Model independent results for the inflationary and reheating epochs

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We address the problem of determining inflationary characteristics in a model independent way and then study constraints for reheating. We start from a recently proposed equation which allows to accurately calculate the value of the inflaton at horizon crossing ϕ_k. We then use an equivalent form of this equation to write a formula that relates the tensor-to-scalar index r to the number of e-folds during inflation N_k, hence a general bound for N_k follows. In particular, at present $r < 0.063$ implies $N_k < 56.3$. We also give an upper bound to the size of the universe, during the inflationary epoch, that gave rise to the current observable universe. The reheating epoch is discussed and a bound is given for the effective number of relativistic degrees of freedom g_{re} which translates into a bound for the reheat temperature. From here bounds for the number of e-folds during reheating and also during the radiation dominated epoch follow. A criteria to know whether the constraint for the effective number of degrees of freedom exists is given in terms of the ratio V_e/V_k where V_e is the potential at the end of inflation and V_k at the horizon crossing scale k. Finally we study two particular models: Starobinsky model, which was studied before and is mostly used here for comparison, and a Mutated Hilltop Inflation (MHI) model. Tables II and III show results for the two specific models of inflation.

I. INTRODUCTION

During the last several years we have seen an extraordinary advance in our knowledge of the universe, its composition, geometry and evolution. The idea of an inflationary universe remains solid some 40 years after its inception [1], [2], (for reviews see e.g., [3], [4], [5]), however the existence of a plethora of models [6] constantly reminds us that our knowledge of that epoch is imprecise, and even more so when we consider the time of reheating after inflation ends, for reviews on reheating see e.g., [7], [8], [9]. Numerous works have been done in our attempt to better understand the reheating era with varying degrees of success [10] - [23]. In this work, we initially address the problem of determining important inflationary characteristics in a model independent way and then study possible constraints for reheating.

The organization of the article is as follows: in Section II we first start from a recently proposed equation [24] which allows us to accurately calculate the value of the inflaton at horizon crossing ϕ_k. We then use an equivalent form of this equation to write a formula that relates the tensor-to-scalar index r to the number of e-folds during inflation N_k, hence a general bound for N_k follows. In particular, at present $r < 0.063$ implies $N_k < 56.3$. We end the section by calculating a bound to the size of the universe, during the inflationary epoch, that gave rise to the current observable universe. In Section III we discuss the reheating epoch and a bound is given for the effective number of relativistic degrees of freedom g_{re} which translates into a bound for the reheating temperature [25] and bounds for the number of e-folds during reheating and also during the radiation dominated epoch. The constraint is given by Eq. (20) as a bound for the ratio V_e/V_k where V_e is the potential at the end of inflation and V_k at the horizon crossing scale k. In Section IV we study two particular models: Starobinsky model [29] - [32] which was studied before [24] and is used now to compare with a Mutated Hilltop Inflation (MHI) model [33], [34] where several interesting aspects occur. Tables II and III show results for the two specific models of inflation. Finally in Section V we give our conclusions on the most important points discussed in the article.

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II. RESULTS FOR THE INFLATIONARY EPOCH

The equation which determines the inflaton field ϕ at horizon crossing is [24]

$$\ln[\frac{a_* H_k}{k_*}] = 2N_k \ , \tag{1}$$

where k is the horizon scale during inflation k_* is the pivot scale and a_* is the corresponding scale factor. The Hubble function at k is given by $H_k = \sqrt{8\pi^2\epsilon_k A_s}$ and $N_k \equiv \ln\frac{a_e}{a_k}$ is the number of e-folds from ϕ_k up to the end of inflation at ϕ_e. Notice that the Hubble function introduces the scalar power spectrum amplitude given here by A_s. Eq. (1) is a model independent equation although its *solution* for ϕ_k requires specifying a model of inflation; H_k and N_k are model dependent quantities. Thus, after finding ϕ_k, we can proceed to determine all inflationary parameters and observables.

An easy way to understand Eq. (1) is by connecting the epoch where the scale k left the horizon during inflation to the pivot scale k_* where we measure the horizon reentry of precisely the same scale k thus, $k = k_*$. This can be expressed by

$$\ln\left(\frac{a_*}{a_k}\right) = 2N_k \ , \tag{2}$$

where $2N_k$ is the number of e-folds from the scale k up to the end of inflation *plus* the number of e-folds from the end of inflation up to the scale k_* which is also equal to N_k. Multiplying the term inside the parenthesis above and below by H_k and setting $a_k H_k \equiv k = k_* \equiv a_* H_{k_*}$ we get Eq. (1). An alternative but equivalent way of obtaining Eq. (1) is given in [24]. To find the value of a_* we solve the Friedmann equation for a_*

$$k_* = H_0 \sqrt{\frac{\Omega_{md,0}}{a_*} + \frac{\Omega_{rd,0}}{a_*^2} + \Omega_{de} a_*^2} \ , \tag{3}$$

where $k_* = 0.05/Mpc \approx 1.3105 \times 10^{-58}$ (see Table I to find the numerical values of the other parameters used in our calculations).

