarrow -150=11-3n+3$

$\Rightarrow 3n=164$

$\Rightarrow n=\frac{164}{3}$

But, n cannot be in fraction.

Therefore, our supposition is wrong. −150 cannot be term in AP.

6. An AP consists of 50 terms of which 3rdterm is 12 and the last term is 106. Find the 29thterm.

Ans. An AP consists of 50 terms and the 50thterm is equal to 106 and $a_3=12$

Using formula $a_n=a+(n-1)d$, to find nth term of arithmetic progression,

$a_{50}=a+(50-1)d$ And $a_3=a+(3-1)d$

$\Rightarrow 106=a+49d$ And $12=a+2d$

These are equations consisting of two variables.

Using equation $106=a+49d$, we get $a=106-49d$

Putting value of a in the equation $12=a+2d$,

$12=106-49d+2d$

$\Rightarrow 47d=94$

$\Rightarrow d=2$

Putting value of d in the equation, $a=106-49d$,

$a=106-49(2)=106-98=8$

Therefore, First term $=a=8$ and Common difference $=d=2$

To find 29^{th} term, we use formula $a_n=a+(n-1)d$ which is used to find n^{th} term of arithmetic progression,

$a_{29}=8+(29-1)2=8+56=64$

Therefore, 29th term of AP is equal to 64

7. How many multiples of 4 lie between 10 and 250?

Ans. The odd numbers between 0 and 50 are 1,3,5,7...49

It is an arithmetic progression because the difference between consecutive terms is constant.

First term = a = 1, Common difference = 3 – 1= 2, Last term = $l=49$

We do not know how many odd numbers are present between 0 and 50.

Therefore, we need to find n first.

Using formula $a_n=a+(n-1)d$, to find nth term of arithmetic progression, we get

$49=1+(n-1)2$

$\Rightarrow 49=1+2n-2$

$\Rightarrow 50=2n$

$\Rightarrow n=25$

Applying formula, $S_n = \frac{n}{2}(a+l)$ to find sum of n terms of AP, we get

$S_{25} = \frac{25}{2}(1+49) = \frac{25}{2}\times 50$

$= 25\times 25 = 625$

8. Which term of the AP: 121, 117, 113, ...is its first negative term?

Ans. Given: 121, 117, 113, ...

Here $a = 121,\quad d = 117 - 121 = 4$

Now, $a_n = a + (n-1)d$

$= 121 + (n-1)(-4) = 121 - 4n + 4 = 125 - 4n$

For the first negative term, $a_n < 0$

$\Rightarrow 125 - 4n < 0$

$\Rightarrow 125 < 4n$

$\Rightarrow \frac{125}{4} < n$

$\Rightarrow 31\frac{1}{4} < n$

n is an integer and $n > 31\frac{1}{4}$.

Hence, the first negative term is 32nd term

9. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of sixteen terms of the AP.

Ans. Let the AP be $a-4d, a-3d, a-2d, a-d, a, a+d, a+2d, a+3d, ...$

Then, $a_3 = a-2d, a_7 = a+2d$

$\Rightarrow a_3 + a_7 = a-2d+a+2d = 6$

$\Rightarrow 2a = 6$

$\Rightarrow a = 3$ ....(i)

Also $(a-2d)(a+2d) = 8$

$\Rightarrow a^2 - 4d^2 = 8$

$\Rightarrow 4d^2 = a^2 - 8$

$\Rightarrow 4d^2 = 3^2 - 8$

$\Rightarrow 4d^2 = 1$

$\Rightarrow d^2 = \frac{1}{4}$

$\Rightarrow d = \pm\frac{1}{2}$

Taking $d = \frac{1}{2}$,

$S_{16} = \frac{16}{2}\left[2\times(a-4d)+(16-1)d\right]$

$= 8\left[2\times\left(3-4\times\frac{1}{2}\right)+15\times\frac{1}{2}\right]$

$= 8\left[2+\frac{15}{2}\right] = 8\times\frac{19}{2} = 76$

Taking $d = \frac{-1}{2}$,

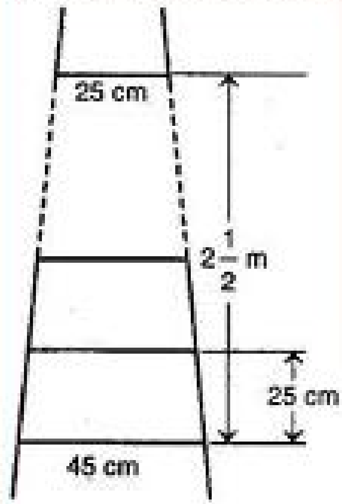
$S_{16} = \frac{16}{2}\left[2\times(a-4d)+(16-1)d\right]$

$= 8\left[2\times\left(3-4\times\frac{-1}{2}\right)+15\times\frac{-1}{2}\right]$

$= 8\left[\frac{20-15}{2}\right] = 8\times\frac{5}{2} = 20$

$\therefore S_{16} = 20$ and 76

10. A ladder has rungs 25 cm apart (see figure). The rungs decrease uniformly in length from 45 cm, at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?

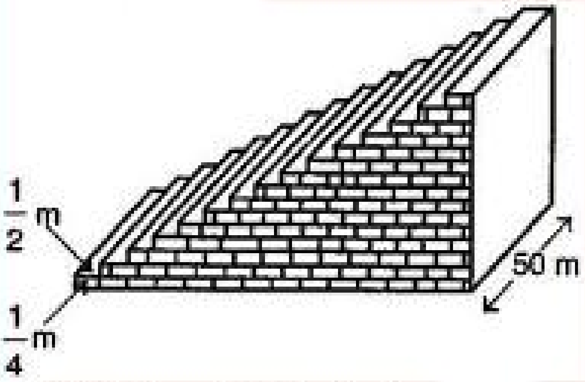


Ans. Number of rungs $(n) = \frac{2\frac{1}{2}\times 100}{25} = 10$

The length of the wood required for rungs = sum of 10 rungs

$= \frac{10}{2}[25+45]$ = 5 x 70 = 350 cm

11. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.



Ans. Here $a=1$ and $d=1$

$\therefore S_{x-1} = \frac{x-1}{2}[2\times1+(x-1-1)\times1]$

$= \frac{x-1}{2}(2+x-2)$

$= \frac{(x-1)x}{2} = \frac{x^2-x}{2}$

$S_x = \frac{x}{2}[2\times1+(x-1)\times1]$

$= \frac{x}{2}(x+1) = \frac{x^2+x}{2}$

$S_{49} = \frac{49}{2}[2\times1+(49-1)\times1]$

$= \frac{49}{2}(2+48)$ = 49 x 25

According to question,

$S_{x-1} = S_{49} - S_x$

$\Rightarrow \dfrac{x^2 - x}{2} = 49 \text{ x } 25 - \dfrac{x^2 + x}{2}$

$\Rightarrow \dfrac{x^2 - x}{2} + \dfrac{x^2 + x}{2} = 49 \text{ x } 25$

$\Rightarrow \dfrac{x^2 - x + x^2 + x}{2} = 49 \times 25$

$\Rightarrow x^2 = 49 \text{ x } 25$

$\Rightarrow x = \pm 35$

Since, x is a counting number, so negative value will be neglected.

$\therefore \ x = 35$

12. Find the first term and the common difference $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \ldots\ldots$

Ans. $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \ldots\ldots$

$a = \dfrac{1}{3}$

$d = \dfrac{5}{3} - \dfrac{1}{3} = \dfrac{4}{3}$

13. Is $\sqrt{3}, \sqrt{6}, \sqrt{9}, \ldots\ldots$ **form an AP?**

Ans. $a_1 = \sqrt{3}, a_2 = \sqrt{6}, a_3 = \sqrt{9}$

$d_1 = \sqrt{6} - \sqrt{3}$
$= \sqrt{3}\left(\sqrt{2} - 1\right)$

$d_2 = \sqrt{9} - \sqrt{6}$
$= 3 - \sqrt{6}$

Since $d_1 \neq d_2$

Hence, it is not an AP.

