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5.5 Reaction-diffusion dynamics: bifurcation and morphogenesis

In the case of two sources of noise operating selectively upon each other at an interface, the formation of a trajectory is possible because of a lock-in, while in the case of three a complex regime can be generated. In general, the recombination of three subdynamics enables us to generate the various species of chaotic behavior (Poincaré, 1905; Li and Yorke, 1975). However, the complex dynamics of three sources of historical variation can be expected also to contain lock-ins between two sources as relative stabilities. The third dynamic induces a life-cycle into these more organized densities. This configuration is consistent with the prediction of local and global optima using Kauffman's (1993) NK-model (Kauffman, 1993; Frenken, 2000, 2001), and was further elaborated in this other context (Leydesdorff, 2002a).

In the above models, the option of return to equilibrium was generated by historical events such as the advent of new technologies on the market. Historical events are exogenous to the evolutionary model; but my claim about the life-cycle induced by a third selection environment implies that return to equilibrium can also be considered as endogenous to the evolutionary model. May "break-out" from "lock-in" or, more generally, a co-evolution also be endogenous to the evolutionary model?

I turn to reaction-diffusion dynamics to understand this process. While the lock-in provides us with a mechanism to lose a degree of freedom in the system, the reaction-diffusion mechanism enables us to understand how a degree of freedom can be gained by a system endogenously. In Chapter One, I discussed this mechanism in intuitive terms, for example, with the example of a coupled process of co-evolution between production and marketing in a small enterprise (or in an industrial district). If the smaller production unit is in a next stage absorbed by a multinational corporation or otherwise internationalized, the tight coupling between production and marketing at the local level may become a constraint on its further development. Under the pressure of the global diffusion dynamics, one may then have to take reallocation decisions about the production system. Thus, while originally the linear sequence of production and then marketing prevailed, in a later stage the feedback of the market may increasingly reshape the production process from the perspective of hindsight.

Reaction-diffusion dynamics have been elaborated in the natural sciences (Rashevsky, 1940; Turing, 1952; Rosen, 1985, at pp. 182 ff.). However, these insights have hitherto not influenced the context of economics or other social sciences (Bruckner *et al.*, 1994). If two systems are tightly coupled (as in a co-evolution; see Figure 8), the simplest coupling mechanism can be specified by the following differential equations:

$$dx_1/dt = -ax_1 + D(x_1 - x_2) + S \qquad (8a)$$
$$dx_2/dt = -ax_2 + D(x_2 - x_1) + S \qquad (8b)$$

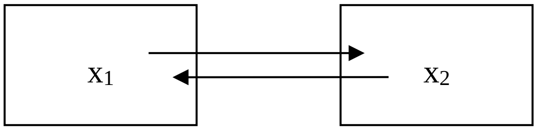

Figure 5.8: Two coupled processes. (Source: Rosen, 1985, at p. 183.)

Let us assume that x is produced in both compartments at a constant and equal rate *S*. The parameter *a* represents the decay of x; *D* is the diffusion constant across the membrane. (For the sake of simplicity, these parameters are assumed to be equal on both sides.) The diffusion is asymmetrical depending on the concentrations of x_1 and x_2 in the two compartments.

This system of equations provides a value for the steady state at:

$$x_1^* = x_2^* = S/a \tag{9}$$

The concentrations of x in the two cells are then equal: the system is homogeneous. The operational stability of the system, however, is determined in general by the eigenvalues of the matrix of the coefficients of x_1 and x_2 in Equations 8a and 8b. This matrix is:

$$\begin{Vmatrix} D - a & -D \\ -D & D - a \end{Vmatrix}$$

The two eigenvalues of this system are (Rosen, 1985, at p. 184):

$$\lambda_1 = -a; \qquad \lambda_2 = 2D - a \tag{10}$$

While the first eigenvalue is always negative ($\lambda_1 = -a$), the second can become positive if $D > a/2$. Thus, if diffusion of the material x to the other system becomes more important than the flux in the production process (divided by 2), a positive and a negative eigenvalue coexist. The system then becomes unstable because a so-called *saddle point* (see Figure 5.1 above) is generated in the phase diagram.

The consequence is that in this case any deviation from homogeneity will be amplified, and the system can go through a phase transition. A phase transition changes the dynamics of the system irreversibly. In the case of two previously coupled dynamics, the bifurcation leads to a polarization, that is, a situation in which all the materials are either in the one cell or the other. Which subdynamic will prevail depends on the initial (and potentially random) deviation from homogeneity.

Note that this mechanism can also explain the lock-in into either Technology *A* or Technology *B* as discussed in the previous section, but then the understanding is generalized. When the equilibrium is disturbed as a specific coupling mechanism between two communication systems, the system can be expected to lock-in on either side. However, the example of reaction-diffusion dynamics enables us also to understand how the resulting lock-in between a single technology and the market can be dissolved in a later stage, for example, when the context has changed. The previous co-evolution along a single trajectory can then be 'unlocked.'

Mutual shaping between production and market dynamics can be reinforced by local conditions, but the lock-in was defined by Arthur (1988, 1989) in terms of a competition between technologies at the level of the global market. However, when a third dimension becomes relevant to any locked-in system, the new configuration may begin to tilt this system as soon as diffusion at the new interface becomes more important than half of the rate along the trajectory of the system. Because an economic production system is attracted by market opportunities, it will tend to exploit a trajectory to gain market share. Thus, the diffusion rate can be expected to increase for the technology that was locked-in. The lock-in can thus be expected to erode the condition for its existence in the longer term. The globalization of the technology triggers another subdynamic to emerge, because later alternatives can build on the lock-in as a historical given and develop that situation further.

For example, given the locked-in situation of the VHS as the dominant technology for VCR in the 1990s, the DVD became increasingly relevant as an alternative, but this did not mean that the lock-in was immediately broken. The system is also resilient (Frenken and Leydesdorff, 2000). After a while, however, when the DVD-share grows independently for other reasons (e.g., data storage), the system can be tilted and the substitution process can then be expected to generate an avalanche. As shown in Figure 3, the expectation is that the newly emerging lock-in will follow the curve of the alternative technology. The alternative technology is in this case a new technology (Nooteboom, 1999).

In other words, the techno-economic system dwells in one regime or another along a trajectory. Each regime can be considered as a suboptimum, but the system may be locked into a given suboptimum because the fitness landscape is rugged (Kauffman, 1993; Frenken, 2001). While the system has also materiality and a history in which it leaves traces, some traces may become more important than others, and a preferential pathway may be locked-in along a trajectory. Along this trajectory the techno-economic system under study is thereafter relatively stabilized against disturbances. However, the trajectory leads the system increasingly into another context. Within this other context, reaction-diffusion dynamics may open the lock-in, but the dimensionalization of the result will be substantively different from the dimensions which went previously into the lock-in because a new environment has been shaped in the meantime. This new context may have been made available to the system because of the lock-in. Using a trajectory as a path-dependency, the system has thus been moved from one regime into another.

5.6 Adding reflexivity to the non-linear dynamics

The formulas used above for the lock-in cannot be made into anticipatory models because Arthur's (1988) model evaluates the function $x = a_r + rN_A$. In this formula, the dependent variable x is not developed recursively over time, but evaluated at each moment in time. (The time axis was considered by Arthur as naturally given.) However, we can add reflexivity to the adopters who make the evaluation. If they were reflexive, one would expect them to be more inclined to stay with their natural preferences than if they were driven only by market forces. This can be modeled, for example, by adding a 5% cost to switching to a technology other than the one naturally preferred. (The additions are again boldfaced in Table 5.9 below.)