Compresence and Coalescence

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I. Introduction

In this paper I shall review the paper "The ambiguity of 'one' and 'two' in the description of Young's experiment" by Henri Bortoft (Bortoft 1970) in the light of the present day quantum model for physics and in terms of points of view about recursions and second order cybernetics. Bortoft's work is linguistic and phenomenological, directed at description and how that description may be related to the observations that are possible in the experimental arrangement. The issues raised by Bortoft are in parallel with considerations of quantum physics and they shed light on quantum and cybernetic epistemology. The distinction between compresence and coalescence is central to Bortoft's work and to this paper. We shall describe this distinction below and then from a number of points of view.

A word about Henri Bortoft: The paper we concentrate upon in this essay was written in 1970. It was preceded by (Bortoft, 1966) and followed the next year by the paper(Bortoft, 1971) "The whole: counterfeit and authentic". This paper is, in the opinion of this author an important companion piece to the 1970 paper and one of the deepest evocations of wholeness that I have encountered. Bortoft wrote a Master's Thesis on the philosophy of quantum theory in 1982 (Bortoft, 1982). This thesis concentrates on the Bohmian theme of "Wholeness and the Implicate Order" and its relationship with Spencer-Brown's "Laws of Form" (Spencer-Brown, 1969), but does not hark back to the paper of 1970 on Young's experiment. After that Bortoft devoted his career to a study of wholeness in relation to the work of Goethe as can be seen in (Bortoft, 1998 and 2012). The papers of 1970 and 1971 appear as the seeds of all his later work.

A key to quantum mechanics is the principle that *trajectories indistinguishable to an observer can give rise to interference at the point of observation*. A key point in the work of Bortoft is that the point of observation giving rise to interference in Young's experiment is that place where the distinction between the two slits is indistinct for the optical observer. Paths from the two slits to that point are not distinguished by the observer. In Bortoft's phenomenology the observer is in coalescence with the observing apparatus and it is in this coalescence that the distinction is not present.

A key point in the foundation of the logic of recursion is that self-reference can arise when the operator of self-action is applied to itself. This application of an operator to itself can be seen to be a description of the act of coalescence where what is seen by an observer is determined by the connection of the observer to the act of perception. In the act of making a distinction the very boundary of that distinction can come to stand for the distinction itself. The boundary makes the distinction and in this sense is the distinction. The boundary stands for the distinction and in this sense refers to the distinction. The boundary is sense and it is reference. Where sense and reference coalesce, the observer comes into being.

The world of actualities is, in the language of cybernetics, a world of eigenforms (Kauffman, 2005a, von Foerster 1981b). It is a world of objects that remain what they are when they are observed and yet the very process of observation can call them into existence. The world of quantum mechanics comes into contact with the world of actuality when a measurement produces an eigenstate, a special eigenform that meets the requirements of a physical model based in possibility. Here we examine the place where eigenstate and eigenform come together.

In this essay we will explore all of these points of view and discuss their relationships. They are not disparate, but the apparent necessity for clarity in scientific discussion has often separated them. Here we make a beginning in bringing these points of view together.

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II. Quantum Mechanics

Quantum Theory is a radical method for modeling and obtaining information about physical processes that was discovered in the early part of the twentieth century. It continues to be the most powerful physical theory presently known, and remarkably can be described very simply. I will give a capsule summary of the theory. While it is not so hard to grasp the essentials of this theory, it uses principles that are different from the way we have been conditioned to think about the world.

We begin with an observer. In a cybernetics context this beginning is natural since cyberneticians accept that everything is said by an observer, and that all phenomena are actual only in the presence of an observer. In cybernetics we conceive that an observer and something seen by that observer arise together in a pair - observer/observed.

In the terms of this essay, the observer/observed pair is a *coalescence*. In looking through a telescope at the moon, the observer is in coalescence with the telescope and the moon. The moon seen is not independent of the position of the telescope relative to itself and the observer. Move the telescope to the right by a foot, keeping the observer fixed, and the relationship of the moon and the observer changes radically. The opposite of coalescence is *compresence,* where two things can be independently in the sight of the observer, and neither of them is integral for his observation of the other. See the discussion in Sections III and IV for Henri Bortoft's use of these terms. In our cybernetic, semiotic, phenomenological point of view we do not usually consider the condition of a world prior to or independent of observation. The world is not seen as independent of the observer. The observer participates in the creation of the world.

In classical physics, models are constructed to describe the evolution of a causal world that is independent of any particular observer. Then one can insert observers into such a world. With an observer present, the classical models explain, indicate or predict what will be seen.

The quantum mechanical model invokes the deterministic evolution (via the Schrodinger equation) of a physical state **|psi>** that is a *superposition of possible observations.* This state is sometimes called the wave function. In fact, the wave function, being a mathematical entity is neither a particle nor is it a wave. It can model both particle-like and wave-like properties of the quantum phenomena. This wave function, a superposition of possibilities, evolves in time.

An observation or *measurement* reduces the state **|psi>** to *exactly one of its possibilities*. Thus a measurement produces an actuality, a definite result in the world of the observer. Physical states can interact with one another independent of an observer. The key mode of combination of states is *addition* where **|psi> + |phi>** is a new state that is a superposition of all the possibilities in the individual states **|psi>** and **|phi>.** Interference can occur in such a summation so that possibilities in one state are cancelled by possibilities in the other.