14. Which is the next term of the AP $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots\ldots\ldots$

Ans. $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots\ldots$

$d = \sqrt{8} - \sqrt{2}$
$= 2\sqrt{2} - \sqrt{2}$
$= \sqrt{2}$

a_5 = a + (5 – 1)d

$= \sqrt{2} + 4 \times \sqrt{2} = 5\sqrt{2}$

Next term is $5\sqrt{2}$ *or* $\sqrt{50}$

15. Find the 11th term from the last term of the AP 10, 7, 4,.........,-62.

Ans. $a = -62, \quad d = -(7 - 10) = 3$

$a_{11} = a + 10d$
$\quad = -62 + 10(3)$
$\quad = -32$

16. If $x + 1, 3x$ and $4x + 2$ are in A.P, find the value of x .

Ans. Since $x+1,\ 3x$ and $4x+2$ are in AP

$$2(3x)=x+1+4x+2$$
$$\Rightarrow 6x=5x+3$$
$$\Rightarrow 6x-5x=3$$
$$\Rightarrow x=3$$

17. Find the sum of first n odd natural numbers.

Ans. $1,\ 3,\ 5,\ 7,.........$

$$a=1, d=3-1=2$$
$$Sn=\frac{n}{2}\left[2a+(n-1)d\right]$$
$$=\frac{n}{2}\left[2\times1+(n-1)2\right]$$
$$=\frac{n}{2}\left[2+2n-2\right]$$
$$=n^2$$

18. Find the 12th term of the AP $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}...$

Ans. $a=\sqrt{2}, d=3\sqrt{2}-\sqrt{2}=2\sqrt{2}$

$$a_{12}=a+11d$$
$$=\sqrt{2}+11\left(2\sqrt{2}\right)$$
$$=\sqrt{2}+22\sqrt{2}$$
$$=23\sqrt{2}$$

19. Find the sum of first 11 terms of AP 2, 6, 10...

Ans. $2,\ 6,\ 10,...$

$a = 2, d = 6 - 2 = 4$

$$S_{11} = \frac{11}{2}\left[2\times 2+(11-1)\times 4\right]$$
$$= \frac{11}{2}[4+40]$$
$$= 11\times 22 = 242$$

20. Find the sum of first hundred even natural numbers divisible by 5.

Ans. Even natural no. divisible by 5 are 10, 20, 30...

$a = 10, d = 10$

$n = 100$

$$S_{100} = \frac{100}{2}\left[2(10)+(100-1).10\right]$$
$$= 50[20+99\times 10]$$
$$= 50500$$

21. Find $a_{30} - a_{20}$ for the A.P $-9, -14, -19, -24, \ldots$

Ans.

$a = -9,$

$d = (-14)-(-9) = -14+9 = -5$

$a_{30} - a_{20} = a + 29d - a - 19d = 10d$

$= 10\times(-5) = -50$

22. Find the common difference and write the next two terms of the AP $1^2, 5^2, 7^2, 73, \ldots$

Ans. $1^2,\ 5^2,\ 7^2,\ 73, \ldots\ldots$

$\Rightarrow 1, 25, 49, 73, \ldots\ldots..$

$d = a_2 - a_1 = 25 - 1 = 24$

$d = 49 - 25 = 24$

$d = 73 - 49 = 24$

Hence, it is AP.

$a_5 = 73 + 24 = 97$

$a_6 = 97 + 24 = 121$

23. Show that sequence defined by $a_n = 3 + 2n$ is an AP.

Ans. $a_n = 3 + 2n$

So $a_1 = 5,\ a_2 = 7,\ a_3 = 9,\ a_4 = 11$

$7 - 5 = 9 - 7 = 11 - 9 = 2$

Hence, it is AP.

24. The first term of an AP is -7 and common difference 5. Find its general term.

Ans. $a = -7,\ \ d = 5$

$$\begin{aligned} a_n &= a + (n-1)d \\ &= -7 + (n-1)(5) \\ &= -7 + 5n - 5 \\ \therefore \quad a_n &= 5n - 12 \end{aligned}$$

25. How many terms are there in A.P? $18, 15\frac{1}{2}, 13, \ldots\ldots, -47$

Ans. $a = 18,\ \ d = \frac{31}{2} - \frac{18}{1} = \frac{-5}{2}$

$a_n = -47$

$a_n = a + (n-1)d$

$$-47 = 18 + (n-1)\left(\frac{-5}{2}\right)$$

$\Rightarrow \quad -47 - 18 = \frac{-5}{2}n + \frac{5}{2}$

$\Rightarrow \quad n = 27$

26. In an AP, the sum of first n terms is $\frac{3n^2}{2} + \frac{13}{2}n$. find its 2nd term.

Ans. $S_n = \frac{3n^2}{2} + \frac{13}{2}n$

Put $n = 1, 2, 3.....$

$S_1 = \frac{16}{2} = 8$

$S_2 = 19$

$a_1 = s_1 = 8$

$a_2 = S_2 - S_1 = 19 - 8 = 11$

27. Show that the progression $4, 7\frac{1}{4}, 10\frac{1}{2}, 13\frac{3}{4}, 17, ...$ is an AP.

Ans. $\frac{29}{4} - \frac{4}{1} = \frac{29-16}{4} = \frac{13}{4}$

And $\frac{21}{2} - \frac{29}{4} = \frac{13}{4}$

And $\frac{17}{1} - \frac{55}{4} = \frac{13}{4}$

Hence, it is an AP.

CBSE Class 10 Mathematics

Important Questions

Chapter 5

Arithmetic Progressions

3 Marks Questions

1. Write first four terms of the AP, when the first term a and common difference d are given as follows:

(i) a = 10, d=10

(ii) a= -2, d=0

(iii) a=4, d=-3

(iv) a=-1, d= ½

(v) a=-1.25, d=-0.25

Ans. (i) First term = a =10, d=10

Second term = a+d = 10 + 10 = 20

Third term = second term + d = 20 + 10 = 30

Fourth term = third term + d = 30 + 10 = 40

Therefore, first four terms are: 10, 20, 30, 40

(ii) First term = a = –2 , d=0

Second term = a+d = –2 + 0 = –2

Third term = second term + d = –2 + 0 = –2

Fourth term = third term + d = –2 + 0 = –2

Therefore, first four terms are:–2, –2, –2, –2

(iii) First term = a = 4, d=–3

Second term = a+d = 4 – 3= 1

Third term = second term + d = 1 – 3= –2

Fourth term = third term + d = –2 – 3= –5

Therefore, first four terms are: 4, 1, –2, –5

(iv) First term = a = –1, d= ½

Second term = a+d = –1+ ½ =–½

Third term = second term + d = –½ + ½ = 0

Fourth term = third term + d = 0 + ½ = ½

Therefore, first four terms are:–1, –½, 0, ½

(v) First term = a =–1.25, d = –0.25

Second term = a+d = –1.25 – 0.25= –1.50

Third term = second term + d = –1.50 – 0.25 = –1.75

Fourth term = third term + d = –1.75 – 0.25 = –2.00

Therefore, first four terms are: –1.25, –1.50, –1.75, –2.00

2. Find the 31^{st} term of an AP whose 11^{th} term is 38 and 16^{th} term is 73.

Ans. Here a_{11}=38 and a_{16}=73

Using formula $a_n=a+(n-1)d$,to find n^{th} term of arithmetic progression,

$38=a+(11-1)(d)$ And $73=a+(16-1)(d)$

⇒ $38=a+10d$ And $73=a+15d$

These are equations consisting of two variables.

We have, $38=a+10d$

$\Rightarrow a=38-10d$

Let us put value of a in equation ($73=a+15d$),

$73=38-10d+15d$

$\Rightarrow 35=5d$

Therefore, Common difference $=d=7$

Putting value of d in equation $38=a+10d$,

$38=a+70$

$\Rightarrow a=-32$

Therefore, common difference = d = 7 and First term = a = –32

Using formula $a_n=a+(n-1)d$, to find nth term of arithmetic progression,

$a_{31}=-32+(31-1)(7)=-32+210=178$

Therefore, 31stterm of AP is 178.

3. If the third and the ninth terms of an AP are 4 and –8 respectively, which term of this AP is zero?

Ans. It is given that 3rd and 9th term of AP are 4 and –8 respectively.

It means $a_3=4$ and $a_9=-8$

Using formula $a_n=a+(n-1)d$, to find nth term of arithmetic progression,

4 = a + (3 – 1)d And, –8 = a + (9 – 1)d

$\Rightarrow 4=a+2d$ And, $-8=a+8d$

These are equations in two variables.

Using equation $4=a+2d$, we can say that $a=4-2d$

Putting value of a in other equation $-8=a+8d$,

$-8=4-2d+8d$

$\Rightarrow -12=6d$

$\Rightarrow d=-2$

Putting value of d in equation $-8=a+8d$,

$-8=a+8(-2)$

$\Rightarrow -8=a-16$

$\Rightarrow a=8$

Therefore, first term $=a=8$ and Common Difference $=d=-2$

We want to know which term is equal to zero.