Note also that Eq. (1) incorporates knowledge from the present universe, in the determination of a_*, of the early universe, when considering the scale k during inflation, and also of the CMB epoch by the presence of the scalar power spectrum amplitude A_s through H_k.

From Eq. (1) and $H_k = \sqrt{8\pi^2 \epsilon_k A_s}$ we can get an expression for $r \equiv 16\epsilon_k$ in terms of the number of e-folds N_k

$$r = \frac{2k_*^2}{\pi^2 a_*^2 A_s} e^{4N_k} \ . \tag{4}$$

Imposing a bound b to r we get a general bound for N_k

$$r < b \ \Rightarrow \ N_k < \frac{1}{4} \ln\left(\frac{\pi^2 a_*^2 A_s}{2k_*^2}\, b\right) \approx 57.016 + \frac{1}{4}\ln b \ , \tag{5}$$

for the particular value $b = 0.063$ [26], [27] we get the present bound for N_k

$$r < 0.063 \quad \Rightarrow \quad N_k < 56.3 \ . \tag{6}$$

This is a model independent result, it follows from Eq. (1), phenomenological parameters and the bound for r without specifying any model of inflation.

We can also calculate a model independent bound to the size of the patch of the universe from which our present observable universe originates. We adapt Eq. (2) to this situation

$$\ln\left(\frac{a_0}{a_k}\right) = 2N_k \ , \tag{7}$$

where a_0 is, as usual, the present scale factor $a_0 = 1$, k is the horizon scale during inflation which gave rise to our observable universe such that $k = k_0$ ($k_0 \equiv a_0 H_0$ is the present scale) and $2N_k$ is the number of e-folds from the scale k up to the end of inflation *plus* the number of e-folds from the end of inflation up to the scale k_0 which is also equal to N_k. Note that the k in Eq. (7) has *not* the same value as the k in Eq. (2). From Eq. (1) and from the bound for N_k follows that at the scale k

$$a_k = a_0\, e^{-2N_k} > a_0\, e^{-132.7} \approx 2.34 \times 10^{-58} \ . \tag{8}$$

Note that we have added $N_* \equiv \ln \frac{a_0}{a_*} \approx 10.05$ e-folds to the upper bound of 56.3 for N_k because there are 10.05 e-folds from the pivot scale $k_* = 0.05/Mpc$ up to the present scale k_0. If the diameter of the observable universe is $8.8 \times 10^{26} m$ then at the scale k the size of the universe from which ours originates was bigger than $2.059 \times 10^{-31} m$. Thus, at the scale k the universe diameter was *at least* 1.274×10^4 times bigger than the Planck length. At the end of inflation it had a size of at least $1.35\, cm$.

III. RESULTS FOR THE REHEATING EPOCH

To establish constraints for the reheating epoch we need in particular a formula for the number of e-folds during reheating. The standard way to proceed is to solve the fluid equation with the assumption of a constant equation of state parameter ω during reheating, this gives the number of e-folds during reheating in terms of the energy densities as follows

$$N_{re} \equiv \ln \frac{a_{re}}{a_e} = [3(1+\omega)]^{-1} \ln[\frac{\rho_e}{\rho_{re}}] \ , \tag{9}$$

where ρ_e is the energy density at the end of inflation and ρ_{re} the energy density at the end of reheating

$$\rho_{re} = \frac{\pi^2 g_{re}}{30} T_{re}^4 \ , \tag{10}$$

with g_{re} the number of degrees of freedom of species at the end of reheating. To proceed we assume entropy conservation after reheating, this assumption establish another expression involving T_{re} which can be substituted in Eq. (10) and then in Eq. (9)

$$g_{s,re} T_{re}^3 = \left(\frac{a_0}{a_{eq}}\right)\left(\frac{a_{eq}}{a_{re}}\right)\left(2T_0^3 + 6 \times \frac{7}{8} T_{\nu,0}^3\right) \ , \tag{11}$$

