The Montessori Contingencies*

A Radical Behaviorist's Perspective

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May 18, 2015

*This paper draws heavily from the paper: "Selectionist Model of Reinforcement" by John W. Donahoe, et al, *Journal of The Experimental Analysis of Behavior*, 1993, 60, pp 17-40.

Contents

1. General Introduction

Children are naturally interested in learning from the time they are born. They continually take action and see the results. However the exact scientific nature of that learning and the best way for a group or society to teach its children has always been a controversial subject.

Marie Montessori as a Scientist

Marie Montessori, nearly a century ago, carried out some very important experimental work that is still applied successfully today in Montessori schools worldwide. Even though there is a network of Montessori schools worldwide and the method has been used successfully for more than a century, the underlying scientific reasons why it is successful, how to measure its success and how it works are not well known.

The reasons for this are many. The main one is that the methods of research most directly comparable to Marie Montessori's original research are largely found today in only one place: behavior analysis. Marie Montessori in her experimental work used protean forms of many of the key ideas and tools of behavior analysis including:

- *Single subject experimentation*: Montessori looked at how changing the behavioral environment affected the behavior of individual children. She did not do statistical comparisons.
- *Multiple exemplar instruction*: The three part lesson[1] is a early formulation of

[1] Lets say we have three rectangular cards that whose colors are "red," "green," and "blue." In Montessori's works corrections are not always given. We have not detailed out how the teacher gives the corrections in the following table for simplicity.

Three part lesson	S_D = {teachers behavior + three cards laid out in front of the child on a rug}	R = {child's behavior}
First Part	Cards are laid out on the rug in front of the child	
	Teacher points at the red card and says: "this color is red, can you say red"	child echos "red"
	Teacher points at the green card and says: "this color is green, can you say green"	child echos "green"
	Teacher points at the green card and says: "this color is blue, can you say blue"	child echos "blue"
Second Part	Cards are moved around and relaid out on the rug	
	Teacher says: "can you point to green"	child points to the green card

one of the mainstays used by behavior analysis in teaching situations or in direct instruction, namely, multiple exemplar instruction.

- *Free operant environments*: The children in a Montessori classroom live in a "free operant environment" because they have multiple behavior chains and response classes that can achieve the same reinforcing events. The classroom is one full of choices for the children.
- *Generalization:* The Montessori environment is very similar to a "real world" environment and hence any behavioral repertoire that is shaped and maintained in it has a very high probability of generalizing to outside of the classroom.
- *Using "things" to control behavior*: She recognized the important role that the "non-humans" could play in controlling behavior (thought, speech and action). She wanted her children to be taught by the works ("non-humans") and not by the teacher.
- *Controlling the teacher's behavioral repertoire*: Both the verbal behavior (internal and external) needed to be precisely controlled as a part of the learning environment.

Today we know a great deal more about behavior and that the fundamental unit of measurement is related to the frequency counts of the occurrence of response classes for an individual organism. Even so the overlap between Montessori and behavior analysis is very striking.

Artificial Walls between Montessori's work and Behavior Analysis

In spite of this fundamental commonality, there has been and continues to be, for many reasons, an artificial wall that separates the Montessori practitioners from those scientists and practitioners that do behavior analysis. Many books that explain Montessori's work usually contrast her work to those offered by "behaviorists" and usually do so to show the latter in a very bad light. The "behaviorists" on the other hand contrast the "constructionist" method of education to the what they claim are more effective ways

	Teacher says: "can you point to blue"	child points to the blue card
Third Part	Cards are again moved around and relaid out on the rug. Teacher gives general prompt: "I am going to point to a card and you are going to tell me the color"	
	Teacher points to red card	child says "red"
	Teacher points to blue card	child says "blue"
	Teacher points to green card	child says "green"

This example is incomplete because the teacher would have to do this with other shapes, and different shapes together, to make sure that the sole stimulus dimension that will control the tact "red" is the red color, etc. Meaning that the child will say "red" only with a red stimulus.

of teaching academic subjects. We believe both of these rhetorical arguments to be misplaced. The "engine under the hood" that drives all learning is set by the genetics of the human race and behavior analysis gives the best platform to understand it–once we define our terms carefully. On the other had, what we will call the "Montessori contingencies" has very many remarkable properties that, in our opinion, could in the future, make it the educational model of choice for almost all children. So we see the disjunction of these two–Montessori and behavior analysis–is artificial and probably harmful to further development of the Montessori methods.

An Unlikely Partnership

My interest in combining Montessori and behavior analysis was strongly shaped by my very intense experience in helping run a Jewish Montessori school for two years with the school founders. Since I am a radical behaviorist, this was a very unusual occurrence. They became my students of behavioral science and I became their student in the art of teaching children. They now Heads of School at the Lamplighters Yeshiva in Crown Heights whose Executive Director is Yocheved Sidof. I have various titles than imply I am a kind of "chief scientist" for Lamplighters.