Using formula $a_n=a+(n-1)d$, to find n^{th} term of arithmetic progression,

$0=8+(n-1)(-2)$

$\Rightarrow 0=8-2n+2$

$\Rightarrow 0=10-2n$

$\Rightarrow 2n=10$

$\Rightarrow n=5$

Therefore, 5^{th}term is equal to 0.

4. Two AP's have the same common difference. The difference between their 100^{th}terms is 100, what is the difference between their 1000^{th}terms.

Ans. Let first term of 1^{st}AP = a

Let first term of 2^{nd}AP = a'

It is given that their common difference is same.

Let their common difference be d.

It is given that difference between their 100^{th}terms is 100.

Using formula a_n=a+(n−1)d, to find n^{th} term of arithmetic progression,

$a+(100-1)d-[a'+(100-1)d]=a+99d-a'-99d=100$

$\Rightarrow a-a'=100$ ... (1)

We want to find difference between their 1000^{th}terms which means we want to calculate:

$a+(1000-1)d-[a'+(1000-1)d]=a+999d-a'-999d=a-a'$

Putting equation (1) in the above equation,

$a+(1000-1)d-[a'+(1000-1)d]=a+999d-a'+999d=a-a'=100$

Therefore, difference between their 1000^{th} terms would be equal to 100.

5. How many three digit numbers are divisible by 7?

Ans. We have AP starting from 105 because it is the first three digit number divisible by 7.

AP will end at 994 because it is the last three digit number divisible by 7.

Therefore, we have AP of the form 105,112,119..., 994

Let 994 is the n^{th} term of AP.

We need to find n here.

First term = a = 105, Common difference = d = 112 – 105= 7

Using formula $a_n=a+(n-1)d$, to find nth term of arithmetic progression,

$994=105+(n-1)(7)$

$\Rightarrow 994=105 + 7n - 7$

$\Rightarrow 896 = 7n$

$\Rightarrow n=128$

It means 994 is the 128th term of AP.

Therefore, there are 128 terms in AP.

6. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Ans. Penalty for first day = Rs 200, Penalty for second day = Rs 250
Penalty for third day = Rs 300
It is given that penalty for each succeeding day is Rs 50 more than the preceding day.
It makes it an arithmetic progression because the difference between consecutive terms is constant.
We want to know how much money the contractor has to pay as penalty, if he has delayed the work by 30 days.
So, we have an AP of the form 200, 250, 300, 350 ... 30 terms
First term = a = 200, Common difference = d = 50, n = 30

Applying formula, $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

$$S_n = \frac{30}{2}\left[400 + (30-1)50\right]$$

$\Rightarrow S_n = 15(400+29 \times 50)$

$\Rightarrow S_n = 15(400+1450) = 27750$

Therefore, penalty for 30 days is Rs. 27750.

7. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g, a section of Class I will plant 1 tree, a section of class II will plant two trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Ans. There are three sections of each class and it is given that the number of trees planted by any class is equal to class number.

The number of trees planted by class I = number of sections × 1 = 3 × 1 = 3

The number of trees planted by class II = number of sections × 2 = 3 × 2 = 6

The number of trees planted by class III = number of sections × 3 = 3 × 3 = 9

Therefore, we have sequence of the form 3, 6, 9 … 12 terms

To find total number of trees planted by all the students, we need to find sum of the sequence 3, 6, 9, 12 … 12 terms.

First term = a = 3, Common difference = d= 6 – 3= 3 and n = 12

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP, we get

$$S_{12} = \frac{12}{2}\left[6 + (12-1)3\right] = 6(6+33) = 6 \times 39 = 234$$

8. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (see figure). Calculate the total volume of concrete required to build the terrace.

Ans. Volume of concrete required to build the first step, second step, third step, ... (in m^2)

are

$$\frac{1}{4}\times\frac{1}{2}\times 50,\left(2\times\frac{1}{4}\right)\times\frac{1}{2}\times 50,\left(3\times\frac{1}{4}\right)\times\frac{1}{2}\times 50,....$$

$$\Rightarrow \frac{50}{8},2\times\frac{50}{8},3\times\frac{50}{8},.......$$

$\therefore$ Total volume of concrete required = $\frac{50}{8}+2\times\frac{50}{8}+3\times\frac{50}{8}+.......$

$= \frac{50}{8}[1+2+3+.......]$

$= \frac{50}{8}\times\frac{15}{2}\left[2\times 1+(15-1)\times 1\right]\ [\because n=15]$

$= \frac{50}{8}\times\frac{15}{2}\times 16$

$= 750\ m^3$

9. For what value of n are the n^{th} term of the following two AP's are same 13, 19, 25,.... and 69, 68, 67 ...

Ans. n^{th} term of 13, 19, 25 ,........... = nth term of 69, 68, 67,.........

$13+(n-1)\times 6 = 69 + (n-1)(-1)$

Therefore, n = 9

10. Check whether 301 is a term of the list of numbers 5, 11, 17, 32,........?

Ans. $d = 11-5 = 6$

$a = 5$

$a_n = a+(n-1)d$

$\Rightarrow 301 = 5+(n-1)d$

$\Rightarrow n = 151$

So, 301 is not a term of the given list.

11. Determine the AP whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by

12.

Ans.

$a_3 = 16$

$\Rightarrow a + 2d = 16 \ldots (i)$

$a_7 = a_5 + 12$

$\Rightarrow a + 6d = a + 4d + 12$

$\Rightarrow 2d = 12$

$\Rightarrow d = 6$

Put the valve of d in eq. (i)

$a + 2 \times 6 = 16$

$\Rightarrow a = 16 - 12$

$\Rightarrow a = 4$

$4, 10, 16 \ldots$

12. Find the sum of AP in $-5 + (-8) + (-11) + \ldots\ldots\ldots + (-230)$

Ans. $a = -5$

$d = -8 - (5)$

$= -8 + 5 = -3$

$a_n = -230$

$a_n = a + (n-1)d$

$\Rightarrow -230 = -5 + (n-1)(-3)$

$\Rightarrow -230 = -5 - 3n + 3$

$\Rightarrow -230 + 2 = -3n$

$\Rightarrow n = 76$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{76} = \frac{76}{2}[2\times(-5)+(76-1)(-3)]$

$= 38[-10+75\times(-3)]$

$= 38[-10-225]$

$= 38\times(-235)$

$= -8930$

13. In an AP, $a_n = 4, d = 2, S_n = -14$ find n and a.

Ans. $a_n = a+(n-1)d$

$\Rightarrow 4 = a+(n-1).(2)$

$\Rightarrow a+2n = 6...(i)$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow -14 = \frac{n}{2}(a+4)$

$\Rightarrow -28 = n[6-2n+4)\ [\because a = 6-2n]$

$\Rightarrow n^2-5n-14 = 0$

$\Rightarrow n = 7, \quad n = -2$

$a = -8$

14. Find $a_{30} - a_{20}$ for the AP in -9, -14, -19, -24...

Ans. $a = -9$

$d = -14-(-9) = -14+9 = -5$

$a_{30} - a_{20} = (a+29d)-(a+19d)$

$= 10d = 10\times(-5) = -50$

15. Find the sum to n term of the AP in 5, 2, -1, -4, -7..........

Ans. $a = 5, d = 2 - 5 = -3$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_n = \frac{n}{2}\left[2\times 5 + (n-1)(-3)\right]$$

$$= \frac{n}{2}[10 - 3n + 3]$$

$$= \frac{n}{2}[13 - 3n]$$

16. Find the sum of first 24 terms of the list of no. whose nth term is given by $a_n = 3 + 2n$

Ans. $a_n = 3 + 2n$

Put $n = 1, 2, 3, \dots$

$a_1 = 5,\ \ a_2 = 7,\ \ a_3 = 9\dots$

$a = 5,\ \ d = 7 - 5 = 2$

$$S_{24} = \frac{24}{2}\left[2\times 5 + (24-1)\times 2\right]$$

$$= 12[10 + 46] = 672$$

CBSE Class 10 Mathematics

Important Questions

Chapter 5

Arithmetic Progressions

4 Marks Questions

1. For the following APs, write the first term and the common difference.

(i) 3, 1, –1, –3 ...

(ii) –5, –1, 3, 7...

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots$

(iv) 0.6, 1.7, 2.8, 3.9...

Ans. (i) 3, 1, –1, –3...

First term = a= 3,

Common difference (d) = Second term – firstterm = Third term – second term and so on

Therefore, Common difference (d) = 1 – 3= –2

(ii) –5, –1, 3, 7...

First term = a= –5

Common difference (d) = Second term –Firstterm

= Third term – Second term and so on

Therefore, Common difference (d) = –1 – (–5)= –1+5 =4

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots$

First term = a= $\frac{1}{3}$

Common difference (d) = Second term –First term

= Third term –Second term and so on

Therefore, Common difference (d) = $\frac{5}{3}-\frac{1}{3}=\frac{4}{3}$

(iv) 0.6, 1.7, 2.8, 3.9...