For example, consider a state **|psi> = |0> + |1>**. Here **0** and **1** stand for two distinct possible observations. We leave exactly what they might be to your imagination. The state |**0> + |1>** is a superposition of the possibilities **0** and **1**. The superposition is not **|0>** and it is not **|1>.** When you observe **|psi>**, you will obtain either **|0>** or **|1>,** but not both.

If **|phi> = |0> - |1>**, then

|psi> + |phi> = |0> + |1> + |0> - |1> = 2 |0>.

So when you observe **|psi> + |phi>** there is no possibility that you will see anything but **|0>**. The possibility for **|1>** has been erased by a destructive interference, just as waves on water, or light waves, can interfere to either add intensity or subtract intensity.

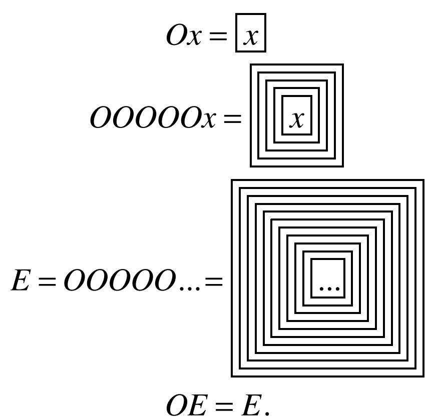
If the possible outcomes are **|0>, |1>**,…, **|n>,** then a state of the system is of the form

$$|S\rangle = z_1|0\rangle + z_2|1\rangle + \ldots + z_n|n\rangle$$

where z_i are complex numbers and $|z_1|^2 + \ldots + |z_n|^2 = 1$. Letting **O** denote the operation of observation one has that $O|S\rangle = |k\rangle$ for some **k** with probability $|z_k|^2$. The probability of observing a particular state is the absolute square of its coefficient in the wave function **|S>.** The new observed state is then an eigenstate. This means that **|k>** is not changed by a further (immediate) observation. We have $O|k\rangle = |k\rangle$ at the next instant. Observation leads to eigenstates in the sense that we produce entities **E** such that **OE = E**.

Recursion can also lead to eigenstates. Formally, if we desire an **E** such that **OE = E** , we can obtain it by forming **E = OOOOO…,** the infinite concatenation of the operator **O** upon itself. Then **OE = E** for the infinite composition. Here we see the result of the observation arising by recursion quite in analogy to the way the interference pattern arises for Bortoft in his phenomenological thought experiment (as we shall see in Sections IV and V).

A diagrammatic example of such an infinite eigenform is shown below.



We may think of the formation of a generalized eigenstate by recursion as the formation of an object for perception or cognition. An *object* is an entity that does not change under the effect of observation, and so if **E** is an object, we expect that **OE = E** where now **O** stands for a general process (not necessarily numerical) of observation made by a human observer. Thus when we view a tree in the forest it remains, for us, a tree and we find stability in both the naming of it as a tree and in the perception of the tree as a whole, and of its parts and their fitting into the whole. Von Foerster (von Foerster 1981b,c,d) suggested in his title "Objects as tokens for eigenbehaviours." that what we call objects have in back of them a recursive process whose stabilization is the perception of the object for a given observer. In some instances we are quite aware of such a process as in what we see when standing between two mirrors. In other situations the objects, for example - a familiar lamp on the desk, appear simply to have presence for the observer.

In a quantum experiment, the state of the system is a summary of the information known about the system. Thus we may have a state of the form

$$|S> = (|Up> + |Down>)/\sqrt{2}$$

where **Up** and **Down** denote two quite opposite possibilities. In the famous Schrodinger's Cat thought experiment, these two possibilities are that a cat is alive or dead. Before measurement, the physical state of the system is the superposition above. The cat is neither alive nor dead. The cat is in a superposition of these states. It might be thought that at least an observer **O** would resolve the difficulty, but alas consider (as did Wigner) that there could be another observer **O^** who does not see the result of **O**'s observation. Then for **O^** the system is in a new superposition

$$|S'> = (|O,Up> + |O,Down>)/\sqrt{2}$$

and it is only when **O^** makes her further observation that she can know Up from Down. Of course we have avoided the notion that **O^** might receive a report from **O** and other complexities.

Note that we might say that our knowledge of another observer is in a superposition of possibilities. But we do not say that our knowledge of a physical actuality is in a superposition of possibilities. We assume that our knowledge can be resolved to definite facts. This is, of course, a modus operandi for doing science. Someone may say that the superpositions are places where our knowledge cannot be so resolved. We prefer to say that only the measurement has actuality in its definiteness and factual nature.

Quantum information does not become actual information until it is finally encountered/measured by a specific human observer.

In speaking of quantum observation, there are two components to that observation. There is the measurement that takes the superposition of states to one particular state of a physical system. And there is that measured state as registered by a human observer and seen as an object, for example as a dot on a phosphor screen or a mark upon a photographic plate.

The quantum model bifurcates into the deterministic Schrodinger evolution of the states, combined with the re-setting of the state to only one of its possibilities by acts of measurement.

We end this section with one more example – the Mach-Zehnder interferometer. View 41. The diagram in this , shows the Mach-Zehnder interferometer to be a device made with two types of mirror, a half-silvered mirror, that we shall refer to as H, is depicted by a white rectangle. An ordinary mirror, that we shall refer to as M, is depicted by a black rectangle. Single qubit (quantum bit) states enter the half-silvered mirror on the left of the device. The half-silvered mirror reflects a |0> to a |1> and reflects a |1> to a -|0>, changing the phase in this case. H transmits |1> to |1> and |0> to |0>. The ordinary mirror M just flips |0> to |1> and flips |1> to |0>.