Combining behavior analysis and Montessori begins with finding a way to compare Montessori methods of education to the other methods of education. Doing this requires a generic model of biological change that can be applied to learning. The Selectionist Model is such a general model and is the model of choice in behavior analysis. Two individuals are credited with "creating" this model: Darwin and BF Skinner. Even though we accept the Torah account of creation, Darwin's ideas do have their place in science and his theory constitutes a very useful metaphor.

Selectionist Model

We are going to apply the selectionist model mostly at the behavior level, since this is relevant to learning, but it also works at the species level and the group level. It has three dimensions: Variation, Selection and Amplification. The difference between Montessori methods and others can occur in any of these three dimensions.

The selectionist model when applied to classrooms as a whole is best studied by considering four integrated dimensions: a) The teacher's behavioral repertoire (what the teacher is able to do and does do); b) The curriculum items (what is taught, scope and sequence and also all the "non-human actors" in the used in the classroom); c) Instructional control and motivating operations (how the students are "disciplined and motivated"); d) the class culture.

A Montessori classroom differs from a normative classroom in all dimensions. For every child there is a optimal mix of these four dimensions. In a Montessori classroom a very fined grained differentiation is possible and each child can get largely exactly what will work best for them. In a normative classroom, teaching to many levels is usually a very difficult task and in some cases is largely impossible.

Benefits of Combining Behavior Analysis with Montessori

I believe that bringing extensive research base of behavior analysis to bear on the "Montessori contingencies" will allow many new Montessori works to be created, additional learning contingencies can be added thus further developing and extending Montessori's work. One of the reasons why this is needed is because many Montessori school are weak in handling remediation for students that are behind and those that did not start their school experience with Montessori. In both of these cases the four integrated dimensions mentioned above may have to be different, for a time, from the standard Montessori contingencies. I believe that further research can disclose how extend the Montessori contingencies to reach all children–including those that need direct instruction and token economies.

There are a number of other important areas that are not well studied scientifically or described in sufficient detail for implementation by Montessori's works and her successors. Namely the details on how to properly arrange space or Montessori works in a classroom. Also not well described are all the details needed to properly set up a class culture. To be sure there are many hints and guidelines in Maria Montessori's work about both of these but even her most ardent supporters admit that many of the practical details are missing. Here too developments outside of the Montessori corpus can be used to good effect within Montessori schools. In particular, I am thinking of the revolutionary works of Christopher Alexander. His work is relevant in two dimensions: a) the understanding of what brings wholeness or life to the physical arrangement of a Montessori classroom; b) the concept of a pattern language to coordinate the design and implementation of Montessori learning environment among a group of teachers, administrators and parents. We will discuss this in due course. A future draft will explore this in more depth.

Part I.

Selectionist Model

2. Generic Change Model for Biological Organisms

There are three levels on which change occurs:

- Species level (in populations of several species across several generations)
- Group level (in groups of organisms of the same species during the same generation)
- Behavior level (or more generally the phenotype a single organism)

These three levels are, of course interconnected. Behavior (anything an organism is doing) has to occur in an intact organism. Organisms, like humans, tend to belong to various groups and can belong to the same species.

Recall that genotype means essentially the genetic sequence of an organism's DNA (or RNA). Phenotype means the physical expression of the genotype. Generally speaking, different genotypes mean different phenotypes but not always. Some genotype differences are not expressed. For a given collection of organisms let us call the gene pool the collection of genotypes weighted according to the number of living organisms that have that genotype.

2.1. Species Level

At the *species level* we have natural selection. Meaning that that for a given species there is natural variability in almost all of its biological characteristics. Some of these variations will increase the probability that the organism will successfully reproduce and pass its genes on to the next generation. These genotypes will over one or more generations occupy a increasing large part of the gene pool. Eventually or even in one generation, some genotypes will disappear from the gene pool. This is extinction. If the genotype of an entire species disappears then that species becomes extinct at the phenotype level and the genotype level.

Darwin's basic idea is that all change in the species can be explained by natural selection over random variations (Darwin's "blind watchmaker" metaphor) in the genotype. For small changes in phenotype this is a proven experimental fact. This so called micro-evolution. For large scale changes, we regard this as largely a modern myth: that all forms of life can be explained by random variations experiencing natural selection over vast periods of time. This is called macro-evolution. There is no evidence that this ever took place (see: "Evolution, a Theory in Crises," by Denton).

The key to natural selection is the natural variability of organisms of a single species. Let us call this the *Variability* dimension. Second, there has to some sort of selection

process that determines what genotypes get passed on the next generation. Let us call this the *Selection* dimension. Third, exponential biological reproduction causes "successful" genotypes to increase very rapidly. Let us call this *Amplification* dimension.

