DOME COOKBOOK OF GEODESIC GEOMETRY

REVIEWS:

"SHOWS ACTUAL DERIVATION OF CHORD FACTORS AND ANGLES"... LLOYD KAHN, WHOLE EARTH EPILOG.

"HERE IS DAVID'S 3-FREQUENCY TRUNCATABLE DOME, WHICH SITS FLAT ON THE GROUND, UNLIKE THE DOMEBOOK 2 — 3 FREQ. DOMES" LLOYD KAHN, SHELTER.

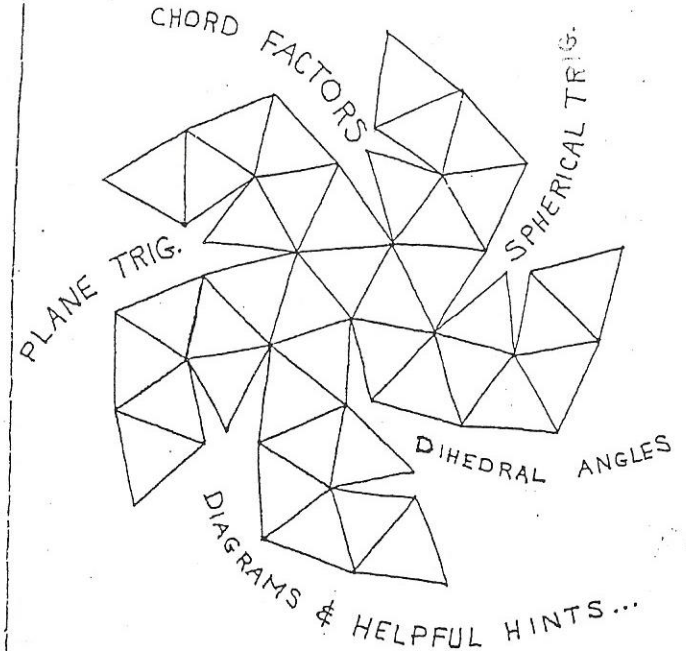
"FOR FULLER FREAKS OF A MATHEMATICAL BENT. AUTHOR GIVES CORRECT CONSTRUCTION OF CHORD FACTORS FOR A 3 FREQUENCY DOME ALL OF WHOSE GROUND LEVEL VERTICES ARE COPLANAR..." "THIS CONSTRUCTION PUTS TO SHAME THE AUTHORS OF VARIOUS BOOKS ON DOMES (EXCEPT FULLER), ESPECIALLY DOMEBOOK TWO, WHICH CLAIMED SUCH A DOME WAS IMPOSSIBLE." PJC, THE AMERICAN MATHEMATICAL MONTHLY (AUG-SEPT, 1974).

APPLICATIONS:

CHORD FACTOR DATA, DIHEDRAL ANGLE DATA, UNDERSTANDING DOME DERIVATIONS, INTRODUCTION TO SPHERICAL MATH, AND THREE DIMENSIONAL MODEL MAKING.

AUTHOR:

A DESIGNER / TEACHER CURRENTLY WORKING ON SOLAR ENERGY TECHNIQUES FOR HOUSING.



DOME COOKBOOK OF GEODESIC GEOMETRY

BY DAVID KRUSCHKE

SECOND EDITION

$3.00

DOME COOKBOOK OF GEODESIC GEOMETRY

BY DAVID KRUSCHKE

TO MY CHILDREN, LEE & JOHN

SPECIAL THANKS TO HANK PHILLIPS & WINDWORKS FOR CORRECTIONS & SUGGESTIONS ...

PREFACE

THE PURPOSE OF THIS BOOK IS TO SHOW THE ACTUAL DERIVATION OF THE CHORD FACTORS AND DIHEDRAL ANGLES WITHOUT THE USE OF JARGON, CO-ORDINATES, AND STRANGE NAMES. ALSO, THERE HAS BEEN AN ATTEMPT TO USE AS FEW FORMULAS AND TRIG. FUNCTIONS AS POSSIBLE (FIVE SPHERICAL TRIG. FORMULAS AND THE SINE & COSINE FUNCTIONS ARE USED FOR THE CHORD FACTORS — THE LAW OF COSINES, SINE & COSINE FUNCTIONS, AND THE PYTHAGOREAN THEOREM FOR THE DIHEDRAL ANGLES).

UNLIKE SOME OTHER DOME BOOKS, THE CHORD FACTOR RESULTS HERE ARE IN CLOSE AGREEMENT WITH BUCKY FULLER'S (A FACT THAT IS IMPORTANT WHEN WORKING WITH THREE FREQUENCY DOMES — SEE PAGE 8).

CONTENTS

HOW TO PROCEED ...	P.1	CHORD FACTORS ...	P. 12
TRIG. REVIEW ...	P.3	COMPARISON WITH FULLER ...	P.29
FREQUENCY ...	P.5	DIHEDRAL ANGLES ...	P.31
SPHERICAL TRIG. ...	P.10	REFERENCES ...	P.45

PUBLISHED BY
DAVID KRUSCHKE
RT 2, BOX 34 A
WILD ROSE, WI.
54984

HOW TO PROCEED:

THE READER SHOULD BE EQUIPPED WITH PENCILS, PAPER AND TRIG. TABLES. SOME OF THE MORE ELABORATE DIAGRAMS SHOULD BE SKETCHED OR TRACED ON A SEPARATE SHEET OF PAPER. THIS WILL ALLOW THE READER TO AVOID HAVING TO FLIP PAGES AROUND SEARCHING FOR A NEEDED OR RELEVANT DIAGRAM.

THE READER IS ALSO URGED TO MAKE UP AND SOLVE ADDITIONAL PROBLEMS (I.E. ANGLES, DISTANCES, & AREAS ASSOCIATED WITH TRIANGLES, GREAT CIRCLE DISTANCES ON A GLOBE, ETC.). EXPERIMENTING WITH SPECIFIC PROBLEMS WILL MAKE THE READER FEEL MORE COMFORTABLE WITH GENERAL FORMULAS.

OFTEN, ONE CHECKS COMPLETED WORK BY EITHER DRAWING TRIANGLES TO SCALE ON PAPER USING A RULER AND A PROTRACTOR OR BY CONSTRUCTING THREE DIMENTIONAL MODELS TO SCALE USING PAPER, CARDBOARD, "GEO-D-STIX", ETC.

CAUTION :

FOR MOST PEOPLE (INCLUDING THIS WRITER), LEARNING GEOMETRY IS NOT A PASSIVE EXPERIENCE REQUIRING ONLY CASUAL OBSERVATIONS. AS A TEACHER, THIS WRITER HAS SEEN MANY BRIGHT STUDENTS FAIL TO SOLVE OR UNDERSTAND PROBLEMS BECAUSE THEY WOULD NOT WRITE DOWN OR SKETCH WHAT THEY WERE THINKING. IT SEEMS THAT PEOPLE FEEL UNEASY OR NERVOUS ABOUT WRITING DOWN SOMETHING THAT MIGHT BE IN ERROR. THIS RELUCTANCE TO WRITE OR SKETCH SHOULD BE RESISTED AS MUCH AS POSSIBLE. THE PERSON WHO CAN WRITE AND SKETCH FREELY AND EASILY WILL FIND IT MUCH EASIER TO ANALYZE, DESIGN, OR CREATE.

WIDELY USED PLANE TRIG

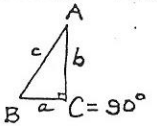
1) NOTATION : $\bar{A} = 90° - A$, $\bar{A}$ IS CALLED "COMPLEMENT OF A" OR "A COMPLEMENT." "A" OF COURSE, IS AN ANGLE. ALSO, "ANGLE A" CAN BE DENOTED BY: $\angle A$.

2) FORMULAS:
GIVEN: ANGLE "$\bar{A}$"

$$\begin{cases} \sin A = \cos \bar{A} \\ \sin \bar{A} = \cos A \\ \tan A = \cot \bar{A} \\ \tan \bar{A} = \cot A \end{cases}$$

EXAMPLE: $\sin 30° = \cos 60° = .5$

3) FORMULAS:
GIVEN: TRIANGLE ABC, ANGLE C ($\angle C$) = 90°



$a = c \sin A$ OR $\sin A = \frac{a}{c}$

$b = c \sin B$ OR $\sin B = \frac{b}{c}$

$a = c \cos B$ OR $\cos B = \frac{a}{c}$

$b = c \cos A$ OR $\cos A = \frac{b}{c}$

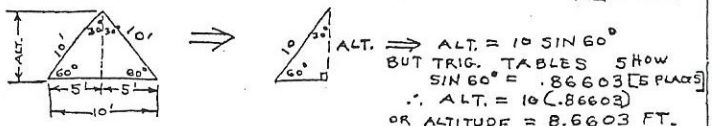
$a = b \tan A$ OR $\tan A = \frac{a}{b}$

$b = a \tan B$ OR $\tan B = \frac{b}{a}$

$\tan A = \frac{\sin A}{\cos A}$ $\quad \cot A = \frac{\cos A}{\sin A} = \frac{1}{\tan A}$

$\tan A = \frac{1}{\cot A}$

EXAMPLE: GIVEN EQUILATERAL TRIANGLE WITH 10 FT. PER EDGE, FIND THE ALTITUDE.



$\Rightarrow$ ALT. $= 10 \sin 60°$

BUT TRIG. TABLES SHOW $\sin 60° = .86603$ [5 PLACES]

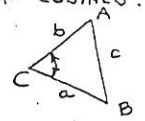
$\therefore$ ALT. $= 10(.86603)$

OR ALTITUDE = 8.6603 FT.

