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Experimental observation of magnetic field effects on VLF propagation at night

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Computations of nighttime field intensity versus distance are made for a 23.4-kHz signal radiated from Hawaii and for propagation paths to Seattle, Ontario (California), Samoa, and Wake Island. The computations were made by using the waveguide computer program developed at the Naval Electronics Laboratory Center to obtain waveguide mode constants, each 2° of arc or 222 km along each path. An exponential electron-density profile defined by $\beta =$ 0.5 km^{-1} and $h' = 85.5$ was assumed, where β and h' are defined by *Wait* [1964]. The resultant field was computed by using a WKB approximation to allow for the variation of mode constants along the paths. Experimental measurements of 23.4-kHz signals from NPM were made aboard an airplane as it flew along these propagation paths. (NPM are the call letters for the Lualualei Navy Radio Station in Hawaii.) Good agreement was obtained, between the theoretical calculations and experimental measurements, strongly supporting the validity of the theoretical approach used (the $\beta = 0.5$, $h' = 85.5$ km profile assumed) and the conclusion that the increased attenuation observed for propagation to the south is an effect of the geomagnetic field.

1. INTRODUCTION

Numerical methods developed at the Naval Electronics Laboratory Center (NELC) have the capability of general solution of the VLF waveguide boundary value problem [*Pappert et al.*, 1967; *Pappert,* 1968; *Snyder and Pappert,* 1969]. This computer program obtains a full-wave solution for arbitrary ionospheric electron- and ion-density distributions with height; adjustable exponential collision frequencies for these distributions; arbitrary (but constant) orientation of the earth's magnetic field; and a lower boundary that is smooth, homogeneous, and characterized by an adjustable surface conductivity. *Pappert et al.* [1967] used this computer program to investigate some classical approximations used in VLF waveguide mode theory. This investigation was continued in a paper [*Pappert,* 1968] which also included a study of the pronounced coupling that can occur between the TE and TM modes for arbitrary orientations of the earth's magnetic field vector below a highly anisotropic ionosphere. *Snyder and Pappert* [1969] used this same NELC waveguide program to compute the parametric variation of the waveguide mode propagation constants (i.e. attenuation rate, phase velocity, and excitation factor) with the azimuthal orientation of the geomagnetic field. The result was a set of propagation constants that were quite variable for the highly anisotropic electron density profile assumed, especially in the north–south direction. Vector sums of the waveguide modes were presented, and the resulting total field indicated greater attenuation for propagation in the southerly direction, with propagation to the east providing the highest signal levels.

To check the validity of the NELC numerical methods, a series of nighttime airborne measurements of VLF amplitude were made in January and February 1969. The flights made are as listed in Table 1. The signals monitored included NPM on 23.4 kHz radiated from Hawaii. The aircraft used was a CK-121 provided by the Naval Research Laboratory.

Theoretical computations made prior to the measurements are described in section 2. In Section 3, the experimental airborne measurements of the VLF signals are described, and a comparison of the measurements with the computations is discussed.

TABLE 1. List of VLF data obtained on aircraft flights

Flight Origin	Flight Destination	Flight Time, GMT Beginning	End	Data Figure Number
Seattle	Hawaii	Jan. 27 from 0522	to 1701	5
Hawaii	Samoa	Jan. 29 from 0240	to 1521	6
Samoa	Hawaii	Jan. 31 from 0755	to 1902	6
Hawaii	Wake Is.	Feb. 2 from 0731	to 1420	7
Wake Is.	Hawaii	Feb. 3 from 0731	to 1756	7
Hawaii	Ontario, Calif.	Feb. 7 from 0341	to 1514	8

In section 4, the time variability of the VLF propagation medium is discussed. Conclusions are stated in section 5.

2. THEORETICAL CALCULATIONS

The purpose of this study was to examine the effect of changes in orientation of the earth's magnetic field at night on the VLF signal level from distant stations. Therefore, it was planned to study all sea-water propagation paths along radials from Hawaii in various directions to suitably located flight terminal points. Possible paths included radials to the west coast of the United States, Tahiti, Samoa, Kwajalein, Wake Island, Adak, and Kodiak. Because of limited availability of the airplane, the most important radials had to be chosen. Computations of the expected signal level along all possible radials were made in order to assist in this selection.