Hence we have three dimensions:

- Variability
- Selection
- Amplification

We will call any process using these three dimensions a *Selectionist Model*. The exact structure and interconnection of these three dimensions will be called the *contingencies of selection*.

Evolution is speeded up by speeding up any one of these three dimensions but since they are connected each one can be a bottleneck for the others. We increase variability by inducing mutations in the genotype by direct genetic manipulation, by mutagens and radiation. We can increase the efficiency of the selection process and increase the potency of the amplification by putting the organisms in special environments.

It is our contention that probably all biological change can be understood as a combination of these three. Many ideas remain to be explored (e.g. differentiation during development) but we will simply confine ourselves to the three areas mentioned at the beginning of the paper.

2.2. Group Level

The *contingencies of selection* is very common in our society at the group level. It is, in fact, the most common way to used change our society. It only works if there is a way of creating variation in the members of the society so that a selection process can select a sufficiently number of people with the desired characteristics. Lets give some common examples of the contingencies of selection at the group level.

Our first example are the universities. Most universities take an incoming population of students and expose them to classroom experiences in-mass. This takes the natural variation present in the group and adds some addition variability due exposure to lectures, homework, writing papers and exams. There is also some of the contingencies of selection at the behavior level as discussed in the next section due to explicit teacher intervention. The selection process is done by giving each student a grade at the end of each course. Those students that "survive" eventually graduate. There some amplification at the group level in the form of imitation and one student teaching another. Students can form organizations that share experiences and exchange notes on various things that worked. That is to say, once one student learns how to "game the system" the rest of the students tend to copy the first successful student to do it.

Any professional licensing program constitutes contingencies of selection at the group level. Of course if there are no qualified candidates then no one will pass the license exam. Hence there are contingencies of selection that operate in training programs and universities in order to graduate enough candidates.

2.3. Behavior Level (or more generally the phenotype level)

This level is more subtle because the variation, selection and amplification occur in a single organism. This was first studied by BF Skinner and what we are calling the contingencies of selection he called the *contingencies of reinforcement*. Recall that by behavior we mean anything the organism is doing. The behavior repertoire of an organism is any behavior that the organism is capable of doing in a particular behavioral enforcement.

Now suppose that element X is a specific behavior in the organism's behavioral repertoire. Of course X does not occur the exact same way each time, but there is always a slight variation, say X1, X2, etc. However all of these are similar to X, and, after the organism performs any of one of them, the event that follows constitutes a similar consequence. Each of these slight variations has a chance of occurring.

A good example is unlocking a door. Lets suppose there is something behind the locked door we desperately want. Unlocking the lock will allow us to open the door–a positive consequence for us. We never do put the key in the lock and turn it exactly the same way (stand in exactly the same place, breath in or out, talking on our cell phone, etc) however these variations make no real difference. We are now able to open the door and hence the positive event always occurs. In other words, we are able to do X–even though there is a slight variation. This is *variability dimension*.

Now if someone changes the lock, or the lock is worn out then then X1, X2, X3 may stop working. We will probably stop doing X1, X2, X3 after trying it many times. . Lets suppose that X4, X5 do work ("jiggle the key just the right way"). We will start doing X4, X5, since these are only ways we know how to unlock the door. This is the *selection dimension*. X1, X2 and X3 are now "extinct species." The surviving species is X4, X5, etc. If we get exceptionally good at doing it using only X4 then this may end up being the only way we do it. This is *amplification dimension*. It is this case it is also called *fluency*. In the language of behavior analysis X is called the *response class*, the event that happens when the behavior X has a successful consequence is called a *positive reinforcement*.

There is considerable additional complexity due to the context where X occurs. For example, if there is a red light that happens to be on when X1, X2, X3 work and is off when they don't then we may find that X1, X2, X3 become extinct when the red light is off and they will survive when the red light is on. Additionally, in the human species, we have the capability of linking all sorts of things together in relational frames and equivalence classes. This is what distinguishes the human species from others. We will not go into these extra complexities right now.

Here is a summary:

Variation: Every behavior never occurs exactly the same way even though it may have very similar consequences. Most of the time this variation will be irrelevant until something in the behavioral environment changes (e.g the lock starts wearing out).

Selection: Those behaviors that do not achieve "success" will go extinct and the ones that "succeed" will survive. Success means a "positive reinforcing event." This is

individual to the organism and can only be determined experimentally.

Amplification: Those behaviors that either that achieve "fluency" will tend to occur most frequently or those that experience success on a intermittent basis will tend to occur most frequently.

So far we have only been talking about positive reinforcement. There is also negative reinforcement and the phenomena of avoidance conditioning, but we will not discuss that here even though it is very common. Even thought the Selectionist Model is at the heart of things, some alternate terminology is usually used because, for technical reasons, it is better suited to experimentation and analysis.