WIDELY USED PLANE TRIG. [CON'T]

1) FORMULA : CALLED "THE LAW OF COSINES".

GIVEN TRIANGEE ABC



$0° \leq \angle C \leq 180°$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

NOTE : IF C IS GREATER THAN 90°

THEN $\cos c = -\cos(180° - c)$

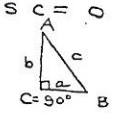
AND $-2ab\cos C = +2ab\cos(180° - C)$

OR $c^2 = a^2 + b^2 + 2ab\cos(180° - C)$

ALSO IF $C = 90°$, THEN $\cos C = 0$

AND THEN $c^2 = a^2 + b^2$

PYTHAGOREAN THEOREM



EXAMPLE 1 : GIVEN TRIANGLE ABC WITH $a = 10$ FT, $b = 10$ FT, & $C = 60°$;

FIND SIDE c.

CONSULTING TRIG. TABLES, WE FIND THAT $\cos 60° = .5$. THEN $c^2 = 10^2 + 10^2 - 2(10)(10)(.5)$

$\therefore c^2 = 10^2 + 10^2 - 10^2 = 10^2$

$c^2 = 10^2$

$\therefore$ $c = 10$ FT

EXAMPLE 2 : GIVEN TRIANGLE ABC WITH $a = 10$ FT, $b = 10$ FT, & $C = 120°$;

FIND SIDE c.

SINCE 120° IS GREATER THAN 90°, $\cos 120° = -\cos(180° - 120°)$

$\therefore \cos 120° = -\cos 60° = -.5$

THEN $c^2 = 10^2 + 10^2 - 2(10)(10)(-.5)$

$c^2 = 10^2 + 10^2 + 10^2 = 3 \cdot 10^2$

$c = \sqrt{3} \cdot 10 = 10\sqrt{3} = 10(1.732)$

$c = 17.32$ FT.

FREQUENCY ...

"FREQUENCY" IS ANOTHER TERM THAT IS USED TO DESCRIBE A KEY PROPERTY OF A DOME. FOR INSTANCE, A DOME WITH A FIXED RADIUS, SAY 10 FT, COULD CONSIST OF A SMALL NUMBER OF LONG STRUTS (LOW FREQUENCY) OR A LARGE NUMBER OF SHORT STRUTS (HIGH FREQUENCY). WHEN SELECTING A PARTICULAR FREQUENCY FOR A DOME OF A GIVEN DIAMETER, ONE TRIES TO COMPROMISE IN ORDER TO AVOID LARGE, BULKY, EXPENSIVE STRUTS & PANELS (LOW FREQUENCY) AS WELL AS NUMEROUS, FLIMSY SMALL STRUTS & PANELS THAT ARE MADE WITH MANY DIFFERENT DIMENSIONS (HIGH FREQUENCY). MOST DOMES HAVE STRUTS THAT ARE LESS THAN 8 OR 9 FEET. THIS BOOKLET WILL COVER 1, 2, 3, & 4 FREQUENCY DOMES. A MORE PRECISE DEFINITION OF "FREQUENCY" FOLLOWS ...

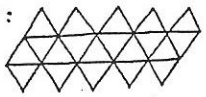
"FREQUENCY" (MORE ACCURATELY DEFINED)

TO BEGIN WITH, A ONE FREQUENCY "DOME" IS SIMPLY AN ICOSAHEDRON, A 20 FACED GEOMETRIC SOLID [EACH FACE IS AN EQUILATERAL TRIANGLE]. WITH AN ICOSAHEDRON, ALL ANGLES (BETWEEN FACES AND ON FACES) AND ALL EDGES OR "STRUTS" ARE EQUAL.

SOLID ICOSAHEDRON

HOLLOW ICOSAHEDRON

THE SOLID ICOSAHEDRON CAN BE CUT APART AND FLATTENED OUT THUSLY:

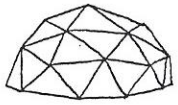


BY PROJECTING NATIONAL BOUNDARIES AS FOUND ON A GLOBE ONTO AN ICOSAHEDRON AND THEN FLATTENING OUT THE ICOSAHEDRON, ONE HAS A MAP THAT SHOWS COUNTRIES AT THE CORRECT SIZE IN RELATION TO EACH OTHER [DEVELOPED BY R. BUCKMINSTER FULLER].

"FREQUENCY" – CONTINUED ...

A TWO FREQUENCY DOME IS A STRUCTURE THAT IS MADE BY TAKING EACH ICOSAHEDRON STRUT AND DIVIDING IT INTO TWO PARTS AND THEN INSERTING ANOTHER TRIANGLE WITHIN THE BASIC ICOSAHEDRON TRIANGLE.

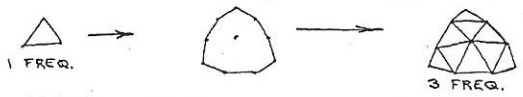
THIS MUST BE DONE IN SUCH A MANNER SO THAT EACH HUB THAT CONNECTS STRUTS IS TOUCHING THE SHELL OF A SPHERE THAT PASSES THROUGH ALL OF THE VERTICES OF THE ORIGINAL ICOSAHEDRON.



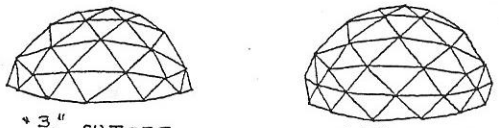
2 FREQUENCY DOME

A THREE FREQUENCY DOME IS A STRUCTURE FORMED BY DIVIDING THE ICOSAHEDRON STRUTS INTO THREE PARTS. THE ICOSAHEDRON TRIANGLE CHANGES AS FOLLOWS:

"FREQUENCY" — CONTINUED



EACH VERTEX OR "HUB" IS STILL IN THE SPHERE "GENERATED" BY THE BASIC ICOSAHEDRON.



3 FREQUENCY DOMES

ALL OF THE DOMEBOOKS THAT THIS WRITER HAS SEEN HAVE 3 FREQUENCY CHORD FACTORS THAT DO NOT ALLOW A PERFECT CUTOFF PLANE AT GROUND LEVEL (ALL GROUND LEVEL VERTICES EXACTLY IN THE SAME PLANE). IN FACT, BOTH DOMEBOOK 1 AND DOMEBOOK 2 HAVE STATEMENTS ABOUT 3 FREQUENCY DOMES THAT COULD BE CONFUSING AND MISLEADING.* NEVERTHELESS, THE CHORD FACTORS IN THIS BOOK YEILD TWO PERFECT CUT OFF PLANES FOR THE 3 FREQUENCY DOME AND THREE PERFECT CUTOFF PLANES FOR THE 4 FREQ. DOME (BUCKY FULLER ACCOMPLISHED THIS YEARS AGO).

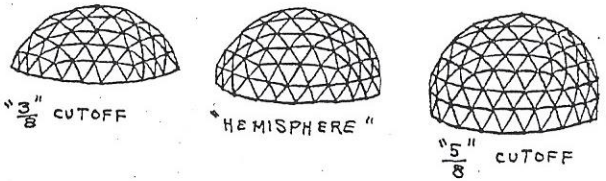
* DOMEBOOK 1, PAGE 19, PACIFIC DOMES; AND DOMEBOOK 2, PAGES 24 & 76, PACIFIC DOMES.

"FREQUENCY" — CONTINUED

IN ANY CASE, THE READER CAN CHECK THIS OUT BY CONSTRUCTING A MODEL OF A THREE FREQUENCY DOME USING THE CHORD FACTORS HEREIN.

A FOUR FREQUENCY DOME IS A STRUCTURE FORMED BY DIVIDING THE ICOSAHEDRON STRUTS INTO FOUR PARTS. THE ICOSAHEDRON TRIANGLE CHANGES AS FOLLOWS:

AS WITH THE OTHER DOMES MENTIONED, EACH VERTEX OR "HUB" IS IN THE SPHERE (SHELL) "GENERATED" BY THE BASIC ICOSAHEDRON.

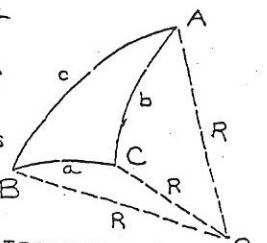


4 FREQUENCY DOMES

SPHERICAL TRIG.

TO DERIVE THE CHORD FACTORS FOR THE ONE (ICOSAHEDRON), TWO, THREE, AND FOUR FREQUENCY DOMES, WE NEED A FEW FORMULAS THAT ARE CONCERNED WITH RIGHT SPHERICAL TRIANGLES.

GIVEN: RIGHT SPHERICAL TRIANGLE ABC



NOTE: 1) ARCS $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, & $\overset{\frown}{AC}$ ARE GREAT CIRCLE ARCS ON A SPHERE WITH RADIUS R AND CENTER O.

2) ANGLES A, B, & C ARE "SURFACE" ANGLES DETERMINED BY THE INTERSECTIONS OF THE THREE GREAT CIRCLE ARCS.

3) ANGLE C [DETERMINED BY THE INTERSECTION OF ARCS $\overset{\frown}{AC}$ & $\overset{\frown}{BC}$] IS A RIGHT ANGLE.

4) a, b, & c ARE ANGLES, WHERE EACH ANGLE IS DETERMINED BY AN ARC AND TWO RADII. ANGLES a, b, & c ARE OFTEN CALLED "CENTRAL" ANGLES AND ARE THE ANGLES USED TO COMPUTE THE CHORD FACTORS OF A DOME.