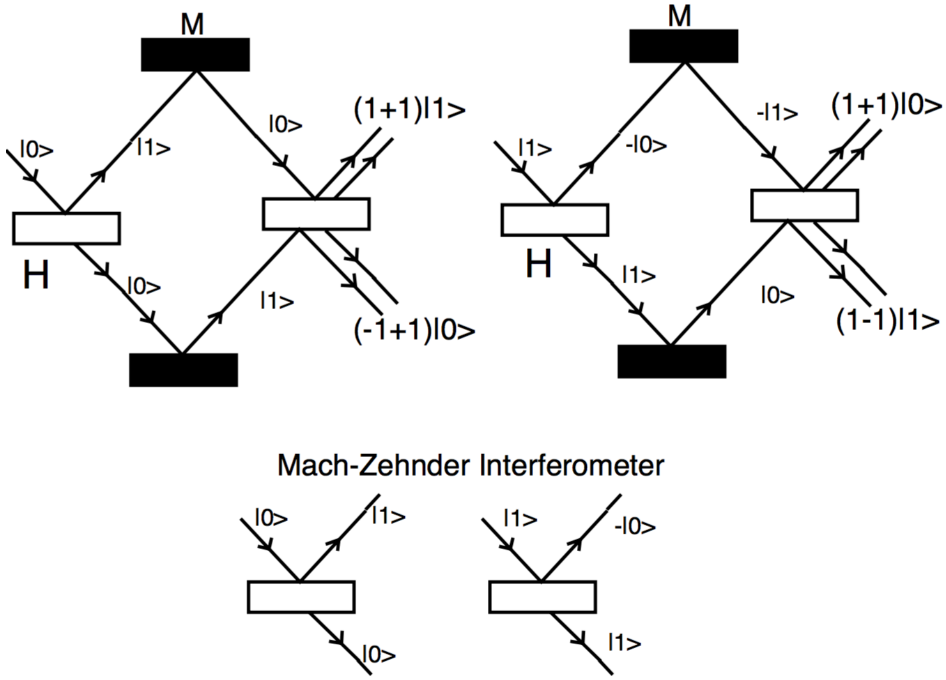


Figure 1 – The Mach-Zehnder Interferometer

The mirrors H and M represent quantum processes, and a mathematical representation of them is given by

$$H\,|0> = (|0> + |1>)/\sqrt{2}$$
$$H\,|1> = (-|0> + |1>)/\sqrt{2}$$
$$M\,|0> = |1>$$
$$M\,|1> = |0>$$

where the summation indicates that that H produces a superposition of states |0> and |1>. The superposition means that an observer of the state of the half-silvered mirror with input |0> will detect either |0> or |1> with equal probability.

The entire interferometer corresponds to the quantum process of first doing H, then doing M, and then doing H. The end result of a preparation of |0> or of |1> is illustrated in the Figure. You can follow the possible paths of the particle through the interferometer. There are a total of four paths, and with input |0> you can see from the diagram that two of them cancel at the top part of the diagram and the other two reinforce one another at the bottom. The conclusion is that for |0> as input, the interferometer will only show |1> as output. It will not be possible to detect |1> at the end of the process. Similarly with |1> in, only |0> will be detectable.

The key point about the interference that occurs in the interferometer is that the measurement that takes place at the end of the process can not involve any discrimination among the possible paths that the particle could take to get through the device. If the observer who makes the measurement had put a detector somewhere inside the interferometer to find out if the particle went on a preferred path, this would completely change our calculation of the contributions of all the paths, and we would get a different answer. For example, suppose that the measurement included a detector at the lower mirror. Then paths going through the lower mirror would be stopped at that mirror, and you can see from this that the detection at the right hand side of the interferometer would come out differently. It would be possible to detect either |0> or |1> while before one of then was forbidden.

The basic principle of quantum mechanics is that if one considers, at a point of observation, the contribution of a collection of paths, then differences among these paths must not be detectable by the observer. From the point of view of the obsever a the multiplicity of paths can only be a unity.

This two part model of quantum mechanics separates the deterministic evolution of the wave function and the measurement, the resetting of the wave function to an eigenstate at the point of observation. The separation is inevitable. The measurement corresponds to the making of a distinction and it is intertwined with the coalescence of an observer with the knowledge of the measurement. From this point of view we see that an exploration of the eigenform creations of the observer is worth the pursuit, and may shed light on the relationship of these two essential parts of the quantum model.

III. Bortoft – A First Look at Young's Experiment

Here are a pin and a lens in compresence. They are each objects and they are related to one another by their proximity in space to one another.

Here below are a pin and a lens in coalaesence. The observer sees the pin through the lens. The pin is seen by the observer through the intermediary of the lens.

To underline the essential difference between these two states, note that the same external relation of pin and lens could result in the pin appearing upside down if the lens were concave rather than convex. The condition of coallesence gives the observer a view that is dependent upon the structure of the coalescence.

In performing the double slit experiment the observer (in Bortoft's description of the optical version of the experiment) is in coalescence with an optical telescope and the slits through which the photons emerge. See Figure 2.

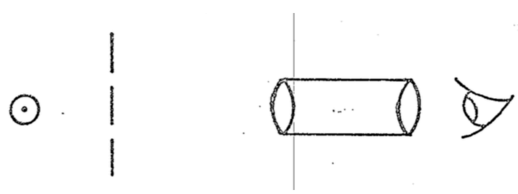


Figure 2 – Young's Optical Experiment

This coalescence may have the property that the observer can no make a distinction between the two slits. At this point the interference pattern emerges. For the optical version of Young's experiement this is the way of its world and Bortoft can state (our paraphrase of his conclusions):

(i) *Young's optical experiment has never been described.*
(without the correct discrimination of compresence and coalescence).
(ii) *Young's optical experiment can never be described in a language with the numerial singular/plural distinction.* (That is, one must have the (plural) slit in the

compresence of the optical bench, but a single slit in the coalescence of the observer.)