2.3.1. Actors and Events

One alternative way of describing things is to conceptualize the behavioral environment as made up of ACTORS and to conceptualize all change that occurs in that behavioral environment as consisting of EVENTS. Lets refer back to our lock example. The primary actors would be the person, the lock, the door, the key. Of course there are many other actors but they are not as relevant to our discussion. The primary events could be: 1) approaching the door; 2) getting out the key; 3) putting the key in the lock; 4) turning the lock; 5) unlocking the door; 6) the door is unlocked; 7) opening the door; and, 8) entering into the next room. We would then classify the events as follows:

Antecedents We could group here many events such as #1-3. These are the events before the behavior X ("unlocking the door").

Behavior We could group here events #4-5 and call this the behavior of "unlocking the door" response class X.

Consequence We could group here the events #6-8. Or, perhaps, simply #6 as this the primary positive consequence of the behavior X.

The grouping of events is subject to judgment by the experimenter or scientist and there can be differences of opinion as to the precise grouping. However grouping them this way is immensely useful and generally well trained observers can generally come to agreement as to how the events should be labeled. Particularly useful is the determination of those events that are the consequences, namely, the positive or negative reinforcements. By changing these "ABC's" one is able to change the *selection dimension* and therefore change future behavior in a similar situation.

Part II.

Doing What Works

3. Montessori versus Non-Montessori

Since the underlying learning model—the selectionist model—is one of adaptive learning, we can compare two learning designs by comparing the contingencies. That is to say, the way a given behavioral environment provides feedback to a child. It connects action to consequences—the child takes action and the sees the results. This will be true for all learning environments. It is convenient to look at the following bins to put the contingencies in:

1. *Behavioral repertoire of the Teacher.* This is what the teacher is able to do. The contingencies that shape and maintain this are quite varied. They involve feedback from many sources: Children, team members, coaches, books, classroom visits at other schools, videos, etc.

2. *Curriculum.* This is all the non-human actors the child interacts with. In a Montessori preschool they usual include the classic Montessori works.

3. *Motivational system and Instructional Control:* This is also quite varied. In a Montessori classroom, ideally, most of the motivation will come from "within" meaning natural reinforcement from doing things independently and seeing the results. In other designs there can be, just like an adult paycheck, prosthetic reinforcement or rewards.

4. *Class culture:* This are all the group contingencies and peer interactions that becomes more and more important as a child gets older. It is always important but for older children it is can become a big component of the contingencies that control the child's behavioral repertoire

These four "bins" can be used to compare any two classrooms. Of course, if we want to succeed with all children, teachers have to constantly do action planning as a team—experimentation—to determine what works and what doesn't. They will continually cycle through the following steps :

1. Collect data through observation

2. Analyze the data

3. Change the learning design

4. Return to step one after implementing the new design

Now lets apply the selectionist model to see what the Montessori contingencies are, why they might not work for a specific child and how behavior analysis helps fix them.

3.1. Montessori

Given the dimensions of variability, selection and amplification at the level of behavior how can we characterize a Montessori behavioral environment? Here we are dealing with a single child. Of course there are contingencies of selection at the group level present as well, since the school can determine, to a certain extent, the population of children they want to accept.

3.1.1. Preschool

First lets look at the preschool:

Variability First there are all the non-human actors present in the form of Montessori works. This creates variability in the children's behavior due to the their colorful and stimulating nature. They were designed by Montessori to create interest in the children. There are the human actors–teachers and other children. Their behavior can create variability in the child's behavior. The teachers show the children works and a child can watch the other children engaged in works. The presence and behavior of all of these actors creates "natural" variability in the child's behavior. To this one should add the impressionistic lessons given by the teachers.

Selection Each Montessori work incorporates within it a "correction of error." This generates a series of events for the child that interacts with the work ("the tower falls down when the blocks are stacked off center, etc."). These events can be positive reinforcements or negative ones. Over time this reshapes the behavior of the child. On selection component is usually avoided is conditioned positive reinforcement by the teacher in the form of approval. Montessori wanted the consequences to come purely from the works and not from the teacher.

Amplification In the course of the school year, the child can interact with a given Montessori work many many times. Over time, the child's behavioral repertoire develops fluency and a particular developmental step becomes firmly part of the child's most frequent behavioral repertoires.

3.1.2. Elementary School

All the above elements are present at the elementary school level as well. However, here the interaction of the teacher affects the selection dimension since the teacher does by their behavior generate reinforcing events for the child in the form of teacher approval or corrections.

3.1.3. What Can Go Wrong

It would seem that a Montessori behavioral environment would work well for all children. However this is not found to be the case. It is clear at all of the dimensions we are making some assumptions about the child that may not be true for every child.