where $g_{s,re}$ is the number of degrees of freedom of species after reheating, $T_0 = 2.725K$ and the neutrino temperature is $T_{\nu,0} = (4/11)^{1/3} T_0$. The number of e-folds during radiation domination $N_{rd} \equiv \ln \frac{a_{eq}}{a_{re}}$ follows from Eqs. (9) and (11)

$$N_{rd} = -\frac{3(1+\omega)}{4} N_{re} + \frac{1}{4}\ln[\frac{30}{g_{re}\pi^2}] + \frac{1}{3}\ln[\frac{11 g_{sre}}{43}] + \ln[\frac{a_{eq}\, \rho_e^{1/4}}{a_0\, T_0}] \ . \tag{12}$$

TABLE I. For easy reference this table collects numerical values of parameters used in the paper. Dimensionless quantities have been obtained by working in Planck mass units, where $M_{pl} = 2.4357 \times 10^{18} GeV$ and set $M_{pl} = 1$, the pivot scale $k_* \equiv a_* H_* = 0.05 \frac{1}{Mpc}$, used in particular by the Planck collaboration, becomes a dimensionless number given by $k_* \approx 1.3105 \times 10^{-58}$. This can be compared with $k_0 \equiv a_0 H_0 \approx 8.7426 \times 10^{-61} h$. To calculate a_* we have to specify h for the Hubble parameter H_0 at the present time. We take the value given by Planck $h = 0.67$ for definitiveness and check that no important changes occur for $N_* \equiv \ln \frac{a_0}{a_*}$ for h in the interval $0.67 < h < 0.73$. The solution of Eq. (3) for a_* is $a_* = 4.3000 \times 10^{-5}$ from where we get $N_* = 10.05$ for the number of e-folds from a_* to a_0.

Parameter	usually given as	*Dimensionless*
H_0	$100\, h \frac{km}{s}/Mpc$	$8.7426 \times 10^{-61}\, h$
T_0	$2.725\, K$	9.6235×10^{-32}
A_s	2.0968×10^{-9}	2.0968×10^{-9}
k_*	$0.05/Mpc$	1.3105×10^{-58}
a_*	$-$	4.3000×10^{-5}
$\Omega_{md,0}$	0.315	0.315
$\Omega_{rd,0}$	7.9×10^{-5}	7.9×10^{-5}
Ω_{de}	0.685	0.685

We can finally obtain an expression for the number of e-folds during reheating N_{re} by combining Eqs. (1) and (12), the result is [24]

$$N_{re} = \frac{4}{1-3\,\omega} \left(N_k - \frac{1}{3} \ln[\frac{11 g_{s,re}}{43}] - \frac{1}{4} \ln[\frac{30}{\pi^2 g_{re}}] - \ln[\frac{a_* \rho_e^{1/4}}{a_0 T_0}] \right) . \tag{13}$$

From this equation we can study the dependence of the degrees of freedom g_{re} on N_{re} and ω. For as long as species have the same temperature and $p \approx \frac{1}{3}\rho$ we have that $g_{s,re} \approx g_{re}$. Thus, we set $g_{s,re} = g_{re}$ in Eq. (13) and proceed to solve for g_{re}, the result is [24]

$$g_{re} = g_{re}(\phi_k)\, e^{-3(1-3\omega)N_{re}} , \tag{14}$$

where

$$g_{re}(\phi_k) = \left(\frac{43}{11}\right)^4 \left(\frac{\pi^2}{30}\right)^3 \left(\frac{H_k}{e^{N_k} \rho_e^{1/4}} \frac{a_0 T_0}{k_*}\right)^{12} . \tag{15}$$

The quantity $g_{re}(\phi_k)$ essentially depends on ϕ_k. Numerical studies of the thermalization phase during reheating suggest that $0 \lesssim \omega \lesssim 0.25$ [28]; here we extended our discussion up to $\omega \approx 1/3$. To extract a constraint for the reheating temperature we observe that for $\omega \lesssim \frac{1}{3}$ the exponential in Eq. (14) is always less than one. Thus, $g_{re}(\phi_k)$ gives the largest possible value of g_{re}

$$g_{re} \lesssim g_{re}(\phi_k) . \tag{16}$$

In certain models $g_{re}(\phi_k)$ and, as a consequence, g_{re} is less than the number of species of the Standard Model of Particles (106.75). If this is the case [25] then a constraint on the reheating temperature T_{re} follows immediately. We have shown before [24] that this is indeed the case for the Starobinsky model and in the next section we show that it is also the case for a mutated hilltop inflation model.