Neither Bortoft nor this author can state that this confluence of One and Two causes the interference pattern. We can only observe that it is at this point, this nexus, that the interference happens. As we point out in the next section, this special place of NotOne/NotTwo can be expressed by a symbolic and self-referential fixed point P=[PP] whose associated recursion does indeed look like an interference pattern. In the next section we will discuss more about this aspect of the analysis.
Here I wish to ask further questions about the role of compresence and coalescence. In order to do this, let us move from Young's optical experiment to the quantum mechanical double slit experiment with electrons.

Now to the double slit experiment. Instead of a source of light one can take a source of electrons, and in the modern version of the experiment one can configure the system so that one electron at a time moves through the system. It is a figure of speech to say that the electron "moves through the system" since one only knows that one has an electron when it is measured for example as an excitation on a phosphor screen. Thus the eye and tube of the Young's experiment is replaced by a screen when the electrons can be detected. See Figure 3.

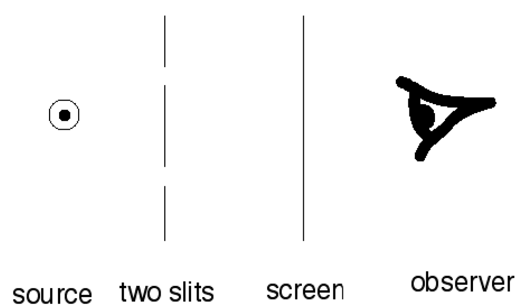


Figure 3 - Double Slit Experiment with a Screen.

In this form of the experiment, single events build up on the screen over time. After some time the pattern of the events on the screen appears as in the Figure 4 taken from the well known Hitachi version of this experiment (Tonomura, 2015).

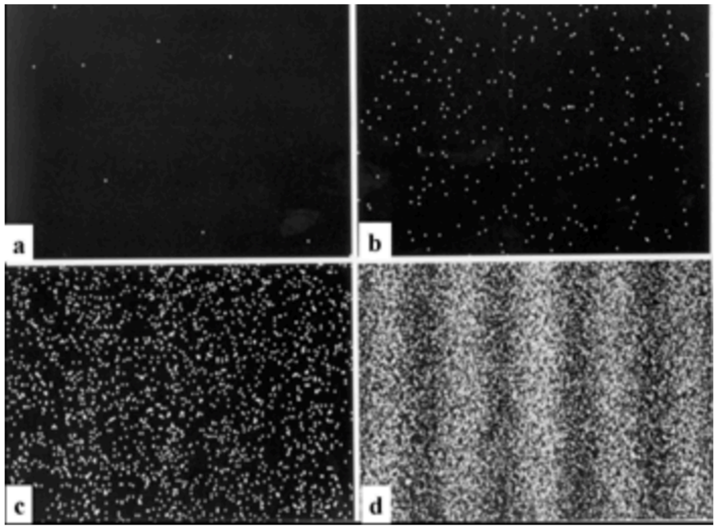


Figure 4 - Hitachi Double Slit Experiment

In the Hitachi photographic record the screen is blank at (a), the initial time of the experiment. Then at (b) we see some pattern of electron events on screen as dots. In (c) more dots are produced and in (d) we see the very remarkable final pattern of the experiment with its apparent interference pattern on the screen.

Indeed the experiment is arranged so that the observer in Figure 4 can never tell whether any given electronic event (dot on the screen) originated at one slit or the other. Furthermore the pattern that is eventually seen by the observer has been built up over time. It would appear that each electronic event on the screen is independently contributing to this interference pattern. The quantum mechanical model (in the usual interpretation) only gives the probability that electrons will appear on the screen at certain points. The statistics of these probabilities do predict the interference pattern, based on the geometry of the paths from the slits to the screen.

Return to Coalescence and Compresence

As we have seen, the standard construction and description of the electronic double slit experiment allows for the compresence of all the elements of the experiment including the screen. The observer need not stand looking at the screen throughout the duration of the experiment. Instead she can wait until the end and then view the screen in the state (d) of Figure 4. This view constitutes a coalsesence with the data and results in the perception of an interference pattern when the experiment has been set up so that no information is available about electrons going through or coming from given slits.

The coalescence of the observer with the screen and the experiment is available to any observer who would care to look at the screen. With this description of the modern legacy of Young's experiment, we can probe further the relationship of Bortoft's fixed point, self-reference, reentry at the point of NotOne/NotTwo and the emergence of quantum interference.

One interpretation of this interchangeability of observers is that physics is only concerned with compresence and not with coalescence. The definite and unchangeable data at level (d) in Figure 4 is available for the examination of any observer. The experiment has been performed in such a way that no distinction was possible to select one slit over the other. Quantal events happened individually at the screen separated by time intervals that show that only one quantum of energy was present in the system during each interval. All of this preparation for the data at level (d) required the laboratory observers who set up the experiment and constructed the equipment. But indeed, this part of the work is repeatable as well, and the apparatus can, after all that work, be set up so that the experiment goes forward just at the flip of one switch.