Variability It could be that the stimulating character of the Montessori works will not be stimulating to that particular child. Or, the child will not have a sufficient imitative behavioral repertoire to make seeing a teacher showing a work a significant source of variability in the future behavior of that child. They may never take down the work or their interaction with it may be extremely different than what the teacher showed them. In all of these cases, the Montessori environment, for that child, may fail in the variability dimension.

Selection The natural correction of error in a Montessori work that exerts a shaping on most children, using the correction of error, may for that particular child simply generate a series of negative reinforcements every time that child interacts with the work ("if the work has ten steps they always make mistakes on eight or nine of the steps"). Eventually the child may simply cease interacting with the work. This then extinguishes the natural variability in the child's behavior. Alternative works may have to be created for a particular child to avoid failure in the dimension of selection.

Amplification If the child never experiences success with a group of Montessori works they may cease using them and there will never be any amplification as they move towards fluency.

3.2. Modification of the Montessori Contingencies

This a very big topic and, for this draft, we will confine ourselves to considering prosthetic reinforcement and token economies. Both of these are present in adult life for most adults. Having an outdoor barn raising followed by a fun keg party is an example of prosthetic reinforcement. There is no intrinsic connection between constructing a barn and drinking beer. Money is the world's largest token economy.

From both of these examples one can immediately see that society doesn't the resources or the skills to totally depend on prosthetic reinforcement or money to create and sustain all the behavioral repertoires needed in a complex society or perhaps in any society. In the barn raising example, the contingent beer drinking in the absence of the natural reinforcement of a job well done would not generate very good barns. Also, "money can't buy love" as the song goes.

3.2.1. Prosthetic Reinforcement

Given the discussion above about the potential dimensions that can fail for particular child in the pure Montessori environment, what are some of the ways of dealing with them? Obviously one way is contingencies of selection at the group level–simply don't admit such children into the school. This is the main way most pure Montessori schools address these issues.

If such children are in the school, however, there are proven techniques that can address some of these issues. If the failure is in the dimension of variability perhaps

some contingent pairing of prosthetic reinforcement can jump start the process of being stimulated by the works. An imitative repertoire can be induced in the child using protocols that may use prosthetic reinforcement.

Obviously, eventually natural reinforcement must ultimately be the dominate way that reinforcing events figure in to the contingencies of selection for a child. It is not practical for it to be any other way–there are not enough resources or designs for it to work otherwise.

One of the technical difficulties of using prosthetic reinforcement in a pure Montessori environment is that the imitative behavior of the other children that don't need it. They may "want it too." It then could disrupt the behavioral environment for them. This technical difficulty has to be solved on a case by case basis.

3.2.2. Token Economy

These designs are extremely useful in certain situations. Much research has been done on them and their advantages and disadvantages are well known.

In essence, receiving a token, like receiving a dollar bill, has no intrinsic value. Its reinforcing potency is maintained purely by what that token or dollar can buy. For some of the same reasons that money is useful so is the use of tokens. For skillful users of a token economy, tokens allow prosthetic reinforcement to be delivered in a time delayed fashion. The child receives a token right now for a specific behavior and that token can be exchanged for a prosthetic reinforcement in the future. This technical feature has many benefits.

Using a token economy in a pure Montessori environment may be needed for specific children where the selection component fails to work. The reinforcing events delivered by the Montessori works are simply not potent enough, for whatever reason. It can, from a technical perspective, be difficult to use a token economy in a pure Montessori environment because "the other children want it too." As the children get older, in a Montessori environment or not, one has to fade out any token economy because, as discussed above, almost all behavior has to ultimately be sustained largely by non-prosthetic reinforcement–it is impractical to do things any other way.

The best way of having a token economy in Montessori environment is to either have specific children have their own private money, with an understanding on the part of other children that this is special to this child alone, or involve the children in designing and running their own economy as an educational experience. At Lamplighters we have found a "Montessori way" to create beautiful token economies that fit into the classroom. Eventually, of course, these have to mutate to self-control systems of self regulation (e.g. a to do list where the person rewards themselves).

Part III.

The Visible Classroom

4. Visualizing the Montessori Contingencies

One of the amazing things about a Montessori classroom is the extent to which the success of the learning environment can be determined by observing the overall ecology of a class. It is a highly visual experience. Referring back to the discussion above, we have four dimensions that determine a successful learning environment:

1. *Teachers Behavioral Repertoire:* What the teacher does and is able to do.
2. *Curriculum:* All the non-human actors in the classroom. This includes Montessori works, tables, rugs, worksheets, etc.
3. *Motivation and Instructional Control:* The system used to maintain instructional control and motivation of the children and teachers.
4. *Class Culture:* This is a less easily described dimension since it is something that is highly dynamic. It is a abstraction from from countless transient interactions in the classroom. For example when one says a girls class is "hard working" what does this mean? Obvious it means, at a minimum, that one sees them on task most all the time, finishing their work plans, not pushing back when the teacher presents challenging opportunities to learn more and different things, etc. Defining it precisely would be difficult but it definitely is something one can "feel."