To make the computations, it is necessary to assume some ionospheric electron-density profile $n(h)$ and effective collision-frequency profile $\nu(h)$. Exponential profiles were used following *Wait and Spies* [1964] who assumed that

$$\omega_r = \omega_p^2/\nu = (2.5 \times 10^5) \exp [\beta(z - h')] \sec^{-1} \quad (1)$$

where $\beta = 0.5$ km^{-1}, $h' = 90$ km, z is the height above ground in kilometers, and ω_p is the angular plasma frequency that is proportional to the square root of the electron density. The collision frequency was assumed to be

$$\nu = (1.816 \times 10^{11}) \exp (-0.15z) \quad (2)$$

It was found that the resulting velocity of the dominant mode at 10.2 kHz was significantly different from nighttime values measured at 10.2 kHz by E. R. Swanson of NELC in 1965 using the Omega navigation system and from an analysis technique developed by J. A. Pierce and described by *Wait* [1962]. Swanson's measurements were made on the Haiku-Forestport Omega 'baseline,' and the average velocity was deduced for round-trip propagation on this path. Therefore, a number of phase-velocity computations were made for a $\beta = 0.5$ km^{-1} profile for various h' values and with magnetic-field orientations and ground electrical parameters associated with the Haiku-to-Forestport-to-Haiku round-trip propagation path. It was found that excellent agreement between the computed average velocity and Swanson's measurements was obtained with a value of $h' = 85.5$ km. Therefore, this height value was used for the computations of NPM signal amplitude for the various radials from Hawaii that were being considered.

Each propagation path was divided into segments arbitrarily chosen to be 222 km long, corresponding to 2° of arc at the earth's surface. The earth's magnetic-field orientation and magnitude were determined for the end of each segment. The magnetic-

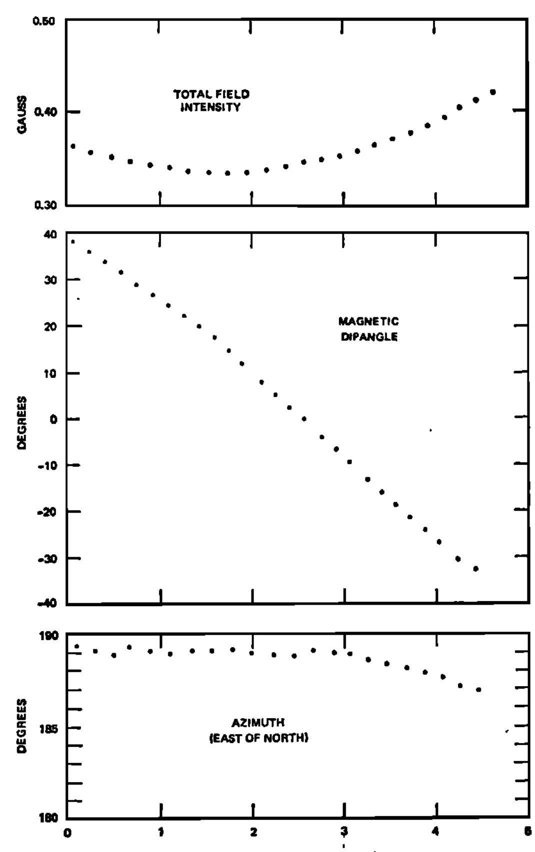

Fig. 1. Magnetic field parameters assumed for the Hawaii-to-Samoa propagation path. Values are plotted at 2° of arc or 222-km increments along the path.

field parameters were determined from maps published by the U. S. Navy Hydrographic Office in 1954 and by the U. S. Navy Oceanographic Office in 1965. These parameters for the Hawaii-to-Samoa path are shown in Figure 1.