Nevertheless, the actual observing of the screen is eventually necessary. We have taken the screen to be the final placement for the human observer. It could have been more distant if we had constructed an artificial intelligence to scan the screen and test it for an interference pattern. Then the observer would only see a yes or a no, or a light that was lit of not lit. The observer would only see a mark or the absence of a mark. And even so the observer must in the perceiving of that mark be in coalescence with the mark.

This analysis shows us that the end place of any experiment will be a coalescence, a place where the observer and the mark of distinction are in the form identical. That place of identity is a self-reference and it can give rise to an interference pattern at the level of this cognition. But we have argued that in this modern version of Young's experiment, the observer can be insulated (by automatic pattern recognition) from the phenomenology described by Bortoft for the optical Young's experiment.

In the modern version of the double slit experiment there remains the activity of the observers. Each observer sees a world in coalescence relative to his or her own being. Pattern recognition is necessary at all levels of an experiment, and even when the observation is an apparently binary one a distinction must be made by the observer. Thus we see that the fundamental property of the quantum model is that it does depend upon a stable observation, a distinction on the part of the observer.

IV. Bortoft – The Two Slit Experiment and the Interpretation of Quantum Mechanics

Bortoft (Bortoft 1970)suggested that the interference pattern that arises from Young's optical two-slit experiment is the result of the observer being placed so that *two* (the two slits) *is indistinguishable from one* (the undetectability of the slit through which the electron did pass).

Place the observer at a distance where it is not possible to resolve the slits.
This is where the interference occurs.

Something curious is going on here. First of all, it is a fact of standard quantum mechanics (as we have explained in Section 2) that if we take all the trajectories that a particle may take from a point A to a point B, and regard all these trajectories as *indistinguishable* to an observer at B, then there is a way (the Feynman path summation or integral) to add up complex number contributions of all the disparate paths to find the interference of them at B, and to obtain the probability of finding a particle at the point B as the absolute square of this summation. Thus Bortoft's principle is directly related to a basic principle of quantum mechanics.

The interference that Bortoft suggests is one that arises from the self-referential recursion of setting "one" equal to "two" as in a formal equation

$$P = \prec P \mid P \succ$$
$$= \prec\prec P \mid P \succ \mid \prec P \mid P \succ\succ$$
$$= \prec\prec\prec P \mid P \succ \mid \prec P \mid P \succ\succ \mid \prec\prec P \mid P \succ \mid \prec P \mid P \succ\succ\succ$$
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As the reader can see, illustrated above, we begin with P as "one" and equate it to a pair of copies of P. Then recursive substitution of the equality

$$P = \prec P \mid P \succ$$

leads to a pattern that we may call the interference pattern of "one = two".
Could this be a formal relative of the physical interference pattern of Figure 4 for the Hitachi double slit experiment?

Bortoft suggests that his recursive pattern is in back of the phenomenon of the Young's double slit experiment. It is not clear how to quantitatively relate the Bortoft recursive pattern with the path sum pattern of the quantum mechanics.
It is Bortoft's suggestion that the recursive and eigenstate properties of the observer finding objects as tokens of eigenbehaviour and the special quantum process of collapsing a superposition to a specific measurement are two sides of one coin.

The peculiarity is not over. We have to confront the question: Who or what is an observer? How is this question informed by Bortoft's discussion of the phenomenological, recursive observer?

What constitutes the knowledge of the observer?
The observer knows that he knows a given item of knowledge.
We shall handle this analysis in a schematic form.
Let us take the well-known quote of Heinz von Foerster (von Foerster 1981):

" I am the observed relation between myself and observing myself."

Let "observing X" be denoted by

$$\boxed{X}$$

and let XY denote "the relation between X and Y".
Then we can write von Foerster's quote directly and symbolically as

$$I = \boxed{I\boxed{I}}$$

and we see that this equation about the self is a direct relative of the Bortoft equation about P.

$$P = \prec P \mid P \succ$$

In the von Foerster statement the self occurs within itself in two levels as the I and the observed I. It is further implicit that the I observes itself. Indeed the von Forester sentence expresses the recursive self-interference that is, in form, identical with a self. We now see that Bortoft implicitly suggests the confluence of the domain of the self as observer and the domain of quantum observation.

It is necessary to see the context of the quantum model. One way to see it is to regard the world of actuality as a world of objects in the sense of eigenform (invariant under the act of observation), and that physical experiments are made in this world in a repeatable way that produces results that are recognizable as objects, such as a mark on a plate or a reading on a meter. Then it happens that certain experiments produce patterns in this world of objects that are fitted well by the quantum model. No interpretation of a "quantum world" is given. It is only that the method of complex superposition and probabilities as absolute squares of complex amplitudes is seen in many cases to give results that are accurate and predictions that are correct.

In this discussion we can take a second look at the Copenhagen interpretation (described above) and take the world of objects from the von Foerster viewpoint. Then each object, each distinction, each distinct entity is an eigenform, an eigenstate of a generalized operator that is, in form, identified with a human observer. That object, if modeled by the quantum model, then comes to have two eigenstates associated with it. One is the perceptual cognitive von Foerster state. The other is the eigenstate that resulted from the collapse of the superposition that described its quantum possibility.

Here, is our description of the dilemma. How does it come about that the quantum model with its eigenstates fits so well into the apparently more general world of the eigenforms and objects as tokens for eigenbehaviour. Here is a new possibility for reformulating the Copenhagen interpretation of quantum mechanics.

There is much to think about here. In the present paper we start, just with the notion of distinction and we unfold patterns that are related both to physics and to the understanding of recursion and re-entry. One can think of the present essay as a reflection on Bortoft's suggestion about the Young's double slit experiment.