As discussed above, these four dimensions have a "sweet spot" for each child. If a child is in a classroom where these four dimensions match the child's needs the child will thrive. When these four dimensions are a mismatch for a child, the child's learning will start degrading. Eventually as the mismatch widens, the child becomes less and less happy. In more extreme cases, the child will "act out" by becoming disruptive and destructive or they will "check out."

4.1. Successful Montessori Classroom "Levers"

The process of having a successful Montessori classroom is one of designing the learning contingencies to match, as close as possible the needs of each child in the class along these four dimensions. Since the match will not ever be prefect and the target constantly changes, there needs to be a continual adjustment of these dimensions each hour, day and week of the school year based on data. Here are some generic "levers" to changing the design for an elementary class:

Montessori Works The teacher can change the selection of Montessori works on the shelves. This is a very big area of design variability. The strongest Montessori

classrooms are the ones where the works on the shelves are beautiful and inviting, match the students interests, match the students scope and sequence, "invite" the child to explore further a subject area beyond the scope and sequence. This will also affect the degree of student independence. Can the student make choices and learn with out full dependence on being given a lesson or direct teacher interaction.

Room Arrangement The placement and type of classroom furnishings can be changed. Obviously this interacts with the Montessori works. The the degree to which the classroom arrangement is alive and has wholeness—in the sense of Christopher Alexander—will affect tremendously the success of the classroom.

Schedule and Type of Lessons The teacher can change the types of lessons and schedule of pulling children to lessons. This obviously interacts strongly with the previous two dimensions. The extent that the teacher can group children for lessons based on interests and ability, design effective lessons and efficiently transition from one group to another, will determine the pace and extent that the children will receive flawless Learn Units[1]. This will interact strongly with the previous dimensions.

Work Plan The type on way the work is tracked both using individual work plans that the students carry around with them and/or tracked on a white board in a way visible to all. This affects how a team of teachers interact with students.

Visibility of Scope Sequence The extent to which the scope and sequence is visible and understandable to both students and teachers.

Tracking and Teamwork This is the extent to which the team of teachers and supporting resources can "be on the same page." Namely to know at all times where a student is in the scope and sequence.

Motivational Systems This can vary tremendously. Ideally most of the students are motivated based on natural reinforcement. This is not alway successful so various forms of a token economy need to be introduced. This is discussed above. Any motivational system can be adjusted dynamically by changing the rules, value of the tokens, types of tokens.

Special Contingencies When a student becomes disruptive and has to be removed from the classroom or temporarily dealt with in a with special contingencies, this involves changing, potentially, all of the above for that child almost immediately.

[1]Recall that a Learn Unit (LU) is made up of several interlocking interactions between a teacher and a student. We discussed some of this above in the introduction when we discussed the three part lesson. Basically, in the case where there is no mistake made, the sequence of a LU is as follows: 1) Teacher gets attention of child; 2) Teacher presents information to student in a way that invites a response; 3) Student responds to information; 4) In the case of a correct response, the teacher signals that the response is correct in a positive way and in the case of a mistake, the teacher prompts with the correct answer and the student corrects themselves. It is important that the information be presented to the child in a way that it is not cluttered by irrelevant details and the consequence the teacher provides to the child be unambiguous and, in the case of a correct response a true positive reinforcement for the child.

Tracking in real time and making ad hoc decisions in all of the above dimensions becomes important.

Group Contingencies This can include: a) field trips; b) walks around the block; c) special treasure chest activities; d) special treats; e) etc.

Most of these areas have a highly visual side to them for experienced observers of Montessori classrooms. All of them need to be documented, data streams extracted from them, analysis in team or ad-hoc meetings, decisions made based on the data, adjustments in the dimensions as "live experiments"—in general a Action Research Teaming design.

4.2. Actors and Events

All of the above dimensions for the "levers" of control in a Montessori classroom can be conceptualized in terms of ACTORS and EVENTS. What are the ACTORS? We can almost read them out from the descriptions above:

- Montessori works
- Room, tables, rugs, chairs, shelves
- Teachers
- Students
- Resource providers
- Work plans
- Lesson materials
- Student materials, personal or otherwise
- Text and visual records of what took place
- etc.