The NELC waveguide computer program was used to compute the waveguide mode propagation constants for each point on the propagation path for which the magnetic-field parameters were determined. Figure 2 shows the variation of the mode constants with distance along the Hawaii-to-Samoa path for 23.4 kHz. A significant variation along the path is apparent, which suggests that an assumption of horizontal homogeneity along this path is not valid. Therefore, the total fields along all the paths were obtained by assuming a WKB-type approximation [*Wait,* 1964]. A vector sum of eight waveguide modes was computed at Δd increments (where $\Delta d = 20$ km) along the propagation path. The relation used was

$$E(d_n) = \frac{K}{(a \sin d_n/a)^{1/2}} \sum_{m=1}^{8} (X_m{}^1 X_m{}^n)^{1/2} \cdot \exp j[\tfrac{1}{2}(\varphi_m{}^1 + \varphi_m{}^n) + \sum_n (\gamma_m{}^n + j\alpha_m{}^n)\,\Delta d] \qquad (3)$$

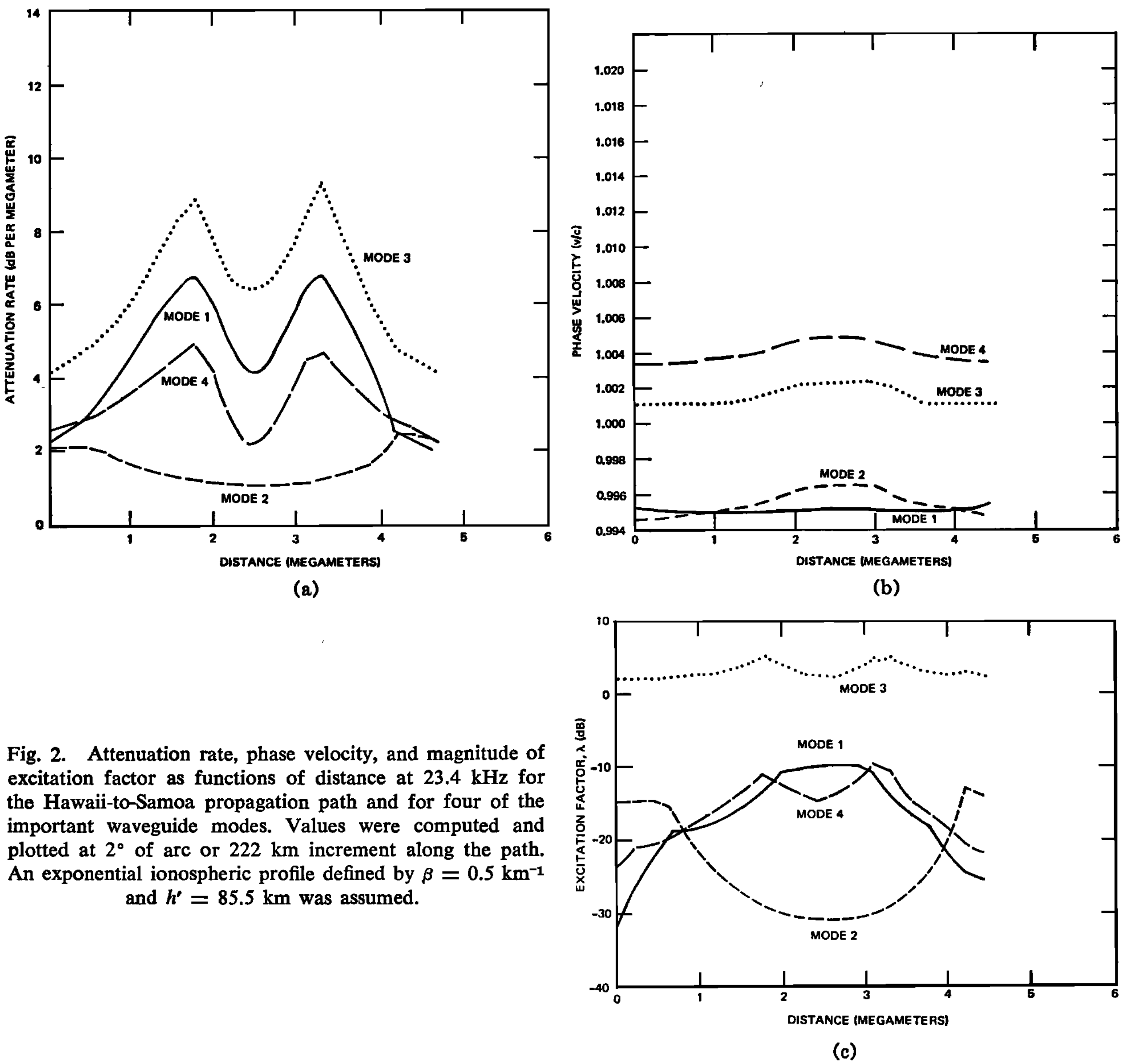

Fig. 2. Attenuation rate, phase velocity, and magnitude of excitation factor as functions of distance at 23.4 kHz for the Hawaii-to-Samoa propagation path and for four of the important waveguide modes. Values were computed and plotted at 2° of arc or 222 km increment along the path. An exponential ionospheric profile defined by $\beta = 0.5$ km^{-1} and $h' = 85.5$ km was assumed.