First term = a= 0.6

Common difference (d) = Second term –Firstterm

= Third term –Secondterm and so on

Therefore, Common difference (d) = 1.7−0.6=1.1

2. The 17th term of an AP exceeds its 10thterm by 7. Find the common difference

Ans. (i) We need to show that $a_1,a_2...a_n$ form an AP where a_n=3+4n

Let us calculate values of a_1,a_2,a_3... using a_n=3+4n

a_1=3+4(1)=3+4=7 a_2=3+4(2)=3+8=11

a_3=3+4(3)=3+12=15 a_4=3+4(4)=3+16=19

So, the sequence is of the form 7,11,15,19...

Let us check difference between consecutive terms of this sequence.

11 – 7= 4, 15 – 11= 4, 19 – 15= 4

Therefore, the difference between consecutive terms is constant which means terms $a_1,a_2...a_n$ form an AP.

We have sequence 7,11,15,19...

First term = a =7 and Common difference = d = 4

Applying formula, $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

$$S_{15}=\frac{15}{2}\left[14+(15-1)4\right]=\frac{15}{2}(14+56)=\frac{15}{2}\times 70=15\times 35=525$$

Therefore, sum of first 15 terms of AP is equal to 525.

(ii) We need to show that $a_1, a_2 \ldots a_n$ form an AP where $a_n = 9 - 5n$

Let us calculate values of $a_1, a_2, a_3 \ldots$ using $a_n = 9 - 5n$

$a_1 = 9 - 5(1) = 9 - 5 = 4$ $a_2 = 9 - 5(2) = 9 - 10 = -1$

$a_3 = 9 - 5(3) = 9 - 15 = -6$ $a_4 = 9 - 5(4) = 9 - 20 = -11$

So, the sequence is of the form 4,−1,−6,−11...

Let us check difference between consecutive terms of this sequence.

$-1-(4) = -5,\ -6-(-1) = -6 + 1 = -5,\ -11-(-6) = -11 + 6 = -5$

Therefore, the difference between consecutive terms is constant which means terms $a_1, a_2 \ldots a_n$ form an AP.

We have sequence 4,−1,−6,−11...

First term = a = 4 and Common difference = d = −5

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP , we get

$$S_{15} = \frac{15}{2}\left[8 + (15-1)(-5)\right] = \frac{15}{2}(8 - 70) = \frac{15}{2} \times (-62) = 15 \times (-31) = -465$$

Therefore, sum of first 15 terms of AP is equal to −465.

3. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If, each prize is Rs 20 less than its preceding term, find the value of each of the prizes.

Ans. It is given that sum of seven cash prizes is equal to Rs 700.

And, each prize is R.s 20 less than its preceding term.

Let value of first prize = Rs. a

Let value of second prize =Rs (a−20)

Let value of third prize = Rs (a−40)

So, we have sequence of the form:

a, a−20, a−40, a – 60...

It is an arithmetic progression because the difference between consecutive terms is constant.

First term = a, Common difference = d = (a – 20) – a= –20

n = 7 (Because there are total of seven prizes)

S_7 = Rs 700 (given)

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP, we get

$S_7 = \frac{7}{2}\left[2a + (7-1)(-20)\right] \Rightarrow 700 = \frac{7}{2}\left[2a - 120\right]$

⇒ 200=2a–120

⇒ 320=2a

⇒ a=160

Therefore, value of first prize = Rs 160

Value of second prize = 160 – 20= Rs 140

Value of third prize = 140 – 20= Rs 120

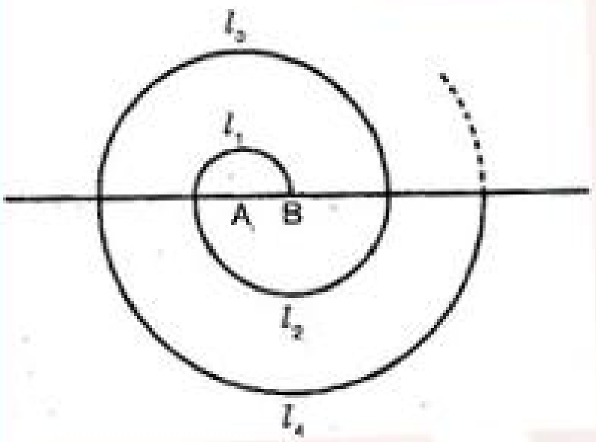
Value of fourth prize = 120 – 20 = Rs 100

Value of fifth prize = 100 – 20 = Rs 80

Value of sixth prize = 80 – 20 = Rs 60

Value of seventh prize = 60 – 20 = Rs 40

4. A spiral is made up of successive semicircles, with centers alternatively at A and B, starting with center at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... What is the total length of such a spiral made up of thirteen consecutive semicircle.



Ans. Length of semi–circle = $\frac{\text{Circumference of circle}}{2} = \frac{2\pi r}{2} = \pi r$

Length of semicircle of radii 0.5 cm = π (0.5) cm

Length of semicircle of radii 1.0 cm = π (1.0) cm

Length of semicircle of radii 1.5 cm = π (1.5) cm

Therefore, we have sequence of the form:

π (0.5), π (1.0), π (1.5) ... 13 terms {There are total of thirteen semicircles}.

To find total length of the spiral, we need to find sum of the sequence π (0.5), π (1.0), π (1.5) ... 13 terms

Total length of spiral = π (0.5)+ π (1.0)+ π (1.5) ... 13 terms

⇒ Total length of spiral = π (0.5+1.0+1.5)... 13 terms ... (1)

Sequence 0.5, 1.0, 1.5 ...13 terms is an arithmetic progression.

Let us find the sum of this sequence.

First term = a= 0.5, Common difference = 1.0 – 0.5 = 0.5 and n = 13

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

$$S_{13} = \frac{13}{2}\left[1+(13-1)0.5\right] = 6.5(1+6) = 6.5\times 7 = 45.5$$

Therefore, 0.5 + 1.0 + 1.5 + 2.0 ...13 terms = 45.5

Putting this in equation (1), we get

Total length of spiral = π (0.5+1.5+2.0+ ...13 terms) = π (45.5) = 143 cm

5. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?

Ans. The number of logs in the bottom row = 20

The number of logs in the next row = 19

The number of logs in the next to next row = 18

Therefore, we have sequence of the form 20, 19, 18 ...

First term = a = 20, Common difference = d = 19 – 20 = –1

We need to find that how many rows make total of 200 logs.

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

$200 = \frac{n}{2}\left[40+(n-1)(-1)\right] \Rightarrow 400 = n(40 - n + 1)$

$\Rightarrow 400 = 40n - n^2 + n$

$\Rightarrow n^2 - 41n + 400 = 0$

It is a quadratic equation, we can factorize to solve the equation.

$\Rightarrow n^2 - 25n - 16n + 400 = 0$

$\Rightarrow n(n-25) - 16(n-25) = 0$

$\Rightarrow (n-25)(n-16)$

$\Rightarrow n = 25, 16$

We discard n = 25 because we cannot have more than 20 rows in the sequence. The sequence is of the form: 20, 19, 18 ...

At most, we can have 20 or less number of rows.

Therefore, n = 16 which means 16 rows make total number of logs equal to 200.

We also need to find number of logs in the 16th row.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP, we get

$200 = 8\,(20 + l)$

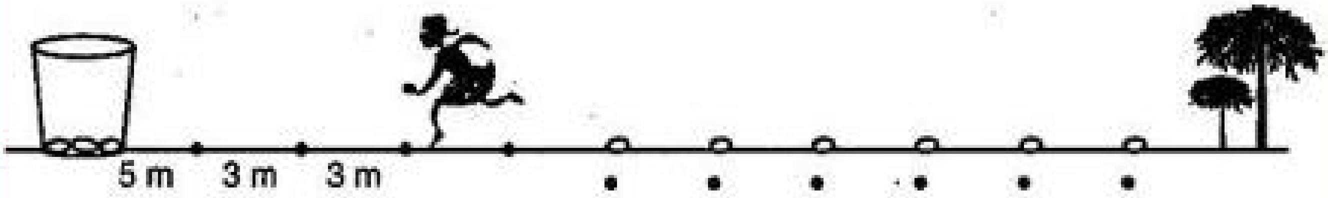
$\Rightarrow 200 = 160 + 8l$

$\Rightarrow 40 = 8l$

$\Rightarrow l = 5$

Therefore, there are 5 logs in the top most row and there are total of 16 rows.