SPHERICAL TRIG. — CONTINUED ...

THE CHORD FACTOR DERIVATIONS IN THIS BOOKLET USE ONLY FIVE SPHERICAL TRIG. FORMULAS:

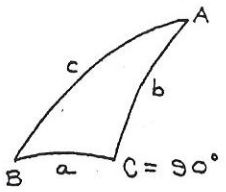
1) $\sin a = \sin c \sin A$ OR $\sin A = \sin a \div \sin c$, $\sin c = \sin a \div \sin A$

2) $\sin b = \sin c \sin B$ OR $\sin B = \sin b \div \sin c$, $\sin c = \sin b \div \sin B$

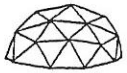
3) $\cos c = \cos a \cos b$ OR $\cos a = \cos c \div \cos b$, $\cos b = \cos c \div \cos a$

4) $\cos A = \cos a \sin B$ OR $\cos a = \cos A \div \sin B$, $\sin B = \cos A \div \cos a$

5) $\cos B = \cos b \sin A$ OR $\cos b = \cos B \div \sin A$, $\sin A = \cos B \div \cos b$



CHORD FACTORS

A CHORD FACTOR IS THE LENGTH OF A STRUT ON A DOME HAVING A ONE FOOT RADIUS. FOR EXAMPLE, A "TWO FREQUENCY" DOME  WITH A ONE FOOT RADIUS HAS TWO DIFFERENT STRUT LENGTHS: 0.547 FT. & 0.618 FT. THAT MEANS THAT THE CHORD FACTORS OF THIS DOME ARE .547 & .618. HOWEVER, IF WE WISH TO KNOW THE PROPER STRUT LENGTHS OF THIS TYPE OF DOME HAVING A 10 FOOT RADIUS INSTEAD OF A ONE FOOT RADIUS, WE WOULD JUST MULTIPLY EACH OF OUR CHORD FACTORS BY OUR DESIRED RADIUS (10 FT.). WE NOW HAVE STRUT LENGTHS OF 5.47 FT. & 6.18 FT. RESPECTIVELY.

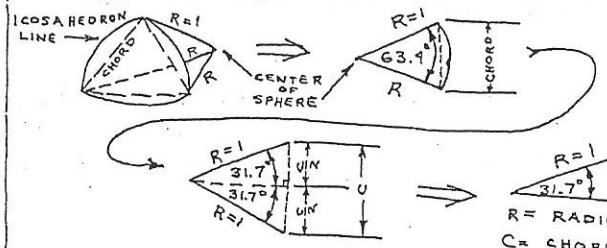
THUS: [redacted]

FOR ALL DOMES DESCRIBED BY CHORD FACTORS.

CHORD FACTORS & CENTRAL ANGLES

AS MENTIONED PREVIOUSLY, "CENTRAL ANGLES" ARE THE ANGLES THAT ARE USED TO COMPUTE THE CHORD FACTORS OF A DOME. FOR EXAMPLE, CONSIDER AN ICOSAHEDRON WITH ONE FOOT RADIUS. IT SO HAPPENS THAT [redacted]

USING THIS INFORMATION, LET US FIND THE CHORD FACTOR.



R = RADIUS = 1 FT.
C = CHORD FACTOR

FROM PLANE TRIG. $\frac{C}{2} = R \sin 31.7°$

$C = 2R \sin 31.7°$

$\sin 31.7° = \sin 31°42' \doteq .525$, $R = 1$ (OR 1 FT)

$\therefore \frac{C}{2} = (1)(.525) = .525$

$\therefore C = 1.05$ (OR 1.05 FT)

$\therefore$ [redacted]

AND [redacted]

TRIG. TABLES — NOTE:

THE READER WILL NOTE THAT ALL ANGLES ARE IN DECIMAL FORM INSTEAD OF THE MORE CONVENTIONAL DEGREES AND MINUTES (I.E. 30.50° INSTEAD OF 30°30'). ANY READER THAT WOULD PREFER TO USE DECIMAL ANGLES CAN PURCHASE THE FOLLOWING BOOK FOR A MERE 50¢: (IF ITS STILL IN PRINT)

TABLE OF SINES AND COSINES TO FIFTEEN DECIMAL PLACES AT HUNDREDTHS OF A DEGREE.

NATIONAL BUREAU OF STANDARDS
APPLIED MATHEMATICS SERIES 5
ISSUED MAY 2, 1949.

FOR SALE BY THE
SUPERINTENDENT OF DOCUMENTS
U.S. GOVERNMENT PRINTING OFFICE
WASHINGTON, D.C. 20402

CHORD FACTORS — 1 FREQUENCY DOME (ICOSAHEDRON)


SOLID

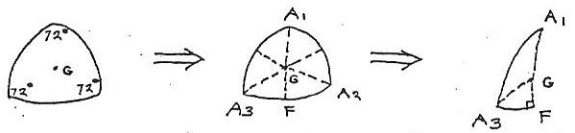

HOLLOW

SINCE EVERY CHORD IS EQUAL ON AN ICOSAHEDRON, ONLY ONE CHORD FACTOR WILL BE FOUND. ALSO, OTHER ANGLES DERIVED HERE WILL BE USED IN THE TWO, THREE, AND FOUR FREQUENCY DERIVATIONS.

TO BEGIN, THE ICOSAHEDRON CONSISTS OF 20 EQUILATERAL TRIANGLES. THEREFORE EACH PLANE TRIANGLE HAS A 60° ANGLE IN EACH VERTEX. HOWEVER, THE DERIVATIONS HERE REQUIRE THE USE OF "SURFACE ANGLES." SURFACE ANGLES ARE ANGLES DETERMINED BY THE TANGENTS TO THE INTERSECTING GREAT CIRCLE ARCS [THE POINT OF TANGENCY IS THE SAME POINT THAT IS DETERMINED BY THE INTERSECTING GREAT CIRCLE ARCS]. THEREFORE, EACH PLANE TRIANGLE HAS A CORRESPONDING SPHERICAL TRIANGLE  AND EACH PLANAR ANGLE HAS A CORRESPONDING SPHERICAL TRIANGLE.

3 FREQUENCY DOME

START WITH THE BASIC SPHERICAL ICOSA-TRIANGLE:



NOTE: G IS THE "CENTER" (CENTROID) OF SPHERICAL TRIANGLE $A_1A_2A_3$ AND F IS THE MIDPOINT OF ARC $\overset{\frown}{A_2A_3}$.

NOW FROM THE 2 FREQUENCY DERIVATION WE KNOW THAT:

$$\overset{\frown}{A_3F} = 31.717474262 3^\circ$$
$$\overset{\frown}{A_1A_2} = 63.434948524 6^\circ$$

AND BY INSPECTION: $\angle A_3A_1F = \angle GA_3F = 36.00\ldots^\circ$

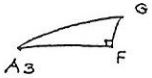
NOW $\sin \overset{\frown}{A_1F} = \sin 72^\circ \sin \overset{\frown}{A_1A_2}$

$\sin \overset{\frown}{A_1F} = (.809016994 9)(.894427187 8)$

$\sin \overset{\frown}{A_1F} = .850650804 8$

$\overset{\frown}{A_1F} = 58.282526741 5^\circ$

NEXT, SOLVE TRIANGLE GA_3F.



$\cos\angle A_3GF = \sin\angle GA_3F \cos \overset{\frown}{A_3F}$

$\cos\angle A_3GF = (.587785251 8)(.850650809 3)$

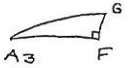
$\cos\angle A_3GF = .500000000 1$

$\angle A_3GF = 60.000000595 8^\circ \doteq 60^\circ$

(NOT SUPRISING)

3 FREQUENCY DOME — CONTINUED ...

SOLUTION OF TRIANGLE GA_3F — CONTINUED



$\sin \overset{\frown}{A_3F} = \sin \overset{\frown}{A_3G} \sin\angle A_3GF$

$\sin \overset{\frown}{A_3G} = \sin \overset{\frown}{A_3F} \div \sin\angle A_3GF$

$\sin \overset{\frown}{A_3G} = (.525731109 3) \div (.866025404)$

$\sin \overset{\frown}{A_3G} = .607061991 7$

$\overset{\frown}{A_3G} = 37.377367152 4$

$\sin \overset{\frown}{GF} = \sin\angle GA_2F \sin \overset{\frown}{A_3G}$

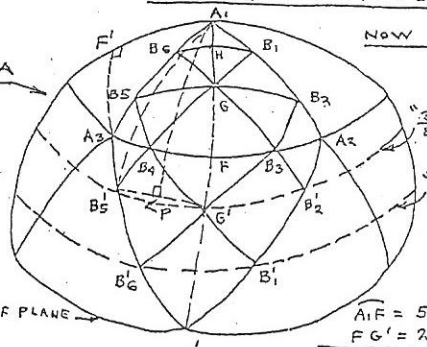
$\sin \overset{\frown}{GF} = (.587785251 8)(.607061991 7)$

$\sin \overset{\frown}{GF} = .356822085 6$

$\overset{\frown}{GF} = 20.905157553 9^\circ$

NOW $\overset{\frown}{GF} = \overset{\frown}{FG'}$

DWG. A



$\overset{\frown}{A_1G'} = \overset{\frown}{A_1F} + \overset{\frown}{GF}$

$\overset{\frown}{A_1F} = 58.282526741 5^\circ$

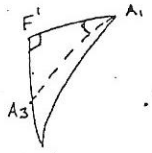
$\overset{\frown}{FG'} = 20.905157553 9^\circ$

$\overset{\frown}{A_1G'} = 79.187684295 4^\circ$

NOW ARC $\overset{\frown}{A_1G'}$ IS THE ARC FROM "VERTICAL" (POINT A_1) TO THE "3/8" CUTOFF PLANE. SO FOR A "PERFECT" CUTOFF PLANE (NO EXTRA "BASE" PANELS) LET POINT B_5' (& PT. B_2') BE IN THE CUTOFF PLANE BY SETTING ARC A_1B_5' = ARC $\overset{\frown}{A_1G'}$.