Let us write in more detail the condition under which a constraint is expected. At the end of inflation when $\omega = -1/3$ an expression for ρ_e follows

$$\rho_e = \frac{3}{2} V_e = \frac{9}{2} \frac{V_e}{V_k} H_k^2 . \tag{17}$$

Together with Eq. (1), Eq. (15) can then be written in a form containing only known quantities with the exception of the last factor involving the ratio of the potential at the scale k and at the end of inflation

$$g_{re}(\phi_k) = \left(\frac{43}{11}\right)^4 \left(\frac{2\pi^2}{270}\right)^3 \left(\frac{a_0 T_0}{\sqrt{a_* k_*}}\right)^{12} \left(\frac{V_k}{V_e}\right)^3 , \tag{18}$$

taking all the numbers from the Table I we get

$$g_{re}(\phi_k) \approx 1.7981 \left(\frac{V_k}{V_e}\right)^3 < 106.75 , \tag{19}$$

where we have written in the r.h.s. of Eq. (19) the number of degrees of freedom of the Standard Model of Particles as an upper limit for $g_{re}(\phi_k)$. When $g_{re}(\phi_k) > 106.75$ no restriction for the reheating temperature coming from the number of degrees of freedom of species arises. In supersymmetric models this number should be twice as big. From the r.h.s. of Eq. (19) and the obvious requirement $V_e < V_k$ follows that

$$0.2563 < \left(\frac{V_e}{V_k}\right) < 1 \,. \tag{20}$$

Whenever V_e/V_k is within these limiting values a constraint on the reheat temperature arises. Notice that all these equations and bounds are model independent although their *solutions* require specifying a model of inflation.

IV. THE STAROBINSKY AND MUTATED HILLTOP INFLATION MODELS

The Starobinsky model revisited.– The potential of the Starobinsky model [29–31] is given by [32]:

$$V = V_0\left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2 , \tag{21}$$

with Hubble function

$$H_k = \sqrt{8\pi^2\epsilon_k A_s} = \sqrt{\frac{32A_s}{3}}\frac{\pi}{e^{\sqrt{\frac{2}{3}}\phi_k} - 1} \,, \tag{22}$$

where ϵ_k is the slow-roll parameter $\epsilon \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2$ at $\phi = \phi_k$. The number of e-folds N_k follows easily

$$N_k = -\int_{\phi_k}^{\phi_e}\frac{V}{V'}d\phi = \frac{1}{4}\left(3e^{\sqrt{\frac{2}{3}}\phi_k} - \sqrt{6}\,\phi_k\right) - \frac{1}{4}\left(3e^{\sqrt{\frac{2}{3}}\phi_e} - \sqrt{6}\,\phi_e\right), \tag{23}$$

where ϕ_e denotes the end of inflation: $\phi_e = \sqrt{\frac{3}{2}}\ln\left(1 + \frac{2}{\sqrt{3}}\right)$. We can solve Eq. (1) for ϕ_k with the result [24]

$$\phi_k = 5.365 \,, \tag{24}$$

from here all inflationary parameters and observables follow. Notice that in the Starobinsky model there are no further parameters apart from the overall V_0 (which is fixed by the scalar amplitude) and we can obtain precise values for all the quantities of interest during inflation. Thus, $N_k = 55.6$ and we can also determine the size of the primordial universe according to Starobinsky model. The scale factor a_k is

$$a_k = a_0\, e^{-2N_k} = a_0\, e^{-131.3} \approx 9.49 \times 10^{-58} \,. \tag{25}$$

Following the previous general discussion around Eq. (8) we have now that at the scale k the size of the patch of the universe from which ours originates was $8.35 \times 10^{-31} m$. At the end of inflation it had a diameter size of $2.71\, cm$.

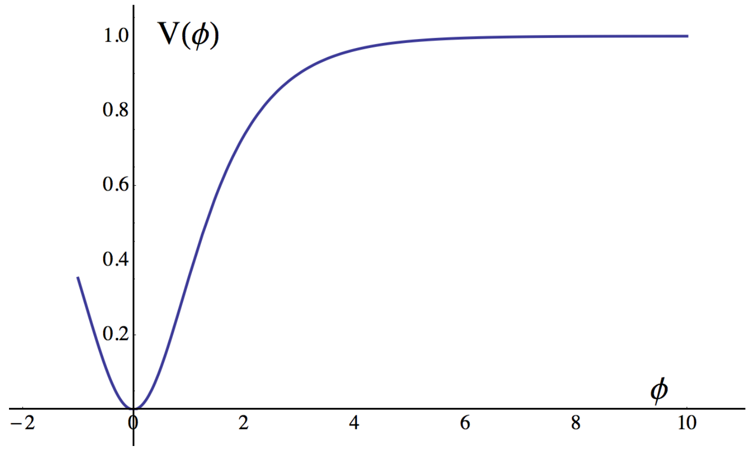

FIG. 1. Schematic plot of the mutated hilltop potential given by Eq. (26) as a function of ϕ for an inflaton field rolling from the right. Characteristics of inflation and reheating for this model are given in Tables II and III.