6. In a potato race, a bucket is placed at the starting point, which is 5 meters from the first potato, and the other potatoes are placed 3 meters apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?



Ans. The distance of first potato from the starting point = 5 meters

Therefore, the distance covered by competitor to pickup first potato and put it in bucket = 5 × 2 = 10 meters

The distance of Second potato from the starting point = 5 + 3 = 8 meters {All the potatoes are 3 meters apart from each other}

Therefore, the distance covered by competitor to pickup 2nd potato and put it in bucket = 8 × 2 = 16 meters

The distance of third potato from the starting point = 8 + 3 = 11 meters

Therefore, the distance covered by competitor to pickup 3rd potato and put it in bucket = 11 × 2 = 22 meters

Therefore, we have a sequence of the form 10, 16, 22...10 terms

(There are ten terms because there are ten potatoes)

To calculate the total distance covered by the competitor, we need to find :

10 + 16 + 22 + ... 10 terms

First term = a = 10, Common difference = d =16 – 10= 6

n = 10 {There are total of 10 terms in the sequence}

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

$$S_{n10} = \frac{10}{2}\left[20+(10-1)6\right] = 5(20+54) = 5\times 74 = 370$$

Therefore, total distance covered by competitor is equal to 370 meters.

7. Which term of the sequence $20, \quad 19\frac{1}{4}, \quad 18\frac{1}{2}, \quad 17\frac{3}{4}, ...$ is the first negative term?

Ans. For first negative term

$a_n < 0$

$20+(n-1).\left(\frac{-3}{4}\right)<0 \left[\because d=\frac{77}{4}-20=\frac{-3}{4}\right]$

$\Rightarrow \frac{20}{1}-\frac{3}{4}n+\frac{3}{4}<0$

$\Rightarrow \frac{80-3n+3}{4}<0$

$\Rightarrow 83-3n<0$

$\Rightarrow -3n<-83$

$\Rightarrow n>\frac{83}{3}$

28^{th} term is first negative term.

8. The p^{th} term of an AP is q and q^{th} term is p. Find its $(p+q)^{th}$ term.

Ans. $q=a+(p-1)d \ldots(i)$

$p=a+(q-1)d \ldots(ii)$

$q-p=(p-1-q+1)d$

$\frac{q-p}{p-q}=d$

$\Rightarrow d=-1$

Put the value of d in eq (i)

$q=a+(p-1)(-1)$

$\Rightarrow q=a-p+1$

$\Rightarrow a=q+p-1$

$a_{p+q}=a+(p+q-1)d$

$= (q + p - 1) + (p + q - 1)(-1)$
$= q + p - 1 - p - q + 1$
$= 0$

9. If m times the m^{th} term of an A.P is equal to n times its n^{th} term, show that the $(m+n)^{th}$ term of the AP is zero.

Ans. $ma_m = n\, a_n$
$m[a + (m-1)d] = n[a + (n-1)d]$
$\Rightarrow ma + m^2d - md = na + n^2d - nd$
$\Rightarrow a(m-n) + (m^2 - n^2)d - md + nd = 0$
$\Rightarrow a(m-n) + (m-n)(m+n)d - (m-n)d = 0$
$\Rightarrow (m-n)[a + (m+n-1)d] = 0$
$\Rightarrow a + (m+n-1)d = 0$
$\Rightarrow a_{m+n} = 0$
Hence proved.

10. The sum of the 4^{th} and 8^{th} terms of an AP is 24 and the sum of the 6^{th} and 10^{th} terms is 44. Find the first three terms of the AP.

Ans. $a_4 + a_8 = 24$ (Given)
$\Rightarrow a + 3d + a + 7d = 24$
$\Rightarrow 2a + 10d = 24$
$\Rightarrow a + 5d = 12 ....(i)$
$a_6 + a_{10} = 44$
$\Rightarrow a + 5d + a + 9d = 44$
$\Rightarrow 2a + 14d = 44$
$\Rightarrow a + 7d = 22 ....(ii)$
On solving equations (i) and (ii)

$d = 5, \quad a = -13$

First three terms are -13,-8,-3.

11. If the sum of n terms of an AP is $3n^2 + 5n$ and its mth term is 164, find the value of m.

Ans. $Sn = 3n^2 + 5n$

Put $n = 1, 2, 3, ...$

$S_1 = 8$

$S_2 = 3 \times 4 + 10 = 22$

$a_1 = S_1 = 8$

$a_2 = S_2 - S_1$

$= 22 - 8 = 14$

$d = a_2 - a_1 = 14 - 8 = 6$

$a_m = 164$

$\Rightarrow a + (m-1)d = 164$

$\Rightarrow 8 + (m-1)(6) = 164$

$\Rightarrow 8 + 6m - 6 = 164$

$\Rightarrow 6m = 164 - 2$

$\Rightarrow 6m = 162$

$\Rightarrow m = \frac{162}{6} = 27$

12. If the sum of three numbers in AP, be 24 and their product is 440, find the numbers.

Ans. Let no. be a – d, a, a + d

$(a-d) + a + (a+d) = 24$ (Given)

$\Rightarrow a = 8$

$(a-d)(a)(a+d) = 440$

$\Rightarrow (8-d).8.(8+d) = 440$

$\Rightarrow 8^2 - d^2 = 55$

$\Rightarrow d = \pm 3$

$a=8$	$a=8$
$d=+3$	$d=-3$
Then AP	Then AP
5, 8, 11,...	11, 8, 5,....

13. If a^2, b^2, c^2 are in AP, then prove that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in AP.

Ans. Given a^2, b^2 ,c^2 are in AP

Then $2b^2 = a^2 + c^2$.........(i)

If $\frac{1}{b+c}; \frac{1}{c+a}; \frac{1}{a+b}$ are in AP then

$$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

$$\frac{(b+c)-(c+a)}{(c+a)(b+c)} = \frac{(c+a)-(a+b)}{(a+b)(c+a)}$$

$$\frac{b-a}{b+c} = \frac{c-b}{a+b}$$

$2b^2 = c^2 + a^2$........(ii)

From (i) and (ii),

Hence proved.

14. If S_1, S_2, S_3 be the sum of n, 2n and 3n terms respectively of an AP, prove that $S_3 = 3(S_2 - S_1)$

Ans. $S_1 = \frac{n}{2}\left[2a+(n-1)d\right]$

$S_2 = \frac{2n}{2}\left[2a+(2n-1)d\right]$

$S_3 = \frac{3n}{2}\left[2a+(3n-1)d\right]$

R.H.S $= 3(S_2 - S_1)$

$= 3\left[\frac{2n}{2}(2a+(2n-1)d) - \frac{n}{2}(2a+(n-1)d)\right]$

$= 3\left[\frac{n}{2}[4a+4nd-2d-2a-nd+d]\right]$

$= 3\left[\frac{n}{2}(2a+3nd-d)\right]$

$= \frac{3n}{2}\left[2a+(3n-1)d\right] = S_3$

15. The ratio of the sums of m and n terms of an AP is $m^2 : n^2$, show that the ratio of the m^{th} and n^{th} term is $(2m-1):(2n-1)$.

Ans. $\frac{S_m}{S_n} = \frac{\frac{m}{2}\left[2a+(m-1)d\right]}{\frac{n}{2}\left[2a+(n-1)d\right]}$

$\Rightarrow \frac{m^2}{n^2} = \frac{m}{n}\left[\frac{2a+(m-1)d}{2a+(n-1)d}\right]$

$\Rightarrow \frac{m}{n} = \frac{2a+md-d}{2a+nd-d}$

$\Rightarrow 2am + mnd - md = 2an + mnd - nd$

$\Rightarrow 2am - 2an - md + nd = 0$

$\Rightarrow 2a(m-n) - (m-n)d = 0$

$\Rightarrow (m-n)(2a-d)=0$

$\Rightarrow 2a=d$

According to question,

$$\frac{a_m}{a_n}=\frac{a+(m-1)d}{a+(n-1)d}$$

$$=\frac{a+(m-1)2a}{a+(n+1)2a}$$

$$=\frac{a[1+2m-2]}{a[1+2n-2]}$$

$$=\frac{2m-1}{2n-1}$$

Hence Proved

16. If the sum of first p terms of an AP is the same as the sum of its first q terms, show that the sum of the first $(p+q)$ term is zero.