3 FREQUENCY DOME – CONTINUED

CONSIDER TRIANGLE $F' A_1 B_5'$



WHERE F' IS LIKE F AND IS LOCATED ON THE GREAT CIRCLE EXTENSION OF ARC $\overset{\frown}{A_1' B_6' B_5' A_3}$.

$\therefore \angle A_1 F' B_5' = 90°$ & $\triangle A_1 F' B_5'$ IS A RIGHT SPHERICAL TRIANGLE.

THAT MEANS THAT: $\overset{\frown}{F'A_1} = 31.717474\,2623°$

$\overset{\frown}{F'A_3} = 58.282526\,7415°$

$\overset{\frown}{A_1 A_3} = 63.434948\,5246°$

$\overset{\frown}{A_1 B_5'} = 79.187684\,2954°$

$\angle F'A_1A_3 = 72.00\ldots°$

$\cos \overset{\frown}{F'B_5'} = \cos \overset{\frown}{A_1 B_5'} \div \cos \overset{\frown}{F'A_1}$

$\cos \overset{\frown}{F'B_5'} = (.187592454\,1) \div (.850650\,8093)$

$\cos \overset{\frown}{F'B_5'} = .220528\,1556$

$\overset{\frown}{F'B_5'} = 77.259\,9437\,3966°$

$-\overset{\frown}{F'A_3} = -58.282\,5267\,415°$

[redacted]

BUT $\overset{\frown}{A_3 B_5'} = \overset{\frown}{A_3 B_5} = \overset{\frown}{A_1 B_6} = \overset{\frown}{A_1 B_1} = \overset{\frown}{A_2 B_2} = \overset{\frown}{A_2 B_3} = \overset{\frown}{A_3 B_4}$

$\therefore$ REFER TO $\overset{\frown}{A_3 B_5'}$ AS $\overset{\frown}{AB}$.

[redacted] A

AND $2 \sin \frac{\overset{\frown}{AB}}{2} = .329\,7064\,628$

$\therefore$ [redacted]

* DENOTES CENTRAL ANGLE

3 FREQUENCY DOME – CONTINUED

NOW $\overset{\frown}{B_5 B_6} = \overset{\frown}{A_1 A_3} - 2\overset{\frown}{AB}$

[redacted] *

BUT $\overset{\frown}{B_5 B_6} = \overset{\frown}{B_1 B_2} = \overset{\frown}{B_3 B_4}$

$\therefore$ REFER TO $\overset{\frown}{B_5 B_6}$ AS $\overset{\frown}{BB_A}$

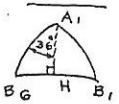
WHERE $\overset{\frown}{BB_A}$ IS AN ARC ALONG AN ICOSALINE (ONE OF THIRTY LINES FORMING A SPHERICAL ICOSAHEDRON).

[redacted] B

$2 \sin \frac{\overset{\frown}{BB_A}}{2} = .441056\,3540$

$\therefore$ [redacted]

FROM DWG. A (DRAWING A) WE HAVE:



$\overset{\frown}{A_1 B_G} = \overset{\frown}{AB} = 18.977\,416\,9951°$

$\sin \overset{\frown}{B_G H} = \sin 36° \sin \overset{\frown}{A_1 B_6}$

$\sin \overset{\frown}{B_G H} = (.587\,785\,2518)(.325\,195\,4548)$

$\sin \overset{\frown}{B_G H} = .191\,145\,0923$

[redacted] C

[redacted] *

BUT $\overset{\frown}{B_G B_1} = \overset{\frown}{B_2 B_3} = \overset{\frown}{B_4 B_5}$

$\therefore$ REFER TO $\overset{\frown}{B_G B_1}$ AS $\overset{\frown}{BB_C}$

WHERE $\overset{\frown}{BB_C}$ IS AN ARC THAT "CUTS" OR SUBDIVIDES THE ICOSALINES.

$2 \sin \frac{\overset{\frown}{BB_C}}{2} = .382\,390\,1844$

$\therefore$ [redacted]

* DENOTES CENTRAL ANGLE

3 FREQUENCY DOME CONTINUED ...

REFERRING BACK TO $\triangle F'A_1B'_5$ (DWG. A),

WE HAVE: $\sin \angle F'A_1B'_5 = \sin \overset{\frown}{F'B'_5} \div \sin \overset{\frown}{A_1B'_5}$

$\sin \angle F'A_1B'_5 = (.9753806079) \div (.9822469503)$

$\sin \angle F'A_1B'_5 = .993009558$

$\angle F'A_1B'_5 = 83.2213490485°$

$-\angle F'A_1A_3 = -72.00...°$

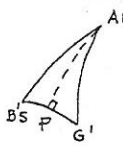
$\angle A_3A_1B'_5 = 11.2213490485°$

$\angle A_3A_1G' = 36.00...°$

$-\angle A_3A_1B'_5 = -11.2213490485°$

$\angle B'_5A_1G' = 24.7786509515°$

LET POINT P BE LOCATED ON GREAT CIRCLE LINE $\overset{\frown}{B'_5G'}$ SUCH THAT $\angle B'_5A_1P = \angle PA_1G' = 12.3893254757°$. [NOTICE THAT POINT P IS NOT IN THE CUT OFF PLANE].



BUT $\overset{\frown}{A_1G'} = 79.1876842954°$

$\therefore \sin \overset{\frown}{PG'} = \sin A_1G' \sin \angle PA_1G'$

$\sin \overset{\frown}{PG'} = (.9822469503)(.2145533638)$

$\sin \overset{\frown}{PG'} = .2107443872$

$\therefore$ [redacted] D

AND [redacted]

BUT $\overset{\frown}{B'_5G'} = \overset{\frown}{B_1G} = \overset{\frown}{B_2G} = \overset{\frown}{B_3G} = \overset{\frown}{B_4G} = \overset{\frown}{B_5G} = \overset{\frown}{B_6G}$

$\therefore$ REFER TO $\overset{\frown}{B'_5G'}$ AS $\overset{\frown}{BG}$.

$2 \sin \frac{BG}{2} = .4214887744$

$\therefore$ [redacted]

✳ DENOTES CENTRAL ANGLE

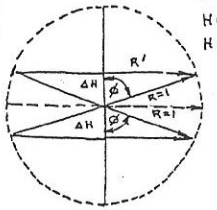
ADDITIONAL 3 FREQUENCY DOME DATA

SPHERICAL (MAX) RADIUS = R = 1.00...

RADIUS OF "SMALL" CIRCLE (BASE) = R'

HEIGHT OF "$\frac{3}{8}$" DOME = H_1

HEIGHT OF "$\frac{5}{8}$" DOME = H_2



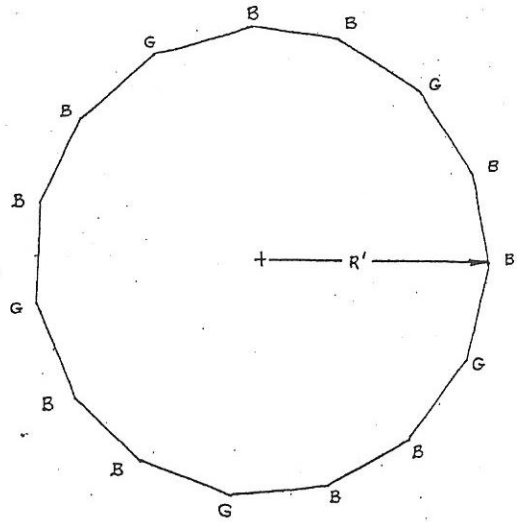
$\phi = 79.1876842954°$

[redacted]

$\Delta H = .1875924541$

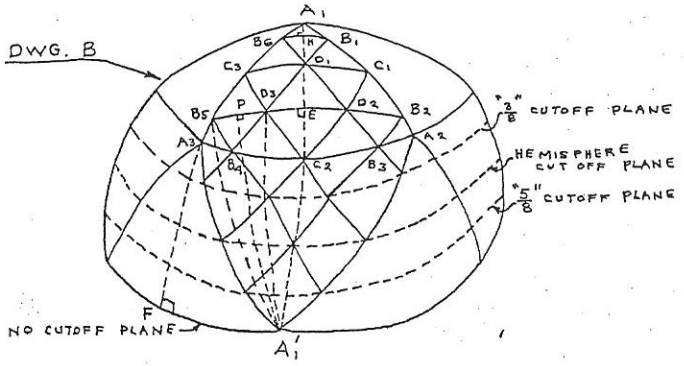
$H_1 = R - \Delta H = .8124075459$

[redacted]

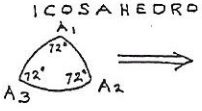


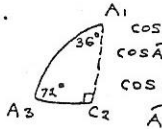
FLOOR PLAN (SEE DWG. A)

THE FOUR FREQUENCY DOME



AGAIN, WE START WITH ONE OF THE SPHERICAL TRIANGLES OF THE SPHERICAL ICOSAHEDRON.