where

$E(d_n)$, field intensity in volts per meter.
d_n, $n\Delta d$.
K, function of frequency and radiated power.
a, radius of the earth in Mm.
X_m^1, X_m^n, excitation factor amplitude of the mth mode at the transmitter and at d_n, respectively (in Figure 2c, $\lambda = 20 \log (kh'X/2)$).
φ_m^1, φ_m^n, excitation factor phase in radians of the mth mode at the transmitter and at d_n, respectively.
γ_m^n, $= (2\pi/c)(1 - c/v_m^n)$ (where c is the speed of light).
α_m^n, attenuation rate of the mth mode over the nth distance increment in nepers per megameter.
v_m^n, phase velocity of the mth mode over the nth distance increment.

The attenuation rates and phase velocities for each Δd increment are determined by assuming linear interpolation between the values computed for the ends of the 222-km segments.

The results of the field-intensity calculations for 23.4 kHz assuming 1 kw of radiated power are are shown in Figure 3 for the propagation paths from

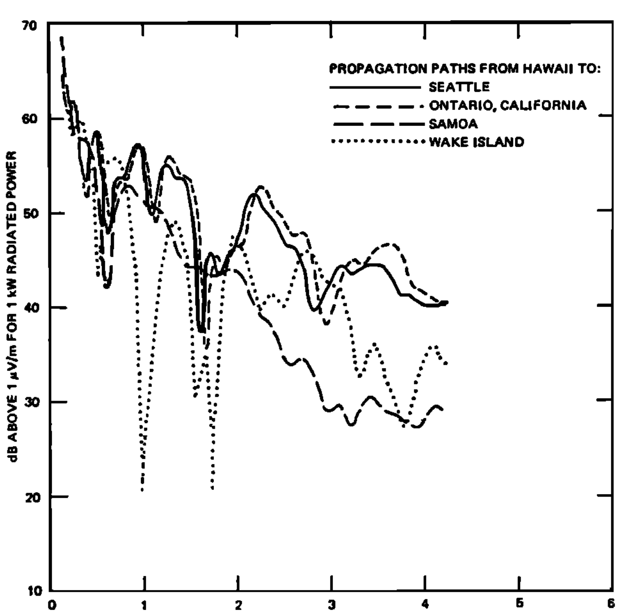

Fig. 3. Computed amplitude as a function of distance of 23.4-kHz signal radiated from Hawaii. Values are computed by using relation (3) for propagation paths to Seattle, Ontario (California), Samoa, and Wake Island.