Ans. $S_p=S_q$

$\Rightarrow \frac{p}{2}[2a+(p-1)d]=\frac{q}{2}[2a+(q-1)d]$

$\Rightarrow p[2a+pd-d]=q[2a+qd-d]$

$\Rightarrow 2ap+p^2d-pd=2aq+q^2d-qd$

$\Rightarrow 2a(p-q)+(p^2-q^2)d-(p-q)d=0$

$\Rightarrow 2a(p-q)+(p-q)(p-q)d-(p-q)d=0$

$\Rightarrow (p-q)[2a+(p+q-1)d]=0$

$\Rightarrow S_{p+q}=0$

17. For the A.P $a_1, a_2, a_3, \ldots$ **if** $\frac{a_4}{a_7} = \frac{2}{3},$ **find** $\frac{a_6}{a_8}.$

Ans. $\frac{a_4}{a_7} = \frac{2}{3} \quad (\text{Given})$

$\Rightarrow \frac{a+3d}{a+6d} = \frac{2}{3}$

$\Rightarrow 3a+9d = 2a+12d$

$\Rightarrow a = 12d - 9d$

$\Rightarrow a = 3d$

$\frac{a_6}{a_7} = \frac{a+5d}{a+6d}$

$= \frac{3d+5d}{3d+6d} = \frac{8d}{9d} = 8:9$

18. In an AP p^th^, q^th^ and r^th^ terms are respectively a, b and c. Prove that $p(b-c)+q(c-a)+r(a-b)=0$

Ans. $A+(p-1)D = a \ldots\ldots\ldots (i)$

$A+(q-1)D = b \ldots\ldots\ldots (ii)$

$A+(r-1)D = c \ldots\ldots\ldots (iii)$

$(ii)-(iii)$

$b-c = (q-1)D-(r-1)D$

$\Rightarrow b-c = D(q-r)$

$\Rightarrow p(b-c) = p \ D(q-r) \ldots\ldots\ldots (iv)$

Similarly,

$q(c-a) = q \ \ D(r-p) \ldots\ldots\ldots (v)$

$r(a-b) = r \ \ D(p-q) \ldots\ldots\ldots (vi)$

Adding (iv), (v) and (vi)

$p(b-c)+q(c-a)+r(a-b)=0$

19. If $(p+1)^{th}$ term of an A.P is twice the $(q+1)^{th}$ term, show that $(3p+1)^{th}$ term is twice the $(p+q+1)^{th}$ term.

Ans. $a_{p-1}=2a_{q-1}$

$\Rightarrow a+(p+1-1)d=2[a+(q+1-1)d]$

$\Rightarrow a+pd=2[a+qd]$

$\Rightarrow a+pd=2a+2qd$

$\Rightarrow pd-2qd=a$

$\Rightarrow (p-2q)d=a$

$\frac{a_{3p-1}}{a_{p-q-1}}=\frac{a+(3p+1-1)d}{a+(p+q+1-1)d}$

$=\frac{(p-2q)d+3pd}{p-2q+(p+q)d}=2$

20. The sum of four numbers in AP is 50 and the greatest number four times the least. Find the numbers.

Ans. Let no. be $(a-3d),(a-d),(a+d),(a+3d)$

$(a-3d)+(a-d)+(a+d)+(a+3d)=50$

$\Rightarrow 4a=50$

$\Rightarrow a=\frac{50}{4}=\frac{25}{2}$

According to question,

$(a+3d)=4\times(a-3d)$

$\Rightarrow a+3d=4a-12d$

$\Rightarrow -3a = -15d$

$\Rightarrow a = 5d$

$\Rightarrow \frac{25}{2} = 5d$

$\Rightarrow \frac{5}{2} = d$

Numbers be 5, 10, 15, 20

21. Find the sum of all integers between 84 and 719 which are multiples of 5.

Ans. 85, 90, 95, ... 715

$a = 85,\ d = 5,\ a_n = 715$

$a + (n-1)d = a_n$

$\Rightarrow 85 + (n-1).5 = 715$

$\Rightarrow n = 127$

$S_{127} = \frac{127}{2}(85 + 715)$

$= 50800$

22. If mth term of an A.P is $\frac{1}{n}$ and the nth term is $\frac{1}{m}$, show that the sum of mn terms is $\frac{1}{2}(mn+1)$.

Ans. $\frac{1}{n} = a + (m+1)d....(i)$

$\frac{1}{m} = a + (n+1)d......(ii)$

On solving (i) and (ii),

$a=\frac{1}{mn}$, $d=\frac{1}{mn}$

$S_{mn}=\frac{mn}{2}\left[2a+(mn-1)d\right]$

$=\frac{mn}{2}\left[2.\frac{1}{mn}+(mn-1).\frac{1}{mn}\right]$

$=\frac{1}{2}(mn+1)$

23. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes 14*th* of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every meter of digging, when it costs Rs 150 for the first meter and rises by Rs 50 for each subsequent meter.

(iv) The amount of money in the account every year, when Rs 10,000 is deposited at compound Interest at 8% per annum.

Ans. (i) Taxi fare for 1st km = Rs 15, Taxi fare after 2 km = 15+8 = Rs 23

Taxi fare after 3 km = 23+8 = Rs 31

Taxi fare after 4 km = 31 +8 = Rs 39

Therefore, the sequence is 15, 23, 31, 39...

It is an arithmetic progression because difference between any two consecutive terms is equal which is 8.(23 – 15=8, 31 – 23=8, 39 – 31=8,...)

(ii) Let amount of air initially present in a cylinder = V

Amount of air left after pumping out air by vacuum pump = $V-\frac{V}{4}=\frac{4V-V}{4}=\frac{3V}{4}$

Amount of air left when vacuum pump again pumps out air =

$$\frac{3}{4}V-\left(\frac{1}{4}\times\frac{3}{4}V\right)=\frac{3}{4}V-\frac{3}{16}V=\frac{12V-3V}{16}=\frac{9}{16}V$$

So, the sequence we get is like $V,\frac{3}{4}V,\frac{9}{16}V...$

Checking for difference between consecutive terms ...

$$\frac{3}{4}V-V=-\frac{V}{4},\frac{9}{16}V-\frac{3}{4}V=\frac{9V-12V}{16}=\frac{-3V}{16}$$

Difference between consecutive terms is not equal.

Therefore, it is not an arithmetic progression.

(iii) Cost of digging 1 meter of well = Rs 150

Cost of digging 2 meters of well = 150+50=Rs 200

Cost of digging 3 meters of well = 200+50 = Rs 250

Therefore, we get a sequence of the form 150, 200, 250...

It is an arithmetic progression because difference between any two consecutive terms is equal.(200 – 150=250 – 200= 50...)

Here, difference between any two consecutive terms which is also called common difference is equal to 50.

(iv) Amount in bank after Ist year = $10000\left(1+\frac{8}{100}\right)$ ... (1)

Amount in bank after two years = $10000\left(1+\frac{8}{100}\right)^2$ ... (2)

Amount in bank after three years = $10000\left(1+\frac{8}{100}\right)^{3}$ ... (3)

Amount in bank after four years = $10000\left(1+\frac{8}{100}\right)^{4}$ ... (4)

It is not an arithmetic progression because (2)−(1)≠(3)−(2)

(Difference between consecutive terms is not equal)

Therefore, it is not an Arithmetic Progression.

24. In the following AP's find the missing terms:

(i) 2,__ , 26

(ii) __, 13, __, 3

(iii) 5, __, __, $9\frac{1}{2}$

(iv) –4. __, __, __, __, 6

(v) __, 38, __, __, __, –22

Ans. (i) 2, __ , 26

We know that difference between consecutive terms is equal in any A.P.

Let the missing term be *x*.

$x-2=26-x$

$\Rightarrow 2x=28 \Rightarrow x=14$

Therefore, missing term is 14.

(ii) __, 13, __, 3

Let missing terms be *x* and *y*.

The sequence becomes x, 13, y, 3

We know that difference between consecutive terms is constant in anyA.P.

$y - 13 = 3 - y$

$\Rightarrow 2y = 16$

$\Rightarrow y = 8$

And $13 - x = y - 13$

$\Rightarrow x + y = 26$

But, we have $y = 8$,

$\Rightarrow x + 8 = 26$

$\Rightarrow x = 18$

Therefore, missing terms are 18 and 8.

