

My optimization problem is as follows,

$$\begin{cases} \min_{\mathbf{X} \in \mathbb{C}^{N \times M}} f(\mathbf{X}) \\ \text{s. t. } |\mathbf{x}_m(i)| = 1, i = 0, \dots, N - 1, m = 1, \dots, M, \end{cases}$$

where the matrix \mathbf{X} is as $\mathbf{X} = [\mathbf{x}_1 \ \mathbf{x}_2 \ \dots \ \mathbf{x}_M]$. If we define the matrix of gradients $\nabla_{\mathbf{X}} f(\mathbf{X}) = [\nabla_{x_1} f(\mathbf{X}), \nabla_{x_2} f(\mathbf{X}), \dots, \nabla_{x_M} f(\mathbf{X})]$ and the matrix of search directions $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_M]$, where $\mathbf{v}_m = -(\nabla_{x_m}^2 f(\mathbf{X}))^{-1} \nabla_{x_m} f(\mathbf{X})$, is it true to write Wolfe conditions as follows

$$\begin{cases} f(\mathbf{X} + \alpha \mathbf{V}) \leq f(\mathbf{X}) + c_1 \alpha \langle \nabla_{\mathbf{X}} f(\mathbf{X}), \mathbf{V} \rangle \\ \langle \nabla_{\mathbf{X}} f(\mathbf{X} + \alpha \mathbf{V}), \mathbf{V} \rangle \geq c_2 \langle \nabla_{\mathbf{X}} f(\mathbf{X}), \mathbf{V} \rangle, \end{cases}$$

where $\langle \mathbf{A}, \mathbf{B} \rangle$ is the inner product of the matrices $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_M]$ and $\mathbf{B} = [\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_M]$ that is defined as

$$\langle \mathbf{A}, \mathbf{B} \rangle = \sum_{i=1}^M \langle \mathbf{a}_i, \mathbf{b}_i \rangle$$