$\cos \widehat{A_3C_2} = \cos 36° \div \sin 72°$

$\cos \widehat{A_3C_2} = (.8090169943) \div (.9510565158)$

$\cos \widehat{A_3C_2} = .8506508093$

$\widehat{A_3C_2} = 31.71747426\,23°$

$\widehat{A_1A_3} = 2\,\widehat{A_3C_2} = 63.43494852\,46°$

$\sin \widehat{A_1C_2} = \sin \widehat{A_1A_3} \sin 72°$

$\sin \widehat{A_1C_2} = (.8944271678)(.9510565158)$

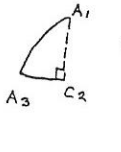
$\sin \widehat{A_1C_2} = .8506508048$

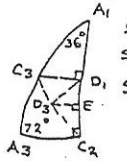
$\widehat{A_1C_2} = 58.28252674\,15°$

SO FAR, THE PROCEDURE HERE IS THE SAME AS THE 3 FREQUENCY DOME.

NOTE: $\widehat{A_1C_3} = \widehat{C_3A_3} = \widehat{A_3C_2} = \widehat{C_2A_2} = \widehat{A_2C_1} = \widehat{A_1C_1}$

4 FREQUENCY DOME — CONTINUED





$\sin \widehat{C_3D_1} = \sin 36° \sin \widehat{A_1C_3}$

$\sin \widehat{C_3D_1} = (.5877852518)(.5257311093)$

$\sin \widehat{C_3D_1} = .3090169924$

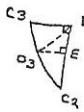
$\widehat{C_3D_1} = 18.000000\,8571° *$

BUT $\widehat{C_3D_1} = \widehat{D_1C_1} = \widehat{C_1D_2} = \widehat{D_2C_2} = \widehat{C_2D_3} = \widehat{D_3C_3}$

$\therefore$ REFER TO $\widehat{C_3D_1}$ AS $\widehat{CD}$.

$2 \sin \frac{\widehat{CD}}{2} = .3128689446$

$\therefore$ CF $_{CD}$ $= .3128689446$



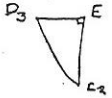
$\widehat{CC} = 2\,\widehat{CD} = 36.000001\,7143°$

$\sin \angle C_3C_2D_1 = \sin \widehat{C_3D_1} \div \sin \widehat{C_2C_3}$

$\sin \angle C_3C_2D_1 = (.3090169924) \div (.5877852760)$

$\sin \angle C_3C_2D_1 = .5257310875$

$\angle C_3C_2D_1 = 31.71747266\,72°$



$\sin \widehat{D_3E} = \sin \widehat{D_3C_2} \sin \angle D_3C_2E$

$\sin \widehat{D_3E} = (.3090169924)(.5257310875)$

$\sin \widehat{D_3E} = .1624598394$

$\widehat{D_3E} = 9.349703843\,3°$

$\widehat{D_3D_2} = \widehat{D_3E}\,D_2 = 2\,\widehat{D_3E} = 18.699407686\,6° *$

BUT $\widehat{D_3D_2} = \widehat{D_2D_1} = \widehat{D_1D_3}$

$\therefore$ REFER TO $\widehat{D_3D_2}$ AS $\widehat{DD}$.

$2 \sin \frac{\widehat{DD}}{2} = .3249196788$

$\therefore$ CF $_{DD}$ $= .3249196788$

$\cos \widehat{C_2E} = \cos \widehat{D_3C_2} \div \cos \widehat{D_3E}$

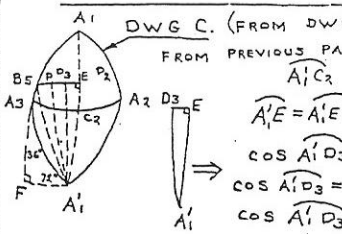
$\cos \widehat{C_2E} = (.9510565118) \div (.9867151546)$

$\cos \widehat{C_2E} = .9638612597$

$\widehat{C_2E} = 15.450433\,0478°$

* DENOTES CENTRAL ANGLE

4 FREQUENCY DOME — CONTINUED ...



DWG C. (FROM DWG. B)

FROM PREVIOUS PAGE $\overset{\frown}{C_2E} = 15.4504390478°$

$\overset{\frown}{A'_1C_2} = \overset{\frown}{A_1C_2} = 58.2825267415°$

$\overset{\frown}{A'_1E} = \overset{\frown}{A'_1EC_2} = 73.7329657893°$

$\cos \overset{\frown}{A'_1D_3} = \cos \overset{\frown}{A'_1E} \cdot \cos \overset{\frown}{D_3E}$

$\Rightarrow \cos \overset{\frown}{A'_1D_3} = (.2801144289)(.9867151546)$

$\cos \overset{\frown}{A'_1D_3} = .2763931520$

$\overset{\frown}{A'_1D_3} = 73.9549467026°$

$\sin \angle D_3A'_1E = \sin \overset{\frown}{D_3E} \div \sin \overset{\frown}{D_3A'_1}$

$\sin \angle D_3A'_1E = (.1624598391) \div (.9610446568)$

$\sin \angle D_3A'_1E = .1690450472$

$\angle D_3A'_1E = 9.7323021117°$

NOW ARC $\overset{\frown}{A'_1D_3}$ IS EQUAL TO THE ARC THAT GOES FROM "VERTICAL" (POINT A_1) TO THE "$\frac{3}{8}$" CUTOFF PLANE. SO FOR A "PERFECT CUTOFF PLANE (NO EXTRA BASE PANELS), LET B_5 (& PT. B_2) BE IN THE SAME PLANE AS D_3 & D_2 BY SETTING ARC $\overset{\frown}{A'_1B_5}$ = ARC $\overset{\frown}{A'_1D_3}$. IT MIGHT HELP TO MOMENTARILY THINK OF POINT A'_1 AS THE TOP OF THE DOME AND ARC $\overset{\frown}{B_5D_3D_2B_2}$ AS PART OF THE "$\frac{3}{8}$" CUTOFF PLANE. IN ANY CASE,

$\overset{\frown}{A'_1D_3} = \overset{\frown}{A'_1B_5} = 73.9549467026°$

$\overset{\frown}{A'_1A_3} = 63.1349485246°$

$\overset{\frown}{FA_3} = 58.2825267415°$

$\overset{\frown}{A'_1F} = 31.7174742623°$

4 FREQUENCY DOME — CONTINUED...

USING DWG. B & DWG. C, CONSIDER RIGHT TRIANGLE $B_5FA'_1$:

$\cos \overset{\frown}{B_5F} = \cos \overset{\frown}{B_5A'_1} \div \cos \overset{\frown}{FA'_1}$

$\cos \overset{\frown}{B_5F} = (.2763931526) \div (.8506508093)$

$\cos \overset{\frown}{B_5F} = .3249196367$

$\overset{\frown}{B_5F} = 71.0392954727°$

BUT $-\overset{\frown}{FA_3} = -58.2825267415°$

$\therefore \overset{\frown}{A_3B_5} = 12.7567687312°$ *

BUT $\overset{\frown}{A_3B_5} = \overset{\frown}{A_1B_6} = \overset{\frown}{A_1B_1} = \overset{\frown}{A_2B_2} = \overset{\frown}{A_2B_3} = \overset{\frown}{A_3B_4}$

$\therefore$ REFER TO $\overset{\frown}{A_3B_5}$ AS $\overset{\frown}{AB}$.

$2 \sin \frac{\overset{\frown}{AB}}{2} = .2221880226$

$\therefore CF_{AB} = .2221880226$

NOW $\overset{\frown}{A_3C_3} = 31.7174742623°$

AND $-\overset{\frown}{A_3B_5} = -12.7567687312°$

$\overset{\frown}{B_5C_3} = 18.9607055311°$ *

BUT $\overset{\frown}{B_5C_3} = \overset{\frown}{B_6C_3} = \overset{\frown}{B_1C_1} = \overset{\frown}{B_2C_1} = \overset{\frown}{B_3C_2} = \overset{\frown}{B_4C_2}$

$\therefore$ REFER TO $\overset{\frown}{B_5C_3}$ AS $\overset{\frown}{BC}$.

$2 \sin \frac{\overset{\frown}{BC}}{2} = .3294187798$

$\therefore CF_{BC} = .3294187798$

ALSO $\sin \angle FA'B_5 = \sin \overset{\frown}{B_5F} \div \sin \overset{\frown}{B_5A'_1}$

$\sin \angle FA'B_5 = (.9457416388) \div (.9610446568)$

$\sin \angle FA'B_5 = .9840766838$

$\angle FA'B_5 = 79.7615938261°$

* DENOTES CENTRAL ANGLE

4 FREQUENCY DOME — CONTINUED

NOW $\angle PA_1' B_5 = 79.76159938264°$ AND $-\angle FA_1' A_3 = -72.00\ldots°$

$\therefore \angle A_3A_1'B_5 = 7.76159938264°$

$\angle D_3A_1'E = 9.732302117°$

$\angle A_3A_1'B_5 + \angle D_3A_1'E = 17.493895938 1°$

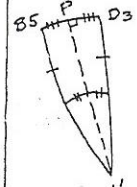
$\angle A_3A_1'E = 36.00\ldots°$

$\angle A_3A_1'B_5 + \angle D_3A_1'E = 17.493895938 1°$

$\angle B_5A_1'D_3 = 18.506104061 9°$

$\frac{\angle B_5A_1'D_3}{2} = 9.253052030 9°$

LET POINT P BE LOCATED ON GREAT CIRCLE LINE $\overset{\frown}{B_5D_3}$ SUCH THAT $\angle B_5A_1'P = \angle PA_1'D_3 = \frac{\angle B_5A_1'D_3}{2} = 9.253052030 9°$.



