Born Rule in Everettian Many Worlds Theory

Everett gives an argument for the Born rule in his 1957 paper. Simon Saunders (in his introduction to the volume of essays: *Many Worlds?: Everett, Quantum Theory, & Reality*, OUP 2010) gives the following summary of Everett's argument:

"But Everett was able to derive at least a fragment of the Born rule. Given that the measure over the space of branches is a function of the branch amplitudes, the question arises: What function? If the measure is to be additive, so that the measure of a sum of branches is the sum of their measures, it follows that it is the modulus square—that was something. The set of branches, complete with additive measure, then constitute a probability space. As such, version of the Bernouilli and other large number theorems can be derived. They imply that the measure of all the branches exhibiting anomalous statistics (with respect to this measure) is small when the number of trials is sufficiently large, and goes to zero in the limit—that was something more."

This account can be criticized on several grounds. Firstly, it relies on the limit of infinitely many trials, whereas in practice, we only ever have a finite number of such trials. Another criticism is that there is not any solid basis for the assumption that the measure should depend only on the branch weights—why should it not depend on the actual structure of the branches themselves? The other main line of objection relates to the simple application of Everett's rule in the case where all possible outcomes occur on each trial. In that case, all possible sequences of results occur, so that predictions using this rule would have been wildly contradicted by the emperical evidence—which only goes to show that the Born Rule, far from being an obvious consequence of the interpretation of the quantum state in terms of many worlds, appears quite unreasonable.

This latter point is made very strongly by Adrian Kent in his contribution to the above cited volume of collected essays (pp. 307–354).

Kent considers a toy multiverse, which is classical, but in which branches are multiplied to record all possible results. The first such world he considers includes conscious inhabitants, but which also includes a machine with a red button on it, and a tape emerging from it, with a sequence of numbers on it, all in the range 0 to $(N-1)$. When the red button is pressed in some universe within the multiverse, that universe is deleted, and N successor universes are then created. All the successors are in the same classical state as the original (and so, by hypothesis, all include conscious inhabitants with the same memories as those who have just been deleted), except that a new number has been written onto the end of the tape, with the number i being written in the i-th successor universe.

"Suppose, further, that some of the inhabitants of this multiverse have acquired the theoretical idea that the laws of their multiverse might attach *weights* to branches, i.e., a number p_i is attached to branch i, where $p_i \geq 0$ and $\Sigma_i p_i = 1$. They might have various different theories about how these weights are defined.... To be clear: this is not to say that the branches have equal weight. Nor are they necessarily physically identical, aside from the tape numbers. However, any such differences do not yield any natural quantitative definition of branch weights. There is just no fact of the matter about branch weights in this multiverse."

Kent goes on the say:

"Everettian quantum theory is essentially useless, as a scientific theory, unless it can explain the data that confirm the validity of the Copenhagen quantum theory within its domain—unless, for example, it can explain why we should expect to observe the Born rule to have been very well confirmed statistically. Evidently, Everettians cannot give an explanation that says that all observers in the multiverse will observe confirmation of the Born rule, or that very probably all observers will observe confirmation of the Born rule. On the contrary, many observers in an Everettian multiverse will definitely observe convincing *discomfirmation* of the Born rule.

"It suffices to consider very simple many-worlds theories, containing classical branching worlds in which the branches correspond to binary outcomes of definite experiments. Consider thus the *weightless multiverse*, with many-worlds of the type outlined above, in which the machine produces only two possible outcomes, writing 0 or 1 onto the tape. Suppose now that the inhabitants begin a series of experiments in which they push the red button on the machine a large number, N, times, at regular intervals. Suppose too that the inhabitants believe (correctly) that this is a series of independent identical experiments, and moreover believe this *dogmatically*: no pattern in the data will shake their faith. Suppose also that they believe (incorrectly) that their multiverse is governed by a many-worlds theory with unknown weights attached to the 0 and 1 outcomes; identical in each trial, and seek to discover the (actually non-existent) values of these weights.

"After N trials, the multiverse contains 2^N branches, corresponding to all 2^N possible binary string outcomes. The inhabitants on a string with pN zero and $(1-p)N$ one outcomes will, with a degree of confidence that tends towards one as N gets large, tend to conclude that the weight p is attached to zero outcome branches and weight $(1-p)$ is attached to one outcome branches. In other words, everyone, no matter what string they see, tends towards complete confidence in the belief that the relative frequencies they observe represent the weights.

"Let's consider further the perspective of inhabitants on a branch with pN zero outcomes and $(1-p)N$ one outcomes. They do not have the delusion that all observed strings have the same relative frequency as theirs: they understand that, given the hypothesis that they live in a multiverse, *every* binary string, and hence every relative frequency, will have been observed by someone. So how do they conclude that the theory that the weights are $(p, 1-p)$ has nonetheless been confirmed?. Because they have concluded that the weights measure the *importance* of the branches for theory confimation. Since they believe they have learned that the weights are $(p, 1-p)$, they conclude that a branch with r zeros and $(N-r)$ ones has importance $p^r(1-p)^{N-r}$. Summing over all branches with pN zeros and $(1-p)N$ ones, or very close to those frequencies, thus gives a set of total importance very close to 1; the remaining branches have total importance very close to zero. So, on the set of branches that dominate the importance measure, the theory that the weights are (very close to) $(p, 1-p)$ is indeed correct. All is well! By definition, the important branches are the ones that matter for theory confimation. The theory is inded confirmed!

"The problem, of course, is that this reasoning applies equally well for all the inhabitants, whatever relative frequency p they see on their branch. All of them conclude that their relative frequencies represent (to very good approximation) the branching weights. All of them conclude that their own branches, together with those with identical or similar relative frequencies, are the important ones for theory confirmation. All of them thus happily conclude that their theories have been confirmed. And, recall, all of them are wrong: there are actually no branching weights."

This argument from Kent completely destroys Everett's attempt to derive the Born rule from his many-worlds approach to quantum mechanics. In fact, it totally undermines most attempts to derive the Born rule from any branching theory, and undermines attempts to justify ignoring branches on which the Born rule weights are disconfirmed. In the many-worlds case, recall, all observers are aware that other observers with other data must exist, but each is led to construct a spurious measure of importance that favours their own observations against the others', and this leads to an obvious absurdity. In the one-world case, observers treat what actually happened as important, and ignore what didn't happen: this doesn't lead to the same difficulty.