V. Laws of Form, Re-entry, Self-Reference and the Structure of the Precursor

Laws of Form (Spencer-Brown 1969) is coextensive with the idea
that the world and existence arise from nothing (no thing).
Non-existence in itself does not exist.
The act of apparent distinction brings forth apparent existence.

Anything can arise from nothing, but a first distinction that would arise,
being first, can have no difference between its sides without further distinction and so is not a distinction.

Being not a distinction it
Has no being, and so
disappears,
and again there is nothing.

This connotes a basic oscillation of the void.

If another distinction should occur beyond the first
(and how could it not?) then
Pandora's Box has opened.

One way to see how recursion/oscillation arises is to begin with the following operator J

$$JX = \boxed{XX}$$

When J is applied to any entity, it produces two copies of that entity. If there exist other entitities than J itself, then this is a prosaic occurrence of two compresent copies of that entity. If we apply J to itself, then we have

$$JJ = \boxed{JJ}$$

and now JJ is creative, producing a distinction around itself.

We have entered into self-reference by taking J to be an operator of self-interaction. J applies to X to give the action of X on itself. When we apply J to itself, J interferes with itself to produce recursion and self-reference. The combination JJ is a *coalescence* of J with J and produces a unique and singular result, just as the coalescence of awaress with itself is the state of awareness.

The Universe is constructed in such a way that it can refer to itself.
In so doing, the Universe must divide itself into a part that is seen and
part that sees. Here we could have taken U = UU, so that UU produces UU and UU collapses to the unity U. The universe becomes a duality that is a unity.

The Universe divides itself into two identical parts each of which refers to the universe as a whole. The universe can pretend that it is two and then let itself refer to the two, and find that it has in the process referred only to the one, that is itself.

The Universe plays hide and seek with herself, pretending to divide
herself into two when she is really only one.

In Section II we have indicated that we can always produce a solution to an equation OE = E by taking E to be an infinite concatentation of O upon itself. There is another way that avoids infinity, but one must allow an entity to act upon itself.

We define

$$Jx = O(xx).$$

Then

$$JJ = O(JJ)$$

and so we let E = JJ and we have O(E) = E.

We have used *a precursor* to the eigenstate E in the form Jx = O(xx).
The precursor to the self-reference or re-entry acts to make a pair of identicals acted upon by the given operator. Into this is inserted the structure J *as a whole*, and the self-reference, re-entry, recursion is the result.

In the quantum mechanical model, a superposition is observed and projects to a specific state that is then observed as that state. At that point of observation, the state has acquired the definiteness of an eigenform, in the moment of observation.

The buck stops with the observer. The observer is a *knower*, a system capable to produce an eigenstate in its knowing of itself as not one/not two.
"I am the observed relation between myself and observing myself."

VI. Discussion

In this essay we have discussed eigenstates as they occur in quantum mechanics where a measurement occurs and there are states **|k>** stable under observation: **O|k> = |k>.** We have pointed out that given any form of observation, there is a natural way to produce an eigenform for that operation, either by concatenating it upon itself in indefinite recursion, or via the precursor **Jx = O(xx),** yielding

JJ = O(JJ).

The precursor construction is a formal model of the emergence of recursion from coalescence since it is the coalescence of **J** as an operator with **J** as an operand that produces the fixed point, the eigenform, **JJ = O(JJ).** The condensed mystery of this fixed point is close to the deeper mystery of our own talent of self-reference and knowledge in observation. We have seen that these forms of observation weave inextricably with the results of physics where indistinguishable trajectories lead to interference patterns at the point of observation. We have seen that the work of Bortoft continues to contribute to this discussion.

In this essay we have indicated that the relationship with oscillation is fundamental because the emergence of a distinction is necessarily related to oscillation. A first distinction requires further distinctions in order to stabilize. Thus in the limit of the emergence from a realm of no-thing, there will be primordial oscillation. It is the structure of oscillation that we have followed in this essay both in the form of Young's double slit experiment and in the structure of distinction.

We have reached the end of this essay. This work harks back to the beautiful papers of Henri Bortoft [Bortoft, 1970, 1971] where he identifies the zero-one oscillation as the condition of an observer who is placed in a condition where he cannot distinguish the whole from the part. It was Bortoft's intuition that this (in the context of Young's double slit experiment) was the nexus and source of the quantum interference. All the ideas from beginning to end are related to one another. The relationships we have articulated are but a hint in the further articulation of the possibility of a distinction.

Remark. The references contain a number papers, all relevant to this discussion as background to its different dimensions.

VII. Appendix on Laws of Form

In this section I will review the ideas behind G. Spencer-Brown's calculus of indications (Spencer-Brown (1969)). The Calculus of Indications (CI) is based on a single symbol and called the *mark*. We shall use the Spencer-Brown form of the mark:

⏋.

In this form, you should think of the mark as a shorthand for a box:

□.

A box has a definite inside and a definite outside in the plane upon which it is drawn, and it is seen to distinguish the inside from the outside. In the same way, the Spencer-Brown mark distinguishes an inside from an outside.

The mark can be seen as the boundary of a distinction and the mark can be seen as that which forms the distinction. The mark can be seen as a symbol of the very distinction that it makes. In this sense the mark is self-referential, and with the participation of the observer, the mark is in coalescence with itself and with the distinction that it makes. Here meaning arises.

What we have said about Laws of Form up to this point is sufficient for the themes of this essay, but the appendix will continue with a concise exposition of the calculus that comes from these considerations of the mark, and how that calculus is related to the production of a J such that JJ = <JJ> as we have described in the body of the paper.