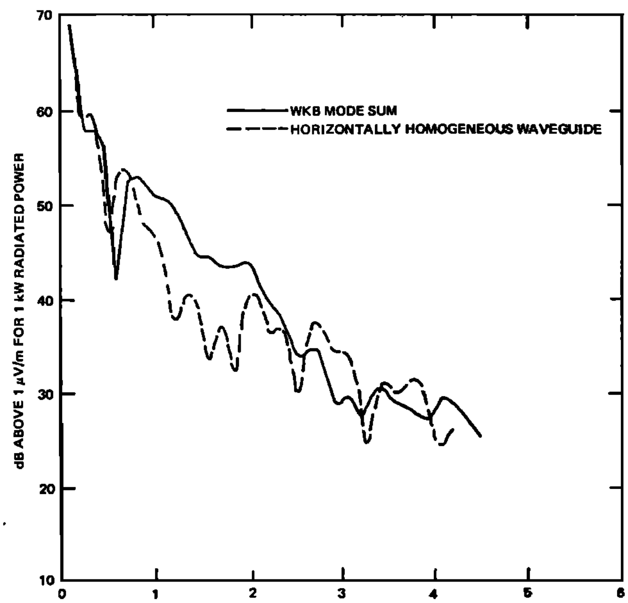

Fig. 4. Comparison of computations made by using the WKB mode sum with that based on assuming horizontal homogeneity. The results are for the Hawaii (NPM, 23.4 kHz)-to-Samoa propagation path.

Hawaii to Seattle, Ontario, Samoa, and Wake Island. These were the paths that were selected for the measurements. The paths include easterly and westerly propagation, and the path to Samoa provided computed fields that indicate the increased southerly attenuation first suggested by *Snyder and Pappert* (1969).

Previous field-intensity calculations at NELC, such as those reported by *Snyder and Pappert* [1969], were based on an assumed horizontally homogeneous waveguide. Similar calculations have also been made for the paths included in this study. The waveguide mode constants computed by using magnetic parameters associated with the midpath are used for the entire path. The results obtained for the Seattle, Ontario, and Wake Island paths are very similar to those obtained by using the WKB approximation and are not shown here. However, a significant difference was obtained for the path to Samoa, as illustrated in Figure 4.

3. AIRBORNE MEASUREMENTS AND COMPARISON WITH THEORETICAL CALCULATIONS

Table 1 gives a list of the flights made and their respective dates and times.

Some flights required as many as 12 hours to complete, and as a result sunrise or sunset occurred along the propagation path during some flights. The flight schedule was arranged so that these transition times occurred while the airplane was near the transmitter. Data recorded within one-half hour of the transition time are not reproduced here.

The antenna used was a 6-foot vertical rod mounted in the upper radome of the airplane. All plotted data are normalized to field intensity for 1 kw of radiated power.

The NPM amplitude recorded on the flight from Seattle to Hawaii is shown in Figure 5. The typical nighttime modal interference pattern is evident. The effects of at least three waveguide modes are apparent. Major modal interference maxima occurred near 1.2, 2.2, and 3.3 Mm. The more rapid oscillatory variation results from the effects of higher-order modes. The theoretical calculation for the path is shown with the data. The agreement between the measurement and the calculation is good.

The two sets of data shown in Figure 6, recorded on flights from Hawaii to Samoa and Samoa to Hawaii, are very similar. The similarity illustrates the degree of reproducibility of propagation conditions on the two different nights. An unusual feature of these data is the relative lack of higher-order modal interference out to about 2.6 Mm coupled with the rapidly varying interference pattern beyond 2.6 Mm.

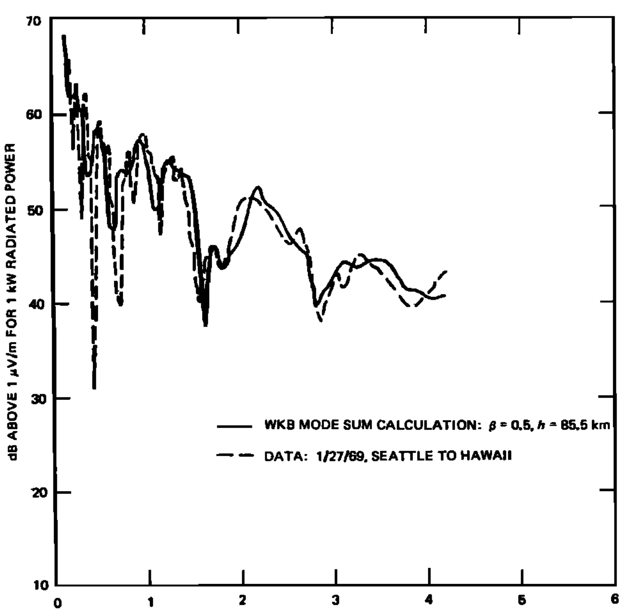

Fig. 5. Comparison of measured data with WKB mode sum calculations for the Hawaii (NPM, 23.4 kHz)-to-Seattle propagation path.