BUT $\overset{\frown}{A_1'D_3} = 73.954946702 6°$

$\therefore \sin \overset{\frown}{PD_3} = \sin \angle PA_1'D_3 \sin \overset{\frown}{A_1'P_3}$

$\sin \overset{\frown}{PD_3} = (.160795140 6)(.961044656 8)$

$\sin \overset{\frown}{PD_3} = .154531310 7$

$\overset{\frown}{PD_3} = 8.889616491 3°$

$\therefore \overset{\frown}{B_5D_3} = 2\overset{\frown}{PD_3} = 17.779232982 6°$ *

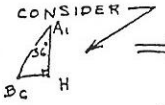
BUT $\overset{\frown}{B_5D_3} = \overset{\frown}{B_4D_3} = \overset{\frown}{B_6D_1} = \overset{\frown}{B_1P_1} = \overset{\frown}{B_2D_2} = \overset{\frown}{B_3D_2}$

$\therefore$ REFER TO $\overset{\frown}{B_5D_3}$ AS $\overset{\frown}{BD}$

$2 \sin \frac{\overset{\frown}{BD}}{2} = .309062621 4$

$\therefore CF_{BD} = .309062621 4$

CONSIDER



$\sin \overset{\frown}{B_6H} = \sin 36° \sin \overset{\frown}{A_1B_6}$

$\sin \overset{\frown}{B_6H} = (.587785251 8)(.220812657 0)$

$\sin \overset{\frown}{B_6H} = .129790423 1$

$\overset{\frown}{B_6H} = 7.457484063 3°$

$\overset{\frown}{B_6B_1} = 2\overset{\frown}{B_6H} = 14.914968126 6°$ *

BUT $\overset{\frown}{B_6B_1} = \overset{\frown}{B_2B_3} = \overset{\frown}{B_4B_5}$. $\therefore$ REFER TO $\overset{\frown}{B_6B_1}$ AS $\overset{\frown}{BB}$.

$2 \sin \frac{\overset{\frown}{BB}}{2} = .258580846 2$

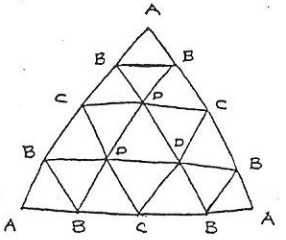
$\therefore CF_{BB} = .259580846 2$

* DENOTES CENTRAL ANGLE

4 FREQUENCY DOME

SUMMARY AND COMPARISON

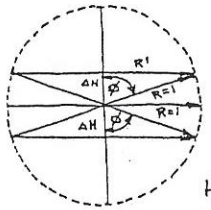
CHORD FACTOR	SUMMARY	COMPARISON WITH R. BUCKMINSTER FULLER'S BOSTON BLUE PRINT DATA ARCHIVE, ITEM "M".
CB	.3294187798	.3294
DD	.3249196788	.3249
CD	.3128689446	.3129
BD	.3090626214	.3091
BB	.2595808462	.2596
AB	.2221880226	.2215



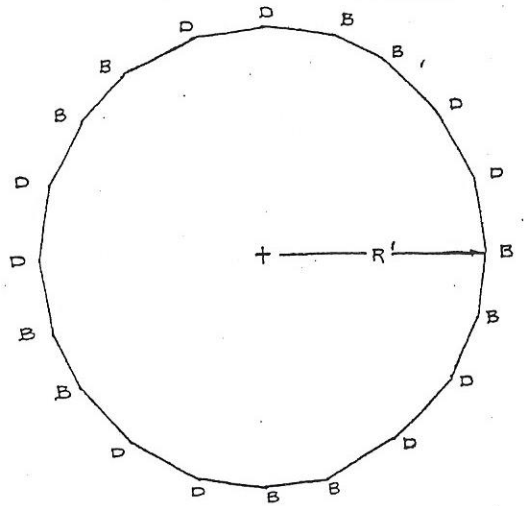
ICOSA-TRIANGLE

ADDITIONAL 4 FREQUENCY DOME DATA

SPHERICAL (MAX.) RADIUS = R = 1.00...
RADIUS OF "SMALL" CIRCLE (BASE) = R'
HEIGHT OF HEMISPHERE DOME = R = 1.00...
HEIGHT OF "$\frac{3}{8}$" DOME = H_1
HEIGHT OF "$\frac{5}{8}$" DOME = H_2



ϕ = 73.954946702 6°
R' = .961044659
ΔH = .276393134
H_1 = R − ΔH = .72360687
H_2 = R + ΔH = 1.27639313



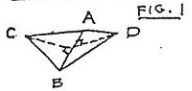
FLOOR PLAN (SEE DWG. B)

DIHEDRAL ANGLES FOR THE FACES OF 1, 2, 3, & 4 FREQUENCY DOMES

CONSIDER THE ONE FREQUENCY DOME (ICOSAHEDRON): 

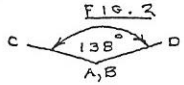
HERE, EACH FACE MAKES APPROXIMATELY A 138° ANGLE WITH ANY OTHER FACE.

FIG. 1



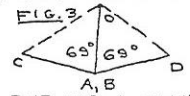
OBLIQUE VIEW

FIG. 2



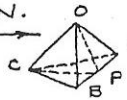
END VIEW

FIG. 3



END VIEW WITH PLANE PASSING THROUGH LINE AB AND CENTER O OF ICOSAHEDRON.
NOTE: OA=OB=OC=OD=1

∴ IN FIGURE 3, THE PLANE DETERMINED BY POINTS O, A, B, & C CUTS THE 138° ANGLE IN HALF. FOR THIS, HOWEVER, IS NOT TRUE FOR ANY PAIR OF TRIANGLES ON A SPHERE THAT HAPPEN TO SHARE AN EDGE. AS A RESULT, WE MUST CONCERN OURSELVES WITH THE ANGLES A FACE MAKES WITH PLANES THAT PASS THROUGH EACH EDGE AND THE CENTER OF THE DOME. THIS MEANS THAT WE ARE CONCERNED WITH PYRAMIDS. FOR EXAMPLE, THE ICOSAHEDRON.



WHERE OA=OB=OC=1
$\overline{AB}$ = BC = CA ≐ 1.05
$\angle$ OPC ≐ 69°

THIS IS AN EXAMPLE OF ONE OF THE DIHEDRAL ANGLES THAT WE ARE LOOKING FOR.

DIHEDRAL ANGLES : GENERAL CASE

ROUGHLY SPEAKING, WE UNFOLD THE PYRAMIDS FORMED BY THE DOME CENTER AND FACES AND THEN AFTER NOTING CERTAIN DIMENSIONS, FOLD THE PYRAMID BACK UP AGAIN AND THEN CALCULATING THE DIHEDRAL ANGLES BY "LOOKING DOWN" EACH EDGE OF EACH TRIANGLE. AN EXACT UNDERSTANDING WILL REQUIRE CAREFUL STUDY OF THE FOLLOWING DIAGRAMS AND FORMULAS (AND A MODEL ??):

FIG. 4

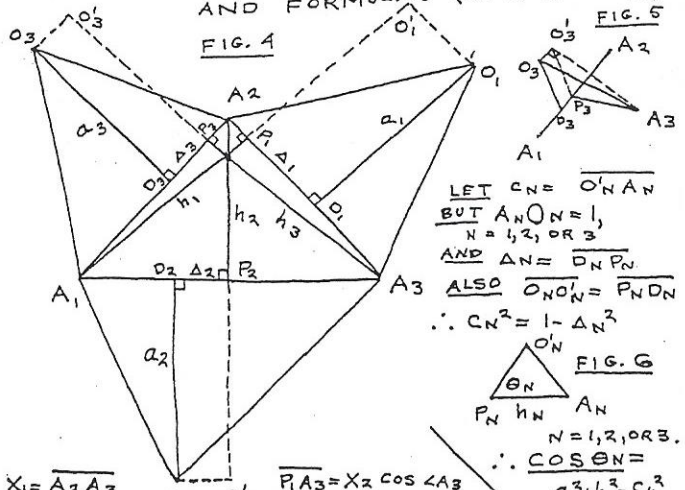


FIG. 5

LET $C_N = \overline{O'_N A_N}$

BUT $A_N O_N = 1$, $N = 1, 2, \text{OR } 3$

AND $\Delta_N = \overline{D_N P_N}$

ALSO $\overline{O_N O'_N} = \overline{P_N D_N}$

$\therefore C_N^2 = 1 - \Delta_N^2$

FIG. 6

$N = 1, 2, \text{OR } 3.$

$\therefore \cos\theta_N = \dfrac{a_N^2 + h_N^2 - C_N^2}{2 a_N h_N}$

OR

$\cos\theta_N = \dfrac{a_N^2 + h_N^2 + \Delta_N^2 - 1}{2 a_N h_N}$

$X_1 = \overline{A_2 A_3}$

$X_2 = \overline{A_1 A_3}$

$X_3 = \overline{A_1 A_2}$

$\overline{XY}$ = DISTANCE BETWEEN PT. X & PT. Y.