On the other hand, the ratio $\left(\frac{V_e}{V_k}\right) \approx 0.2945$ is well inside the limiting values given by Eq. (20) and thus, a constraint follows for the number of degrees of freedom which translates into a constraint for the temperature at the end of reheating, from Eq. (15) or Eq. (19) it follows that $g_{re} \approx 70.39$.

In models where there is at least one free parameter (different from an overall parameter which is fixed by the scalar amplitude), one can investigate whether there is a range of values of the parameter such that ϕ_k gets closer to ϕ_e in such a way that the ratio V_e/V_k falls within the limits of Eq. (20). Of course, the value of the parameter should be consistent with the other requirements of inflation, in particular a tensor-to-scalar ratio r and spectral index n_s within the bounds for these observables. We discuss this possibility with a model of mutated hilltop inflation.

Mutated Hilltop Inflation model.– This model is given by the potential [33], [34]

$$V = V_0\left(1 - \mathrm{sech}\left(\frac{\phi}{\mu}\right)\right). \tag{26}$$

The number of e-folds N_k can be calculated in closed form with the result

$$N_k = \mu^2\left(2\ln\frac{\cosh\left(\frac{\phi_e}{2\mu}\right)}{\cosh\left(\frac{\phi_k}{2\mu}\right)}\right) + \cosh\left(\frac{\phi_k}{\mu}\right) - \cosh\left(\frac{\phi_e}{\mu}\right). \tag{27}$$

The field at the end of inflation ϕ_e is given by the solution to the condition $\epsilon = 1$. The solution is very involved and is given by

TABLE II. Characteristics of the inflationary and reheatings epoch are given for the Starobinsky and Mutated Hilltop Inflation (MHI) models. The second column quotes the value of ϕ_k for each model, with an specified model dependent parameter in the case of MHI. From this value of ϕ_k follows all inflationary quantities. Three values of the parameter μ are given such that a constraint to the reheating temperature follow. The constarint follows if Eq. (19), equivalently Eq. (20), is satisfied.

$Model$	ϕ_k	r	n_s	α	N_k	$g_{re}(\phi_k)$	T_{re}(GeV)
Starobinsky	5.3653	0.00343	0.96534	-6.1×10^{-4}	55.6	70.39	0.5
MHI $\mu=9.8107$	12.8844	0.06300	0.96599	-6.16×10^{-4}	56.3	>106.75	–
MHI $\mu=1.4118$	5.8665	0.00420	0.96601	-5.93×10^{-4}	55.6	106.75	10^3
MHI $\mu=1$	4.7910	0.00228	0.96539	-6.08×10^{-4}	55.5	40.10	0.18
MHI $\mu=0.5196$	3.1405	0.00067	0.96444	-6.35×10^{-4}	55.2	10.75	0.01

TABLE III. The table shows the number of relativistic degrees of freedom $g_{re}(\phi_k)$, the reheating temperature T_{re}, number of e-folds during reheating N_{re} and during radiation N_{rd}. The reheating temperature follows from the history of $g_{re}(T)$ an corresponds to the approximate mass of the particle when annihilation begins. The value of the parameter μ in the second row corresponds to the bound on r ($r<0.063$, [27]) and the other three values are such that a constraint occurs according to the discussion following Eqs. (19) and (20). The lower bound for T_{re} comes from nucleosynthesis considerations and the bounds for N_{re} and N_{rd} follow from Eqs. (12) and (13).