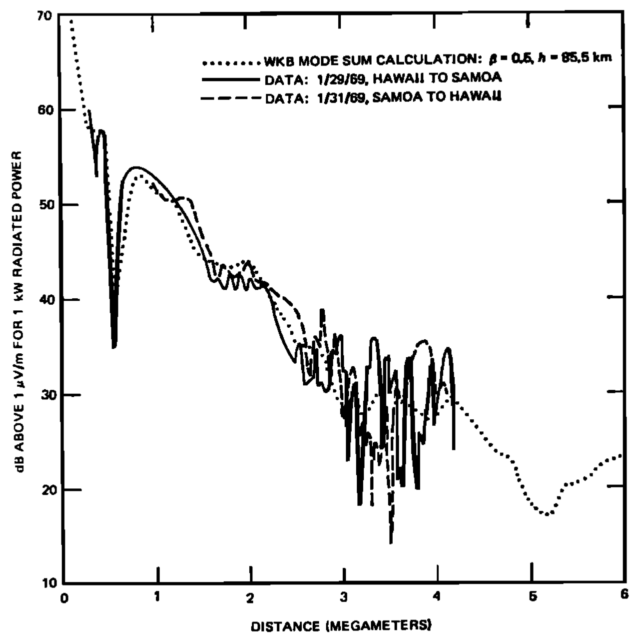

Fig. 6. Comparison of measured data with WKB mode sum calculations for the Hawaii (NPM, 23.4 kHz)-to-Samoa propagation path.

Note that the signal level is from 10 to 15 decibels lower than the data recorded to the east of Hawaii in the distance range from 3 to 4 Mm. The more distant data for the Hawaii-to-Samoa flight were recorded after local midnight and for the Samoa-to-Hawaii flight before local midnight.

The theoretical amplitude curve shown with the data in Figure 6 has all the above general characteristics except for the rapid 15 to 20 decibel oscillation beyond 2.6 Mm. However the theoretical curve agrees well with the mean field intensity in this region. The field-intensity calculation for the horizontally homogeneous waveguide shown in Figure 4 does not agree as well with the data. Note that the calculated values are about 7 to 10 decibels lower than the data in the 1 to 2 Mm range. From this comparison, it is concluded that the WKB approximation is the more appropriate method of calculation to apply to this propagation path.

The data recorded during the two flights from Hawaii to Wake and Wake to Hawaii differ significantly, as shown in Figure 7. The Hawaii-to-Wake data indicate higher attenuation and show the more rapid oscillatory variation with distance and/or possibly time. The data recorded on the return flight have many of the characteristics of the WKB calculations shown. A profile 2 or more km higher would provide computed results with a modal interference pattern that would fit this particular data better.

The data and calculations for the Hawaii-to-Ontario path are compared in Figure 8. The characteristics are relatively similar. However, again, a relative shift of the modal interference pattern, such as would be produced by an increase of the ionospheric profile height by about 1 or 2 km, is needed to obtain better agreement.

4. DISCUSSION

The VLF data reported here indicate some variation of propagation conditions from night to night and throughout a given night. Such nighttime variability is well known. The variability of phase of 10.2 kHz, as measured by Swanson using the Omega system, is equivalent to a change of phase velocity of the dominant waveguide mode of the order of ± 2 to 3 parts in 10^{-4}. This change can be produced by a change in h' of about ± 2 to 3 km. This height variability is within the range of the profile change needed to provide better agreement between computations and data recorded during the flights from Wake to Hawaii (Figure 7) and Hawaii to Ontario (Figure 8). The former data indicate that the ionospheric reflection height was changing throughout the flight, and the latter data indicate that the reflection height was relatively stationary.