$\overline{P_1 A_3} = X_2 \cos \angle A_3$

$\overline{P_2 A_3} = X_1 \cos \angle A_3$

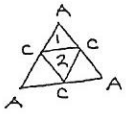
$\overline{P_3 A_2} = X_1 \cos \angle A_2$

UNIQUE PANELS AND THEIR ASSOCIATED CHORD FACTORS FOR THE 1, 2, 3, & 4 FREQ. DOMES.

ICOSA

$\overline{AA} = 1.051462218 6$

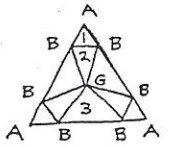
2 FREQ



$\overline{AC} = .5465330548$

$\overline{CC} = .6180339848$

3 FREQ.



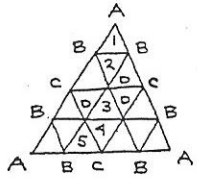
$\overline{AB} = .3297064628$

$\overline{BB_C} = .3822901844$

$\overline{BG} = .4214887744$

$\overline{BB_A} = .4410563540$

4 FREQ.



$\overline{AB} = .2221880226$

$\overline{BB} = .2595808462$

$\overline{BD} = .3090626214$

$\overline{CD} = .3128689446$

$\overline{DD} = .3249196788$

$\overline{BC} = .3294187798$

ICOSA

$\overline{A_2A_3} = X_1 = 1.0514622186$

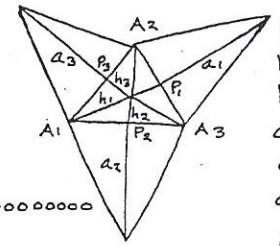
$\overline{A_1A_3} = X_2 =$ "

$\overline{A_1A_2} = X_3 =$ "

$\frac{X_1}{2} = .5257311093$

$\frac{X_2}{2} =$ "

$\frac{X_3}{2} =$ "



$h_1 = .9105929924$

$h_2 =$ "

$h_3 =$ "

$a_1 = .8506508101$

$a_2 =$ "

$a_3 =$ "

$\Delta_1 = .000...$

$\Delta_2 =$ "

$\Delta_3 =$ "

$\cos \angle 1 = .5000000000$

$\cos \angle 2 =$ "

$\cos \angle 3 =$ "

$\angle 1 = 60.00...^\circ$

$\angle 2 =$ "

$\angle 3 =$ "

$\cos \theta_1 = .3568220869$

$\cos \theta_2 =$ "

$\cos \theta_3 =$ "

$\overline{P_1A_3} = .5257311093$

$\overline{P_2A_3} =$ "

$\overline{P_3A_2} =$ "

$\theta_1 = 69.09484379\,82^\circ$

$\theta_2 =$ "

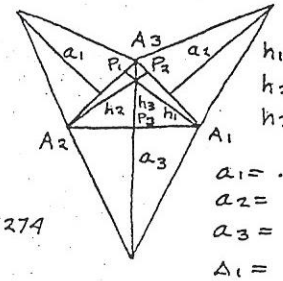
$\theta_3 =$ "

2 FREQ.

$\overline{A_2A_3} = X_1 = .5465330548$

$\overline{A_1A_3} = X_2 =$ "

$\overline{A_1A_2} = X_3 = .6180339848$



$h_1 = .5097592622$

$h_2 =$ "

$h_3 = .4507847362$

$a_1 = .9619383582$

$a_2 =$ "

$a_3 = .9510565169$

$\Delta_1 = .0761780334$

$\Delta_2 =$ "

$\Delta_3 = .000...$

$\frac{X_1}{2} = .2732665274$

$\frac{X_2}{2} =$ "

$\frac{X_3}{2} = .3090169924$

$\cos \angle 1 = .5654131796$

$\cos \angle 2 =$ "

$\cos \angle 3 = .3606158719$

$\angle 1 = 55.5690107900^\circ$

$\angle 2 =$ "

$\angle 3 = 68.8619781722^\circ$

$\cos \theta_1 = .1947387387$

$\cos \theta_2 =$ "

$\cos \theta_3 = .1256238733$

$\overline{P_1A_3} = .1970884940$

$\overline{P_2A_3} =$ "

$\overline{P_3A_2} = .3090169924$

$\theta_1 = 78.7705404132^\circ$

$\theta_2 =$ "

$\theta_3 = 82.7832160269^\circ$

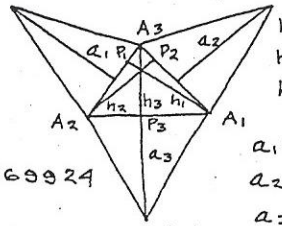
2 FREQ

2

$\overline{A_2A_3} = X_1 = .6180339848$

$\overline{A_1A_3} = X_2 = \,''$

$\overline{A_1A_2} = X_3 = \,''$



$\frac{X_1}{2} = .3090169924$

$\frac{X_2}{2} = \,''$

$\frac{X_3}{2} = \,''$

$\cos\angle 1 = .5000000000$

$\cos\angle 2 = \,''$

$\cos\angle 2 = \,''$

$\angle 1 = 60.0000000000^\circ$

$\angle 2 = \,''$

$\angle 3 = \,''$

$\overline{P_1A_3} = .3090169924$

$\overline{P_2A_3} = \,''$

$\overline{P_3A_2} = \,''$

$h_1 = .5352331312$

$h_2 = \,''$

$h_3 = \,''$

$a_1 = .9510565169$

$a_2 = \,''$

$a_3 = \,''$

$\Delta_1 = .000\ldots$

$\Delta_2 = \,''$

$\Delta_3 = \,''$

$\cos\theta_1 = .1875924724$

$\cos\theta_2 = \,''$

$\cos\theta_3 = \,''$

$\theta_1 = 79.1876841121^\circ$

$\theta_2 = \,''$

$\theta_3 = \,''$

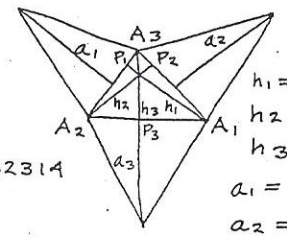
3 FREQ.

1

$\overline{A_2A_3} = X_1 = .3297064628$

$\overline{A_1A_3} = X_2 = \,''$

$\overline{A_1A_2} = X_3 = .3822901844$



$\frac{X_1}{2} = .1648532314$

$\frac{X_2}{2} = \,''$

$\frac{X_3}{2} = .1911450922$

$\cos\angle 1 = .5797432371$

$\cos\angle 2 = \,''$

$\cos\angle 3 = .3277955570$

$\angle 1 = 54.5675136590^\circ$

$\angle 2 = \,''$

$\angle 3 = 70.8649716474^\circ$

$\overline{P_1A_3} = .1080763136$

$\overline{P_2A_3} = \,''$

$\overline{P_3A_2} = .1911450922$

$h_1 = .3114897463$

$h_2 = \,''$

$h_3 = .2686445707$

$a_1 = .9863181089$

$a_2 = \,''$

$a_3 = .9815617931$

$\Delta_1 = .0567769178$

$\Delta_2 = \,''$

$\Delta_3 = .000\ldots$

$\cos\theta_1 = .1189229130$

$\cos\theta_2 = \,''$

$\cos\theta_3 = .0675666318$

$\theta_1 = 83.1700558513^\circ$

$\theta_2 = \,''$

$\theta_3 = 86.1257657222^\circ$

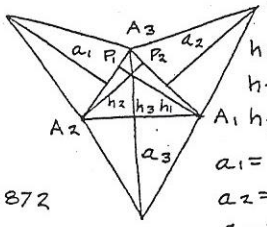
3 FREQ.

2

$\overline{A_2A_3} = X_1 = .4214887744$

$\overline{A_1A_3} = X_2 = $ "

$\overline{A_1A_2} = X_3 = .3822901844$



$h_1 = .3407185571$

$h_2 = $ "

$A_1h_3 = .3756545497$

$a_1 = .9775412028$

$a_2 = $ "

$a_3 = .9815617931$

$\Delta_1 = .0373758492$

$\Delta_2 = $ ",

$\Delta_3 = .000\ldots$

$\frac{X_1}{2} = .2107443872$

$\frac{X_2}{2} = $ "

$\frac{X_3}{2} = .1911450922$

$\cos\angle 1 = .4534998407$

$\cos\angle 2 = $ "

$\cos\angle 3 = .5886757882$

$\angle 1 = 63.0315470000°$

$\angle 2 = $ "

$\angle 3 = 53.93690470216°$

$\overline{P_1A_3} = .2481202369$

$\overline{P_2A_3} = $ "

$\overline{P_3A_2} = .1911450922$

$\cos\theta_1 = .1096971744$

$\cos\theta_2 = $ "

$\cos\theta_3 = .1418116478$

$\theta_1 = 83.7021403811°$

$\theta_2 = $ "

$\theta_3 = 81.8473082873°$

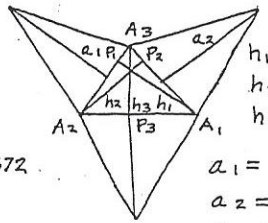
3 FREQ.