$Model$	$g_{re}(\phi_k)$	T_{re}	N_{re}	N_{rd}
Starobinsky	70.39	$10MeV<T_{re}<500MeV$	$40.84>N_{re}>36.30$	$16.69<N_{rd}<21.23$
MHI $\mu=9.8107$	$\gg 106.75$	$10\,MeV<T_{re}$	$41.57>N_{re}$	$16.69<N_{rd}$
MHI $\mu=1.4118$	106.75	$10MeV<T_{re}<10^3GeV$	$40.89>N_{re}>28.61$	$16.69<N_{rd}<28.97$
MHI $\mu=1$	40.10	$10MeV<T_{re}<180MeV$	$40.30>N_{re}>37.41$	$17.13<N_{rd}<20.02$
MHI $\mu=0.5196$	10.75	$10MeV$	40.43	16.69

$$\mathrm{sech}\left(\frac{\phi_e}{\mu}\right)=\frac{1}{6\mu^2(3+2\mu^2)^2}\left(-8\mu^6+4\mu^4(-6+5\times2^{1/3}R_2^{1/3})-2\mu^2(9+2^{1/3}(-15+2^{1/3}R_1)R_2^{1/3}+4\times2^{2/3}R_2^{2/3})-3\times2^{2/3}R_1R_2^{1/3}+3\times2^{2/3}R_2^{2/3}+2R_1R_2^{2/3}\right). \tag{28}$$

where $R_1=2^{1/6}\mu\sqrt{12\mu^4+66\mu^2-3}$ and $R_2=\mu^3(4\mu(9+\mu^2)+3\sqrt{6}\sqrt{4\mu^4+22\mu^2-1}$. We cannot solve in general Eq. (1) for ϕ_k and arbitrary μ thus, we resort to the following strategy: Eq. (4) is equivalent to Eq. (1) although written in terms of r rather than H_k. We solve the equation $r=16\epsilon_k=b$, where b is an upper bound on r (e.g., $b=0.063$ at present). With this solution for ϕ, let us say $\phi_k(\mu,b)$

$$\phi_k=\phi_k(\mu,b)\,, \tag{29}$$

we solve the equation $N_k=N_k(b)$ where $N_k(b)$ is the value of N_k in Eq. (1) at the bound ($e.g., N_k(0.063)=56.3$). In conclusion, we form a system of two equations for two unknowns: ϕ_k and μ, consistently with Eq. (1). There are, of course, some others equally valid variations on this strategy. Once we have the values of ϕ_k and μ at the bound we can investigate from there the behaviour of the solution for various values of the parameter μ. What we find in this particular model is that at the bound $b=0.063$, $\mu=9.8107$ with $\phi_k=12.8844$. Smaller values of μ go in the right direction: lowering the value of r and, in this case, also lowering the spectral index n_s. Due to Eq. (4) N_k follows the behaviour of r, also diminishing as r diminish (see Table II).

From the Table II we see in row number four the case $\mu=1$ where g_{re} satisfies the bounds in Eq. (20)

$$g_{re}(4.791)=40.10\,. \tag{30}$$

Rows numbers three and five correspond to the limiting values 106.75 and 10.75 for the effective number of degrees of freedom. The first limit saturates the degrees of freedom for Standard Model of Particles and the second corresponds to an approximated lower bound for the reheating temperature of 10 MeV coming from nucleosynthesis considerations [28] although a lower T_{re} for the lower bound has been discussed in the literature [35]. The solution for $\phi_k(\mu)$ can be easily obtained by solving Eq. (19) in the two limiting situations

$$V_k=\alpha(g_{re}(\phi_k))\,V_e\,, \tag{31}$$

where $\alpha\approx3.9011$ when taking $g_{re}(\phi_k)=106.75$ and $\alpha\approx1.8149$ when $g_{re}(\phi_k)=10.75$. After having determined

$\phi_k(\mu)$, Eq. (1) gives the solution for μ. In Table III we summarize our results for this section.

V. CONCLUSIONS

We have studied model independent results for the inflationary and reheating epochs following from the formulas given by Eqs. (1) and (20). We have, in particular, establish an equation (Eq. (4)) for the tensor-to-scalar ratio in terms of the number of e-folds $N_k \equiv \ln \frac{a_e}{a_k}$ during inflation. From a bound b for r follows a general bound for N_k (Eq. (5)) which at present is $r < 0.063$ implying $N_k < 56.3$. These are all model independent results in the sense that no model of inflation is used to obtain them. At the end of Section II we also give a bound to the size of patch of the universe from where our observable universe comes from. Section III discuses the reheating epoch and a bound is given for the effective number of relativistic degrees of freedom g_{re} which translates into a bound for the reheating temperature and bounds for the number of e-folds during reheating and also during the radiation dominated epoch. The constraint is given in Eq. (20) in terms of a ratio V_e/V_k of the potential at the end of inflation over the potential at horizon crossing. Tables II and III show results for two specific models of inflation.

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