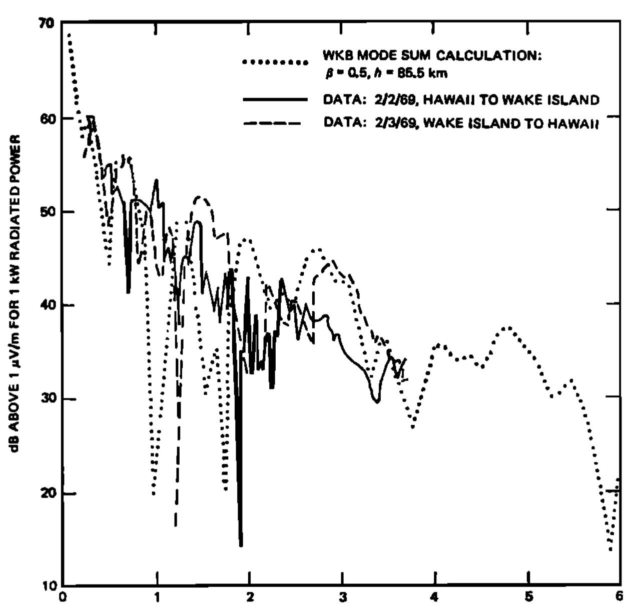

Fig. 7. Comparison of measured data with WKB mode sum calculation for the Hawaii (NPM, 23.4 kHz)-to-Wake Island propagation path.

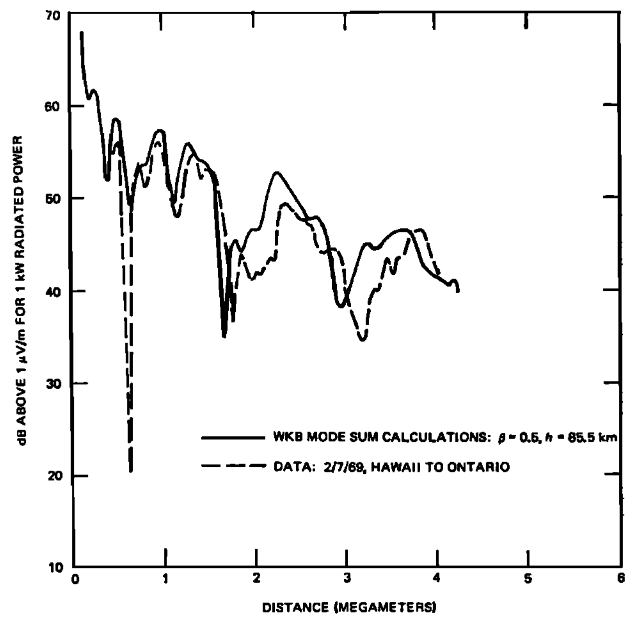

Fig. 8. Comparison of measured data with WKB mode sum calculation for the Hawaii (NPM, 23.4 kHz)-to-Ontario (California) propagation path.

The data recorded on the flight from Hawaii to Wake (Figure 7), which deviate significantly from the computations for this path and from the other data recorded on this same path, indicate that the ionospheric conditions differed significantly and possibly changed with time. A geomagnetic storm was reported to have started at 1503 UT on February 2, which was about 40 minutes after the flight terminated, and a pre-magnetic storm disturbance of the nighttime *D* layer may have occurred.

The relatively rapid oscillatory amplitude variation with distance observed beyond 2.6 Mm on the Hawaii-to-Samoa propagation path is not understood. It may be a spatial variation or a time variation. It is interesting to note that both the geographic and the geomagnetic equators intersect this path at about 2.45 Mm.

5. CONCLUSIONS

On the basis of the data and the discussion of the preceeding sections, it is concluded that the use of the NELC waveguide computer program for computing waveguide mode constants and the application of the WKB approximation for computing the sum of modes adequately accounts for many features of the data. The large attenuation observed for propagation to the south and some of the other amplitude pattern differences observed for the different propagation paths are theoretically accounted for by the differences in the associated geomagnetic field parameters. Therefore, it is concluded that these observed differences were due primarily to the effect of the geomagnetic field on the propagation.

It appears that an exponential electron-density profile described by $\beta = 0.5$ km^{-1} and 84 km $< h'$ $<$ 87 km (Wait's definitions) is a useful representation for nighttime conditions in that it accounts for Omega phase velocity measurements at 10.2 kHz and many features of the amplitude measurements reported here.

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