(iii) 5, __, __, $9\frac{1}{2}$

Here, first term = $a=5$ And, 4^{th}term $= a_4 = 9\frac{1}{2}$

Using formula $a_n = a + (n-1)d$, to find n^{th}term of arithmetic progression,

$a_4 = 5 + (4-1)d \Rightarrow \frac{19}{2} = 5 + 3d$

$\Rightarrow 19 = 2(5 + 3d)$

$\Rightarrow 19 = 10 + 6d$

$\Rightarrow 6d = 19 - 10$

$\Rightarrow 6d = 9 \Rightarrow d = \frac{3}{2}$

Therefore, we get common difference = d = $\frac{3}{2}$

Second term = a+d = $5+\frac{3}{2}=\frac{13}{2}$

Third term = second term + d = $\frac{13}{2}+\frac{3}{2}=\frac{16}{2}=8$

Therefore, missing terms are $\frac{13}{2}$ and 8

(iv) –4, __, __, __, __, 6

Here, First term = a = –4 and 6th term = a_6 = 6

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$a_6 = -4 + (6-1)d \Rightarrow 6 = -4 + 5d$

$\Rightarrow 5d = 10 \Rightarrow d = 2$

Therefore, common difference = d = 2

Second term = first term + d = a+ d = –4 + 2 = –2

Third term = second term + d = –2 + 2 = 0

Fourth term = third term + d = 0 + 2 = 2

Fifth term = fourth term + d = 2 + 2 = 4

Therefore, missing terms are –2, 0, 2 and 4.

(v) __, 38, __, __, __, –22

We are given 2nd and 6th term.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$a_2=a+(2-1)d$ And $a_6=a+(6-1)d$

$\Rightarrow 38=a+d$ And $-22=a+5d$

These are equations in two variables, we can solve them using any method.

Using equation ($38=a+d$), we can say that $a=38-d$.

Putting value of a in equation ($-22=a+5d$),

$-22=38-d+5d$

$\Rightarrow 4d=-60$

$\Rightarrow d=-15$

Using this value of d and putting this in equation $38=a+d$,

$38=a-15$

$\Rightarrow a=53$

Therefore, we get $a=53$ and $d=-15$

First term = a = 53

Third term = second term + d = 38 – 15= 23

Fourth term = third term + d = 23 – 15= 8

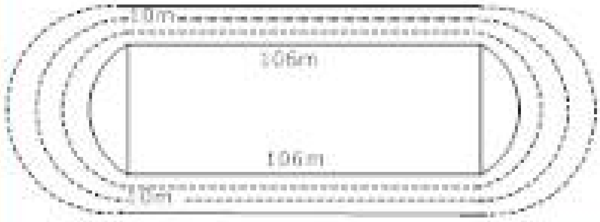
Fifth term = fourth term + d = 8 – 15= –7

Therefore, missing terms are 53, 23, 8 and –7.

25. The given figure depicts a racing track whose left and right ends are semi-circular. The difference between the two inner parallel line segments is 60m and they are each 106 m long. If the track is 10m wide, find:

(i) The distance around the track along its inner edge,

(ii) The area of the track



Ans. (i) The distance around the track along the inner edge

$=106+106+(\pi\times30+\pi\times30)$

$=212+\frac{22}{7}\times60=212+\frac{1320}{7}$

$=\frac{2804}{7}m$

(ii) The area of the track $=106\times80-106\times60+2.\frac{1}{2}\pi\left[40^2-30^2\right]$

$=106\times20+\pi(70)(10)$

$=2120+700\times\frac{22}{7}=2120+2200$

$=4320m^2$

26. Which of the following are APs? If they form an AP, find the common difference d and write threemore terms.

(i) 2,4,8,16...

(ii) $2,\frac{5}{2},3,\frac{7}{2}\ldots$

(iii) −1.2,−3.2,−5.2,−7.2...

(iv) −10,−6,−2,2...

(v) $3,3+\sqrt{2},3+2\sqrt{2},3+3\sqrt{2}\ldots$

(vi) 0.2,0.22,0.222,0.2222...

(vii) 0,−4,−8,−12...

(viii) $-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\ldots$

(ix) 1,3,9,27...

(x) $a,2a,3a,4a\ldots$

(xi) $a,a^2,a^3,a^4\ldots$

(xii) $\sqrt{2},\sqrt{8},\sqrt{18},\sqrt{32}\ldots$

(xiii) $\sqrt{3},\sqrt{6},\sqrt{9},\sqrt{12}\ldots$

(xiv) $1^2,3^2,5^2,7^2\ldots$

(xv) $1^2,5^2,7^2,73\ldots$

Ans. (i) 2,4,8,16...

It is not an AP because difference between consecutive terms is not equal.

As $4-2\neq 8-4$

(ii) $2,\frac{5}{2},3,\frac{7}{2}\ldots$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow \frac{5}{2}-2=3-\frac{5}{2}=\frac{1}{2}$

Common difference (d) = ½

Fifth term = $\frac{7}{2}+\frac{1}{2}=4$ Sixth term = 4+ ½ = $\frac{9}{2}$

Seventh term = $\frac{9}{2}+\frac{1}{2}=5$

Therefore, next three terms are 4, $\frac{9}{2}$ and 5.

(iii) $-1.2,-3.2,-5.2,-7.2\ldots$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow$ -3.2-(-1.2)=-5.2-(-3.2)=-7.2-(-5.2)=-2

Common difference (d) = −2

Fifth term = −7.2 – 2=−9.2

Sixth term = −9.2 – 2=−11.2
Seventh term = −11.2 – 2=−13.2

Therefore, next three terms are −9.2,−11.2 and −13.2

(iv) −10,−6,−2,2…

It is an AP because difference between consecutive terms is equal.

$\Rightarrow$ −6−(−10) =−2−(−6) =2−(−2)=4

Common difference (d) = 4

Fifth term = 2 + 4 = 6 Sixth term = 6 + 4 = 10

Seventh term = 10 + 4 = 14

Therefore, next three terms are 6,10 and 14

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}\ldots$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow 3+\sqrt{2}-3=\sqrt{2}, 3+2\sqrt{2}-(3+\sqrt{2})=3+2\sqrt{2}-3-\sqrt{2}=\sqrt{2}$

Common difference (d) = $\sqrt{2}$

Fifth term = $3+3\sqrt{2}+\sqrt{2}=3+4\sqrt{2}$

Sixth term = $3+4\sqrt{2}+\sqrt{2}=3+5\sqrt{2}$

Seventh term = $3+5\sqrt{2}+\sqrt{2}=3+6\sqrt{2}$

Therefore, next three terms are $\left(3+4\sqrt{2}\right),\left(3+5\sqrt{2}\right),\left(3+6\sqrt{2}\right)$

(vi) 0.2,0.22,0.222,0.2222...

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow 0.22-0.2 \neq 0.222-0.22$

(vii) 0,−4,−8,−12...

It is an AP because difference between consecutive terms is equal.

$\Rightarrow -4-0=-8-(-4)=-12-(-8)=-4$

Common difference (d) = −4

Fifth term = −12 − 4=−16 Sixth term = −16 − 4=−20

Seventh term = −20 − 4=−24

Therefore, next three terms are −16, −20 and −24

(viii) $-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\ldots$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow -\frac{1}{2}-\left(-\frac{1}{2}\right)=-\frac{1}{2}-\left(-\frac{1}{2}\right)=0$

Common difference (d) = 0

Fifth term = $-\frac{1}{2}+0=-\frac{1}{2}$ Sixth term = $-\frac{1}{2}+0=-\frac{1}{2}$

Seventh term = $-\frac{1}{2}+0=-\frac{1}{2}$

Therefore, next three terms are $-\frac{1}{2},-\frac{1}{2}\ and -\frac{1}{2}$

(ix) 1,3,9,27...

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow 3-1 \neq 9-3$

(x) $a, 2a, 3a, 4a \ldots$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow 2a - a = 3a - 2a = 4a - 3a = a$

Common difference (d) = a

Fifth term = $4a + a = 5a$ Sixth term = $5a + a = 6a$

Seventh term = $6a + a = 7a$

Therefore, next three terms are $5a, 6a$ and $7a$

(xi) $a, a^2, a^3, a^4 \ldots$

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow a^2 - a \neq a^3 - a^2$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \ldots \Rightarrow \sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow 2\sqrt{2} - \sqrt{2} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

Common difference (d) = $\sqrt{2}$

Fifth term = $4\sqrt{2} + \sqrt{2} = 5\sqrt{2}$ Sixth term = $5\sqrt{2} + \sqrt{2} = 6\sqrt{2}$

Seventh term = $6\sqrt{2} + \sqrt{2} = 7\sqrt{2}$

Therefore, next three terms are $5\sqrt{2}, 6\sqrt{2}, 7\sqrt{2}$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12} \ldots$

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow \sqrt{6}-\sqrt{3} \neq \sqrt{9}-\sqrt{6}$

(xiv) $1^2, 3^2, 5^2, 7^2 \ldots$

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow 3^2-1^2 \neq 5^2-3^2$

(xv) $1^2, 5^2, 7^2, 73 \ldots \Rightarrow 1, 25, 49, 73 \ldots$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow 5^2-1^2=7^2-5^2=73-7^2=24$

Common difference (d) = 24

Fifth term = 73+24=97 Sixth term = 97+24=121

Seventh term = 121 + 24 = 145

Therefore, next three terms are 97,121 and 145

Let the three parts of 207 be $(a-d)$, a and $(a+d)$

$\therefore$ $a-d+a+a+d = 207$

$\therefore$ $3a = 207$

$\Rightarrow$ $a = 69$

Now, $(a-d) \times a = 4623$

$\therefore$ $69(69-d) = 4623$

$\therefore$ $69-d = \frac{4623}{69}$ 67

$\therefore$ $69-d = 67$

$\therefore$ $d = 69-67$

$\Rightarrow$ $d = 2$

$\therefore$ Parts of 207 are 67, 69 and 71

Question-10

Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers.

