MCA – 11

I Semester M.C.A. Examination, February/March 2010
MATHEMATICS

Time: 3 Hours Max.Marks : 100

Instructions *: 1) Answer **all** questions in Part **A**, **5** out of **8** questions in Part **B** and **4** out of **6** questions in Part **C**.*
*2) Part **A** : Questions from **1** to **8** carry **1** mark **each** and **9** to **14** carry **2** marks **each**.*
*3) Part **B** : **Each** question carries **8** marks.*
*4) Part **C** : **Each** question carries **10** marks.*

PART – A

1. $\sin 3\theta$ = _____.

2. $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

3. State Cayley-Hamilton theorem.

4. If $\vec{a} = 3\hat{i} - 2\hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - 4\hat{k}$ find $\vec{a}.\vec{b}$.

5. If $y = \sin(m\sin^{-1}x)$ find $\frac{dy}{dx}$.

6. What is the nth derivative of $e^{ax}\cos(bx + c)$.

7. Evaluate $\int xe^x\,dx$

8. Evaluate $\int \log x\,dx$

9. Find the modulus and argument of the complex number $\frac{1+2i}{1-3i}$

10. Use Cramer's rule to solve $8x + 5y = 21$
$3x + 10y = 16$

P.T.O.

11. If $\vec{a}=\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-2\hat{k}$ then find $\left|\vec{a}\times\vec{b}\right|$.

12. If x = acost y = asint then find $\frac{d^2y}{dx^2}$.

13. If $y^2 = 4ax$ then S.T. $\frac{d^2y}{dx^2}=\frac{-4a}{y^3}$

14. Evaluate $\int_1^2 (x^4-3x^2+5)\,dx$.

PART – B

1. Solve the following system of equations $3x - y + 2z = 13$, $2x + y - z = 3$, $x + 3y - 5z = -8$

2. Simplify $\frac{(\cos 4\theta+i\sin 4\theta)^3(\cos 5\theta+i\sin 5\theta)^2}{(\cos 6\theta+i\sin 6\theta)^2(\cos\theta-i\sin\theta)^5}$.

3. a) If $y = (1 + x^2)\tan^{-1}x$, Find $\frac{d^2y}{dx^2}$ at x = 1

   b) Find the nth derivative of $e^{2x}\cos^2 x$

4. Find the angle between the curves $r = a\log\theta$, $r = \frac{a}{\log\theta}$

5. If $u=\frac{x^2+y^2}{x+y}$ then S.T $\left(\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right)^2 = 4\left(1-\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right)$.

6. Evaluate $\int_0^a \frac{x^7\,dx}{\sqrt{a^2-x^2}}$

7. Solve $(x - 4y - 9)\,dx + (4x + y - z)\,dy = 0$

8. a) Solve $\frac{dy}{dx} = e^{2x-3y} + 4x^2e^{-3y}$

b) Evaluate $\int_0^{\pi} x\cos^6 x\,dx$.

PART – C

1. Diagonalise the matrix $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$.

2. a) Show that the cube root of unity form an Abelian group under multiplication.

b) Find the sine of the angle between the vectors $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+2\hat{k}$.

3. If $y^{1/m} + y^{-1/m} = 2x$. Show that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$

4. Find the Pedal equation for $\frac{1}{r} = 1+e\cos\theta$.

5. Evaluate :

a) $\int \frac{dx}{3-5\cos\theta}$

b) $\int \frac{dx}{2x^2+x-1}$

6. Solve $(2x + y - 3)dy = (x + 2y - 3)dx$.

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