The Calculus of Indications
A plane space with a mark drawn upon it is said to be *marked*. The reference is to that part of the plane that is outside the mark. The inside of a mark is empty and is said to be unmarked. Thus we have

$$\overline{}| = \overline{u}|\,m$$

where it is understood that "u" stands for the unmarked state (the empty inside of the mark) and "m" stands for the marked state (the outer space of the mark is marked by the very presence of that mark in the space).

In this way, we see the *law of calling:*

$$\overline{}|\overline{}| = \overline{}| \,.$$

The presence of two marks in the outer space of a mark makes that space marked no more than the presence of a single mark. With respect to markedness, two adjacent marks indicate the same state as one mark.

We make the following choice:

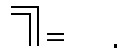
$\overline{a}|$ *denotes the state obtained by crossing from the state indicated by* a.

Note how this works.

$\overline{}|$ denotes the state obtained by crossing from the unmarked state.
Hence $\overline{}|$ denotes the marked state.

$\overline{\overline{}|}|$ denotes the state obtained by crossing from the marked state.
Hence $\overline{\overline{}|}|$ denotes the unmarked state.

We shall write the *law of crossing:*

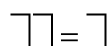


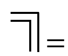
We allow two nested marks, with the innermost mark empty, to vanish from the notational plane. An apparent distinction, transfixed by the absence of any difference between its sides indicates nothing.

With this interpretation of the mark as a transformation from the state indicated on its inside to the state of its outside, we obtain clarity of evaluation. The mark is seen as making a distinction in the plane, as indicating the outside of the distinction that it makes, and as a transformation from the state on its inside to the state on its outside. All three of these interpretations are mutually compatible and compatible with the creation of a first distinction from nothing.

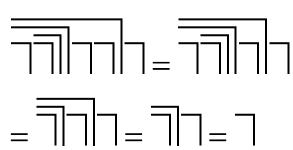
One watched carefully for a distinction to appear, capturing it in a plane space where its sides would be distinct. Without those actions, the distinction, like a fold in a silk scarf, would vanish as quickly as it had come forth. From whence came this apparent ability to capture evanescent events? This is a mystery in the shadow of nothing. Waiting for the next thought.

And so we have an arithmetic, without counting, that is generated by the laws of calling and crossing.

Calling: 

Crossing: 

It is an arithmetic in the sense that we can now calculate the value (marked or unmarked – only two values) of more complex expressions of distinction. Expressions in the mark are patterns of distinction so that any two marks in a given expression are either nested or adjacent to one another. Finite expressions can be reduced by calling and crossing uniquely to either the marked state or to the unmarked state.
For example:



The reader will observe that each change is mediated by an application of either calling or crossing. See (Spencer-Brown (1969)) for the proof of reduction and uniqueness.

The arithmetic that we have constructed (Spencer-Brown's *primary arithmetic*) is no ordinary counting arithmetic. Once the laws of calling and crossing are in place, every expression has only one value, marked or unmarked and that value is uniquely determined by reducing the expression as has been indicated.

Furthermore, in the context of the arithmetic the mark can be seen as an operator. If E is an expression in the arithmetic, then $\overline{E}|$ is another expression, and we find that whenever E is unmarked, then $\overline{E}|$ is marked. And whenever E is marked, then $\overline{E}|$ is unmarked, by an application of the law of crossing. Thus the mark, as an operator, is analogous to logical negation.

Recursion and Self-Reference

Having understood this structure of distinction and the basic interpretation of the mark as both operator and operand, it is natural to point out that there are ways to go into the inbetween of neither marked nor unmarked. One way is to consider an apparently paradoxical equation such as $G = \overline{G}|$. For then the equation asserts itself to be marked when unmarked, and unmarked when marked. It is tempting to construct an infinity of nested marks as in $G = \overline{\overline{\overline{\overline{\ldots}|}|}|}|$ and then note that placing one more mark around the outside will no longer make a difference, hence $G = \overline{G}|$. This is the most elemental eigenform that occurs in this story. We have in this essay pointed out another way to make such a reentering mark by defining an operator J so that $Jx = \overline{xx}|$, and then $JJ = \overline{JJ}|$ and we could take G = JJ. The verification that $JJ = \overline{JJ}|$ happens so quickly that one feels that some magic trick has been performed.

In this essay we have seen that this magic trick is a formal image of the distinction of between compresence and coalescence. The two identical J's in the equation $JJ = \overline{JJ}|$ are in coalescence with each other with one acting on the other.
In the interior world of an individual one acts on oneself.

References

Spencer-Brown, G. (1969) . "Laws of Form"; George Allen and Unwin Ltd.: London, UK, 1969.

Bohm, D. (1980), "Wholeness and the Implicate Order", Routledge & Kegan Paul, London.

Bortoft, H. (1966), The resolution by a rigorous descriptive method of some dilemmas in physical science, Systematics, Vol. 4, September (1966), pp. 93 – 134.

Bortoft, H. (1970), The ambiguity of 'one' and 'two' in the description of Young's experiment, Systematics, Vol. 8, December (1970), pages 221-224.

Bortoft, H. (1971), The whole: counterfeit and authentic, Systematics, Vol. 9, September (1971), pages 43-73.

Bortoft, H. (1982), A non-reductionist perspective for the quantum theory – Masters Thesis, Birkbeck College, London, UK.