Solution:

Let the three numbers be a − d, a, a+ d.

$a - d + a + a + d = 27$

$3a = 27$

$a = 9$

$(a - d)(a)(a + d) = 648$

$a(a^2 - d^2) = 648$

$9(9^2 - d^2) = 648$

$9^2 - d^2 = 72$

$d^2 = 81 - 72$

$d^2 = 9$

$d = 3$

The numbers are 6, 9, 12.

Question-8

An A.P consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32nd term.

Solution:

Given, $n = 60$, $a_1 = 7$,

and $a_{60} = 125$

$\Rightarrow a_1 + 59d = 125$

$7 + 59d = 125$

$59d = 118$

$d = 118/59 = 2$

$a_{32} = a_1 + 31d = 7 + 31(2) = 7 + 62$

$\therefore a_{32} = 69.$

Question-7

The sum of the first six terms of an A.P is zero and the fourth term is 2. Find the sum of its first 30 terms.

Solution:

Let the sum of first 30 terms be S_{30}, first term be a, fourth term be a_4 and the sum of first six terms be S_6.

Given that $S_6 = 0$ and fourth term $a_4 = 2$

$\Rightarrow a + 3d = 2$ ....(i)

$S_6 = 0$

$\frac{n}{2}(2a + 5d) = 0$

$\Rightarrow 2a + 5d = 0$ ....(ii)

(i) × 2,

$2a + 6d = 4$ ....(iii)

(iii) – (ii),

$\therefore d = 4$

Substituting the value of d = 4 in (i),

$a + 3 \times (4) = 2$

$\Rightarrow a = 2 - 12 = -10$

$\therefore a_{30} = a + 29d$

$= -10 + 29 \times (4)$

$= -10 + 116$

$= 106$

$\therefore$ Sum to first 30 terms $= S_{30} = \frac{n}{2}(a + l)$

$= \frac{30}{2}(-10 + 106)$

$= 15 \times 96$

$= 1140.$

Question-9

Find the sum of the series 51 + 50 + 49 + ... + 21.

Solution:

51 + 50 + 49 + ... + 21

$a = 51, d = -1, a_n = 21$

$\therefore a + (n - 1)\,d = a_n$

$51 + (n - 1)(-1) = 21$

$(n - 1)(-1) = 21 - 51$

$n - 1 = 30$

$\therefore n = 31$

$\therefore$ Sum of the series $= \frac{31}{2}(51 + 21)$

$= \frac{31}{2} \times 72$

$= 1116$

$\therefore$ The sum of the series 51 + 50 + 49 + ... + 21 = 1116.

Question-3

In a certain A.P the 24th term is twice the 10th term. Prove that the 72nd term is twice the 34th term.

Solution:

Given, $a_{24} = 2a_{10}$

$a_{24} = a + 23d$ and $a_{10} = a + 9d$

To prove: $a_{72} = 2a_{34}$

$a_{72} = a + 71d$

$a_{34} = a + 33d$

$a_{24} = 2a_{10}$ (Given)

$a + 23d = 2(a + 9d)$

$a + 23d = 2a + 18d$

$a - 5d = 0$

$a = 5d$.........................(i)

$a_{72} = 2a_{34}$

$a + 71d = 2(a + 33d)$

$a + 71d = 2a + 66d$

$a - 5d = 0$

$a = 5d$.........................(ii)

from, (1) and (2) $a_{72} = 2a_{34}$

Hence proved.

Question-2

Find the A.P. whose 10^{th} term is 5 and 18^{th} term is 77.

Solution:

given, 10^{th} term of an A.P= 5

Þ $a + (10 - 1)\,d = 5$

$\Rightarrow a + 9d = 5$ ....(i)

and 18^{th} term = 77

Þ$a + (18 - 1)\,d = 77$

$\Rightarrow a + 17d = 77$ ....(ii)

(ii) − (i), $8d = 72$

$\therefore d = 9$

Substituting the value of $d = 9$ in (i),

$a + 81 = 5$

$a = 5 - 81 = -76$

$\therefore$ The A.P. is $-76, -67,$ ....

Question-5

If 9^{th} term of an A.P. is zero, prove that its 29^{th} term is double the 19^{th} term.

Solution:

9^{th} term = 0

$a_1 + 8d = 0$

$a_{29} = a_1 + 28d = a_1 + 8d + 20d = 0 + 20d = 20d$

$a_{19} = a_1 + 18d = a_1 + 8d + 10d = 0 + 10d = 10d$

$a_{29} = 2a_{19}$.

Question-11

How many terms of A.P -10, -7, -4, -1, ... must be added to get the sum -104?

Solution:

-10, -7, -4, -1, ...

a = - 10, d = 3

$S_n = \frac{1}{2}n\{2a + (n - 1)\ d\}$

$-104 = \frac{1}{2}n\ \{2(-10) + (n - 1)\ 3\}$

$= \frac{1}{2}n\ (-20 + 3n - 3)$

$-208 = n\ (3n - 23)$

$3n^2 - 23n + 208 = 0$

$3n^2 - 39n + 16n + 208 = 0$

$3n\ (n - 13) + 16\ (n - 13) = 0$

$(n - 13)\ (3n + 16) = 0$

$\therefore n = 13$

$\therefore$ 13 terms must be added to get the sum of the A.P − 104.

Question-12

If the sum of p terms of an A.P is $3p^2 + 4p$, find its n^{th} term.

Solution:

$S_p = 3p^2 + 4p$

$t_n = S_n - S_{n-1}$

$= (3n^2 + 4n) - [3(n-1)^2 + 4(n-1)]$

$= (3n^2 + 4n) - [3(n^2 - 2n + 1) + 4(n-1)]$

$= (3n^2 + 4n) - [3n^2 - 6n + 3 + 4n - 4]$

$= (3n^2 + 4n) - [3n^2 - 2n - 1]$

$= 3n^2 + 4n - 3n^2 + 2n + 1$

$= 6n + 1$

Therefore the n^{th} term is $6n + 1$.

Question-6

Determine the A.P whose third term is 16 and the difference of 5^{th} from 7^{th} term is 12.

Solution:

Let the A.P. be a, a + d, a + 2d, ...

⇒ The third term = $a_3 = a + 2d = 16$ ....(i)

and seventh term = $a_7 = a + 6d$

Given that $a_7 - a_5 = 12$

⇒ $(a + 6d) - (a + 4d) = 12$

⇒ $a + 6d - a - 4d = 12$

⇒ $2d = 12$

⇒ $d = 6$

Substituting the value of d = 6 in (i),

$a + 12 = 16$

$a = 4$

∴ The first term of the A.P. is 4 and the common difference is 6.

∴ The A.P. is 4, 10, 16, 22, 28, 34, ...

∴ The fifth term = $a_5 = a + 4d$.

Question-1

Find the sum of the following A.P. 1, 3, 5, 7, ..., 199.

Solution:

Given, $a = 1$, $d = 2$, $a_n = l = 199$,

$a + (n - 1)d = 199$

$1 + (n - 1)2 = 199$

$\Rightarrow 1 + 2n - 2 = 199$

$\Rightarrow 2n = 200$

$\therefore n = \frac{200}{2}$

$n = 100.$

$S_n = n/2\,(a + l)$

$= 50(1 + 199)$

$= 50(200)$

$= 10000$

Question-4

a, b and c are in A.P. Prove that b + c, c + a and a + b are in A.P.

Solution:

Given, a, b and c are in A.P.

$\therefore\ b - a = c - b$

To prove : **b + c, c + a and a + b are in A.P.**

$c + a - (b + c) = a + b - (c + a)$

$\Rightarrow\ c + a - b - c = a + b - c - a$

$a - b = b - c$

$\Rightarrow b - a = c - b$

$\therefore$ a, b, c are in A.P.

$\therefore$ b + c, c + a and a + b are in A.P.