3

$\overline{A_2A_3} = X_1 = .4214887744$

$\overline{A_1A_3} = X_2 = $ "

$\overline{A_1A_2} = X_3 = .4410563540$



$h_1 = .3758692304$

$h_2 = $ "

$h_3 = .3591936944$

$a_1 = .9775412028$

$a_2 = $ "

$a_3 = .9753806042$

$\Delta_1 = .0200217914$

$\Delta_2 = $ "

$\Delta_3 = .000\ldots$

$\frac{X_1}{2} = .2107443872$

$\frac{X_2}{2} = $ "

$\frac{X_3}{2} = .2205281770$

$\cos\angle 1 = .5232124563$

$\cos\angle 2 = $ "

$\cos\angle 3 = .4524974506$

$\angle 1 = 58.4520155000°$

$\angle 2 = $ "

$\angle 3 = 63.0959685143°$

$\overline{P_1A_3} = .1907225958$

$\overline{P_2A_3} = $ "

$\overline{P_3A_2} = .2205281770$

$\cos\theta_1 = .1323598684$

$\cos\theta_2 = $ "

$\cos\theta_3 = .1147242811$

$\theta_1 = 82.3940192833°$

$\theta_2 = $ "

$\theta_3 = 83.4122778320°$

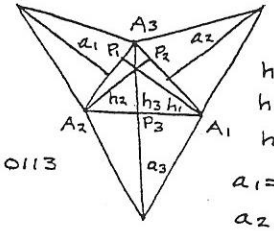
4 FREQ.

△ 1

$\overline{A_2A_3} = X_1 = .2221880226$

$\overline{A_1A_3} = X_2 = \,''$

$\overline{A_1A_2} = X_3 = .2595808462$



$h_1 = .2106882806$

$h_2 = \,''$

$h_3 = .1803384692$

$\frac{X_1}{2} = .1110940113$

$a_1 = .9938099016$

$\frac{X_2}{2} = \,''$

$a_2 = \,''$

$\frac{X_3}{2} = .1297904231$

$a_3 = .9915414495$

$\cos \angle 1 = .5841468028$

$\Delta_1 = .0405393106$

$\cos \angle 2 = \,''$

$\Delta_2 = \,''$

$\cos \angle 3 = .3175450230$

$\Delta_3 = .000\ldots$

$\angle 1 = 54.2572610000^\circ$

$\cos \theta_1 = .0804528797$

$\angle 2 = \,''$

$\cos \theta_2 = \,''$

$\angle 3 = 71.4854770386^\circ$

$\cos \theta_3 = .0438346301$

$\overline{P_1A_3} = .0705547007$

$\theta_1 = 85.3854028807^\circ$

$\overline{P_2A_3} = \,''$

$\theta_2 = \,''$

$\overline{P_3A_2} = .1297904231$

$\theta_3 = 87.4876557449^\circ$

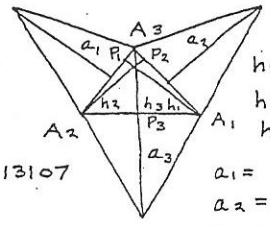
4 FREQ.

▽ 2

$\overline{A_2A_3} = X_1 = .3090626214$

$\overline{A_1A_3} = X_2 = \,''$

$\overline{A_1A_2} = X_3 = .2595808462$



$h_1 = .2355820463$

$h_2 = \,''$

$h_3 = .2804891264$

$\frac{X_1}{2} = .1545313107$

$a_1 = .9879878916$

$\frac{X_2}{2} = \,''$

$a_2 = \,''$

$\frac{X_3}{2} = .1297904231$

$a_3 = .9915414495$

$\cos \angle 1 = .4199486254$

$\Delta_1 = .0455206909$

$\cos \angle 2 = \,''$

$\Delta_2 = \,''$

$\cos \angle 3 = .6472863031$

$\Delta_3 = .000\ldots$

$\angle 1 = 65.1686560000^\circ$

$\cos \theta_1 = .0723754839$

$\angle 2 = \,''$

$\cos \theta_2 = \,''$

$\angle 3 = 49.6626872480^\circ$

$\cos \theta_3 = .1111558911$

$\overline{P_1A_3} = .2000520016$

$\theta_1 = 85.8495613864^\circ$

$\overline{P_2A_3} = \,''$

$\theta_2 = \,''$

$\overline{P_3A_2} = .1297904231$

$\theta_3 = 83.6180483304^\circ$

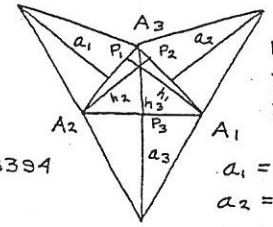
4 FREQ.

△3

$\overline{A_2A_3} = X_1 = .3249196788$

$\overline{A_1A_3} = X_2 = $ "

$\overline{A_1A_2} = X_3 = $ "



$h_1 = .2813886961$

$h_2 = $ "

$h_3 = $ "

$\frac{X_1}{2} = .1624598394$

$\frac{X_2}{2} = $ "

$\frac{X_3}{2} = $ "

$a_1 = .9867151567$

$a_2 = $ "

$a_3 = $ "

$\Delta_1 = .000\ldots$

$\Delta_2 = $ "

$\Delta_3 = $ "

$\cos\angle 1 = .5000000000$

$\cos\angle 2 = $ "

$\cos\angle 3 = $ "

$\cos\theta_1 = .0950590761$

$\cos\theta_2 = $ "

$\cos\theta_3 = $ "

$\angle 1 = 60.0000000000^\circ$

$\angle 2 = $ "

$\angle 3 = $ "

$\theta_1 = 84.5452808475^\circ$

$\theta_2 = $ "

$\theta_3 = $ "

$\overline{P_1A_3} = .1624598394$

$\overline{P_2A_3} = $ "

$\overline{P_3A_2} = $ "

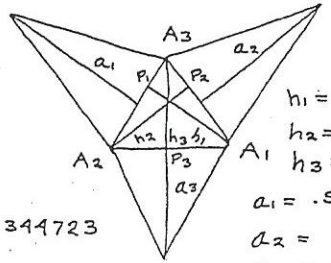
4 FREQ

△4

$\overline{A_2A_3} = X_1 = .3128689446$

$\overline{A_1A_3} = X_2 = $ "

$\overline{A_1A_2} = X_3 = .3249196788$



$h_1 = .2776819683$

$h_2 = $ "

$h_3 = .2673832027$

$\frac{X_1}{2} = .1564344723$

$\frac{X_2}{2} = $ "

$\frac{X_3}{2} = .1624598394$

$a_1 = .9876883394$

$a_2 = $ "

$a_3 = .9867151567$

$\Delta_1 = .0122828128$

$\Delta_2 = $ "

$\Delta_3 = .000\ldots$

$\cos\angle 1 = .5192584374$

$\cos\angle 2 = $ "

$\cos\angle 3 = .4607413488$

$\cos\theta_1 = .0962330898$

$\cos\theta_2 = $ "

$\cos\theta_3 = .0854724562$

$\angle 1 = 58.7174780000^\circ$

$\angle 2 = $ "

$\angle 3 = 62.5650438344^\circ$

$\theta_1 = 84.4777040985^\circ$

$\theta_2 = $ "

$\theta_3 = 85.0968081725^\circ$

$\overline{P_1A_3} = .1441516595$

$\overline{P_2A_3} = $ "

$\overline{P_3A_3} = .1624598394$

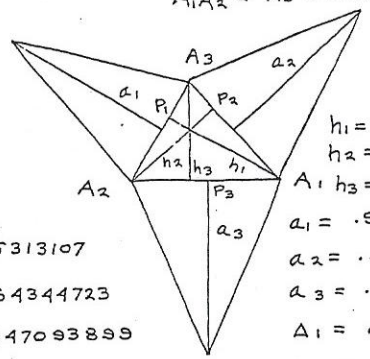
4 FREQ.

5

$\overline{A_2A_3} = X_1 = .3090626214$

$\overline{A_1A_3} = X_2 = .3128689446$

$\overline{A_1A_2} = X_3 = .3294187798$



$h_1 = .2811159897$

$h_2 = .2776959689$

$A_1 h_3 = .2637446619$

$a_1 = .9879878916$

$a_2 = .9876883394$

$a_3 = .9863181089$

$\frac{X_1}{2} = .1545313107$

$\frac{X_2}{2} = .1564344723$

$\frac{X_3}{2} = .1647093899$

$\Delta_1 = .0171967675$

$\Delta_2 = .0207707234$

$\Delta_3 = .0035931052$

$\cos \angle 1 = .5379329604$

$\cos \angle 2 = .5213062778$

$\cos \angle 3 = .4389523013$

$\cos \theta_1 = .0998094673$

$\cos \theta_2 = .0967538582$

$\cos \theta_3 = .0814911169$

$\angle 1 = 57.456963015 6°$

$\angle 2 = 58.580086909 6°$

$\angle 3 = 63.962950074 8°$

$\theta_1 = 84.271802690 7°$

$\theta_2 = 84.447726947 5°$

$\theta_3 = 85.325719381 4°$

$\overline{P_1A_3} = .1373345432$

$\overline{P_2A_3} = .1356637489$

$\overline{P_3A_3} = .1611162847$

NOTE

SINCE THIS IS A "COOK BOOK" AND NOT A MATH BOOK, PROOFS OF FORMULAS ALONG WITH EXTRA FORMULAS & DIAGRAMS HAVE NOT BEEN INCLUDED HERE. ANYONE DESIRING MORE THAN A COOKBOOK APPROACH SHOULD SEEK OUT A "PLANE & SPHERICAL TRIG." BOOK.

THE RESOURCE BOOK USED BY THIS WRITER WAS:

PLANE AND SPHERICAL TRIGONOMETRY

BY

KELLS, KERN, & BLAND.

SECOND EDITION

McGRAW-HILL BOOK CO., INC.

1940

THIS BOOK APPEARED RATHER COMPLETE AND HAS EXCELLENT DIAGRAMS. IT ALSO CONTAINS APPLIED TECHNIQUES OF GLOBAL NAVIGATION & ASTRONOMY.