The EVENTS are any change in the configuration, composition, type, interaction between ACTORS, etc. In general changes in the ACTORS or their configuration. Clearly we are not so much interested in all the details of an event (e.g. where exactly a chair is located or how it was moved down to the inch) but are interested in categories of events—in technical terms its EQUIVALENCE CLASS or RELATIONAL FRAME CLASS with other EVENTS and ACTORS. A LEARN UNIT for example is made up of a sequence of EVENTS—as discussed in a footnote above. If we can visualize the ACTORS and EVENTS we pretty much now everything worth knowing about a Montessori class and its effect on each student and teacher.

4.3. "Centers" as The Natural Conceptual Unit

The actual conceptual entity that is natural for measuring and understanding a Montessori classroom is an extension of the concept of a CENTER as defined by Christopher Alexander (CA) in his four volume work *On the Nature of Order*. He defines it geometrically and this is what we would use for the actors. However, there are also combined collections of EVENTS and ACTORS that define the EQUIVALENCE CLASSES relevant to learning. Each such collection, provided it possess a kind of coherence, forms a CENTER. The collection of centers combine to create larger centers. The wholeness of the centers determines the degree that the Montessori school or classrooms are successful for children and staff members.

This has to be defined, as CA does in his work, using a series of examples. It is hard to define in a formal sense. It requires a person to access their "deep feelings" to determine the presence of CENTERS. This would mean, if we are using computers to assist us, that we need the counterpart of a Proof Assistant—as done with Coq for mathematics. It is not detectable solely by machines with the present state of AI. A proof assistant helps a mathematician search for a mathematical proof and can detect whether the proof is correct. It can also document the proof. However since mathematical proofs are, in general, undecidable there is no machine algorithm that can determine whether a proof is possible and find it for every situation. This is possible only for a very very limited types of mathematical problems or domains.

5. Software Needed for Supporting the Visible Montessori Classroom

5.1. Accumulating the Data

Lets do a thought experiment. Suppose we have a series of fish-eye cameras, with excellent sound pickup—in every classroom and every nook and cranny of the school. Lets suppose the visual resolution was so good we can observe and read every word on every page that a teacher or student is holding. Lets image further that the resolution is so good the we can tell exactly what is happening for each interaction between students in a lesson with a teacher. Also the sound is so good that we can filter it to hear what any person is saying clearly. Lets suppose further that this can all be recorded and stored indefinitely. In addition to this data, lets suppose we have stored all the emails and texting between all the teachers, staff members, administration and parents. Suppose further that any electronic information entered any where relevant to the school is stored and accessible also.

It is clear that all this will have all the data on EVENTS and ACTORS that is scientifically relevant. It is also clear that storing it, making sense and order of all this data will be a massive task. For this to be possible, automated highly intuitive and visual tools are needed and massive compression of the data is needed for storage. Lets tackle the analysis first.

5.2. Determining the Centers

Since CENTERS are the natural unit of analysis for a Montessori classroom or school, how do we take the overwhelming data discussed in the last section and make sense of it? Lets first discuss what types of queries we would be making of the data.

5.2.1. Natural Inquiry Dimensions

There are many things a teacher or staff member might want to know:

1. For a given Montessori work and child. What were all the interactions of the child with that work over the last week (or other time interval)?

2. For any two children, what were all their interactions over the last week (or other time interval)?

3. Given a specific child enumerate all the LEARN UNITS or MONTESSORI LEARN UNITS—and their effectiveness—over the last week that they experienced (or other time interval)?
4. For a given LEARN UNIT, rate its quality.
5. Be able to replay in fast speed, like time lapse photography, the activities of one child over a school day. Be able to stop the action at any point and see the action in slower speed.
6. For a given Montessori work, determine how it was used during one week (or other time interval).
7. For a given scope and sequence, determine where a child is on the sequence.
8. For a given teacher, see selected interactions with children.
9. For a given child, see all the electronic conversations between staff, parents or teachers that relates to the child or the child's class.

5.2.2. Centers

Each of the above inquiry areas has collections of ACTORS and EVENTS associated with it. These collections naturally group themselves into CENTERS. To some extent a raw list of ACTORS and EVENTS can be associated with each of the inquiry areas, but some human-machine interaction will be needed to organize these fully into the natural CENTERS. It may even be true that that organization of the data into CENTERS is really a set of discussions about the data among a set of UNIVERSAL OBSERVERS[1] that are competent enough to engage in the discussion.

5.3. Software and Hardware

In general the massive amount of data that the above thought experiment entails can be simplified considerably since much of it doesn't change much from day to day:

1. Most of the non-human actors in a room don't change all that much from day to day:
   a) Room furnishings

[1] A UNIVERSAL OBSERVER is a theoretical category of individuals that are able to make objective scientific observations. Of course, a truly objective observer is a fiction. After all, they are a behaving organism also subject to the same laws as the people they are observing. There is another category called the UNIVERSAL AUDIENCE that is made up of all those that agree to be bound by the conclusions of rational inquiry. A person that believes the world is thousands of years old can be member of the group of universal observers but can not be a member of the universal audience since that group believes rational argumentation leads to a an estimate of billions of years old. In general the wikepedia is a good approximation of the verbal behavior of the universal audience.

   b) Montessori works on the shelves
   c) Room background

2. Most of the human actors don't change all that much from day to day:
   a) Students
   b) Teachers
   c) Resource staff

Since there are not a lot of daily changes for the collection of the ACTORS, probably the data can be subject to a great deal of compression for storage purposes. The actors may change position and their interaction changes, but the main "essence" of the ACTOR remains largely the same.