Bortoft, H. (1998), “Goethe’s Scientific Consciousness”, Institute for Cultural Research, Monograph Series No. 22 (First published 1986), The Institute for Cultural Research, PO Box 2227, London NW2 3BW.

Bortoft, H. (2012), “Taking Appearance Seriously- the dynamic way of seeing in Goethe and in European thought”, Floris Books, 15 Harrison Gardens, Ediborough, Scotland.

Hiley, B. (2013) The arithmetic of wholeness, Holistic Science Journal, Vol. 2, Issue 2, March 2013. Published by Earthlinks, UK, ISSN:2044-4370, Pages 23 – 30.

Kauffman, L.H. (1978a). Network Synthesis and Varela's Calculus, International Journal of General Systems 4,(1978), 179-187.

Kauffman, L.H. (1978b). DeMorgan Algebras - Completeness and Recursion. Proceedings of the Eighth International Conference on Multiple Valued Logic(1978),. IEEE Computer Society Press, 82-86.

Kauffman, L.H. and Varela F. (1980). Form dynamics. Journal of Social and Biological Structures (1980), 171-206.

Kauffman, L.H. (1985a) Sign and Space , In Religious Experience and Scientific Paradigms. Proceedings of the 1982 IASWR Conference, Stony Brook, New York: Institute of Advanced Study of World Religions, (1985), 118-164.

Kauffman, L.H. (1987a). Imaginary values in mathematical logic. Proceedings of the Seventeenth International Conference on Multiple Valued Logic, May 26-28 (1987), Boston MA, IEEE Computer Society Press, 282-289.

Kauffman, L.H. (1987b). Self-reference and recursive forms. J. Soc. Biol. Struct. 1987, 53--72.

Kauffman, L.H. (1987c). Special relativity and a calculus of distinctions. In Proceedings of the 9th Annual International Meeting of ANPA, Cambridge, UK, 23--28 September 1987; pp.290--311.

Kauffman, L.H. (1988). Space and time in discrete physics. Intl. J. Gen. Syst. (1998), 241--273.

Kauffman, L.H. Knot Logic. (1994). In “Knots and Applications”; Kauffman, L., Ed.; World Scientific Pub. Co.: Singapore, 1994; pp. 1--110.

Kauffman, L. H. (1996). Virtual logic, Systems Research Vol 13 No. 3, pp. 293=310 (1996).

Kauffman, L. H. (2001). The mathematics of Charles Sanders Peirce. *Cybernetics and Human Knowing, 8*(1-2), 79-110.

Kauffman, L.H. (2002a) Biologic. AMS Contemp. Math. Ser. , American Mathematical Society (2002), 313--340.

Kauffman, L.H. (2002b) Time imaginary value, paradox sign and space. In "Computing Anticipatory Systems", CASYS--- Fifth International Conference, Liege, Belgium, 13--18 August 2001; Dubois, D., Ed.; AIP Conference Publishing: Melville, NY, USA, 2002, AIP Conference Proceedings Volume~627.

Kauffman L.H. (2003), "Eigenforms - objects as tokens for eigenbehaviours", in Cybernetics and Human Knowing, Volume 10, No. 3-4, pp. 73-89.

Kauffman L.H. (2005a), Eigenform, Kybernetes - The Intl J. of Systems and Cybernetics, 34, No. 1/2 (2005), Emerald Group Publishing Ltd, p. 129-150.

Kauffman L.H. (2009), Reflexivity and Eigenform -- The Shape of Process. - Kybernetes, Vol 4. No. 3, July 2009.

Kauffman, L.H. (2012a) "Knots and Physics"; World Scientific Pub., Co.: Singapore, 2012.

Kauffman, L. H. (2012b) Categorical pairs and the indicative shift. Applied Mathematics and Computation, 218:7989-8004, 2012.

Kauffman, L.H. (2016a). Knot logic and topological quantum computing with Majorana fermions. In "Logic and Algebraic Structures in Quantum Computing and Information"; Lecture Notes in Logic; Chubb, J., Eskandarian, A., Harizanov, V., Eds.; Cambridge University Press: Cambridge, UK, 2016; pages 223 – 335..

Kauffman, L. H., (2016b) Cybernetics, Reflexivity and Second Order Science. *Constructivist Foundations*, 11(3), 489-507.

Kauffman, L. H. (2017) Imaginary Values, **Special Issue on Laws of Form, Cybernetics and Human Knowing, Vol. 24, No. 3-4, 2017. Pages** 189-223

Tonomura, A. (2015) Quantum Measurement, Hitachi Corporation, https://www.hitachi.com/rd/research/materials/quantum/index.html

von Foerster H. (1981b) Objects: Tokens for (eigen)behaviors. In: Foerster H. von (1981) Observing systems. Intersystems, Salinas CA: 274–285. Originally published as: von Foerster H. (1976) Objects: Tokens for (eigen)behaviors. ASC Cybernetics Forum 8(3/4): 91–96.

von Foerster H. (1981c) Notes on an epistemology for living things. In: von Foerster H. (1981) Observing systems. Intersystems, Salinas CA: 258–271. Originally published as: von Foerster H. (1972) Notes on an epistemology for living things. Biological Computer Laboratory Report 9.3, BCL Fiche No. 104/1. University of Illinois, Urbana.

von Foerster H. (1981d) On constructing a reality. In: von Foerster H. (1981) Observing systems. Intersystems, Salinas CA: 288–309. Originally published as: von Foerster H. (1973) On constructing a reality. In: Preiser F. E. (ed.) Environmental design research. Volume 2. Dowden, Hutchinson & Ross, Stroudberg: 35–46.