5.3.1. Hardware

1. Smart phones (largely Android and iPhone).
2. Tablets.
3. Fish-eye cameras in each room that give complete coverage of the school.
4. Sound recording devices.
5. Cheap multiprocessor "supercomputers" made out many of cheap single processor boards for storing and manipulating the data. There would be a number of these depending on network topology and data processing needs.
6. Networking devices to allow for proper bandwidth for the data collection and data movement.
7. Remote and local virtual machines for servers needed for accessing the data on smart phones, tablets and desktop.

5.3.2. Software Backend

There are several dimensions to this:

Raw Video Data This consists of compressed video and audio.

Actor Tracking As discussed above there are a limited number of ACTORS present during the course of a school day and even over a school year. They could be cataloged periodically and then tracked automatically based on the raw data. Some human interaction will be needed to for the AI to learn fully the identities of the actors and to reliably guess at where they are in the video/audio feed.

Visualization Once specific or several ACTORS are tracked over a time interval, there is also the need for visualization of actors interaction during the time interval. This requires heavy processing that can not usually be wholly done on a client device.

Text and Media The staff, teachers, and parents are constantly engaging in electronic communication using various electronic devices. This can include text, images, video, sound files, design documents, PDF's, etc. There needs to be some backend processing that allows a client device to easily search, view and reorganize this information in multiple ways. Ideally wiki pages could be generated for editing based on the raw data that either contain the content of the information or have links to it. Since abbreviations, code words and natural text are used in texting, emails and document production, the backend would have to have natural language processing capability that "learns" more about the school, children, teachers, staff and parents as more data is accumulated. It then can suggest ways to organize the data that can be edited by a knowledgeable person using a "Proof Assistant" type of software with an appropriate interface.

5.3.3. Software Front End

A person interacting with the backend could be using multiple devices each with a different purpose. There seems to be a number of different types of generic user experiences:

1. *Live Communication:* This will primarily be texting about children using text with associated media files to illustrate points or provide background information. It will also be largely a "text-ring" of different HUMAN ACTORS. Each one of the human actors will want to know whether the information has been viewed by others and also, when they are off-line, the information will still be available for them to read when they come on-line again.

2. *Longer Term Documentation:* This could involve emails, documents, wiki pages. In general, since this also tends to have natural ways of organizing the information by child, class, teacher, etc., there would be an attempt to automatically generate "wiki" type pages with links to the various texts as the various users create documents and/or emails.

3. *Searching and Browsing the data:* The user here is simply trying to look at the documentation trail for children, teachers and parents. They want to be able to see, as visually as possible, the results of the inquiry areas mentioned above.

4. *Annotating the Data:* Here a user is reviewing the data for the purpose of improving upon the default organization and increasing the meaning of the data. In general the AI component of the backend will attempt a preliminary organization for the data. However, this is simply a good first guess for the natural CENTERS. A human UNIVERSAL OBSERVER will be needed to make it better organized and more meaningful.

5. *Live Classroom Viewing*: Here one is viewing a classroom in real time. There should be the possibility to drill down into understanding fine level details of what is happening in the classroom. At the simplest level, this can simply be a video feed.

Because there are so many ACTORS present in a school, these five areas would have to be accessible from many different views. For example having a text-ring ("group chat) for each child where teachers, parents and staff can discuss a particular student can not be a flat hierarchy—there are simply too many children. Hence there would have to be a user interface that hides this complexity by putting the information into a hierarchy. This means, for example, a simple list view for the all text-rings becomes unworkable when there so many text-rings. It would mean scrolling through a list of dozens of text-rings. Similarly for live classroom viewing. The interface should allow easy navigation using some sort of natural hierarchy.

5.3.4. Device User Experiences

It will be expected that the different devices will have different user experiences for each of the just listed five areas of generic user experiences. For example, much of the *live communication* will be done using a smart phone since that is what most people will carry with them at all times. However, *longer term documentation*, and *annotating the data* will probably be done largely using either a tablet or desktop interface.

This means that apps for mobile devices and a desktop web or desktop application will have to exist with some sort of role based security (e.g parents, teachers, resource providers, chinuch admin, organizational admin, etc.) for access and manipulation of data. The user experiences of the different roles could be quite different.