Standard quantum mechanics needs no collapse

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Introduction

First I recall some standard definitions and fix my notation.

I will use "cdot" ($\cdot$) to denote inner product, applied to 3-vectors, but also to a 3-vector and an ordered list of three operators. I will use "otimes" ($\otimes$) to denote outer (or tensor) product. An identity matrix will be denoted by I, its dimension will be evident from the context.

$(|1\rangle,\ |2\rangle)$ stands for the standard orthonormal basis of $\mathbb{C}^2$, while $|12\rangle := |1\rangle \otimes |2\rangle$ is an element of $\mathbb{C}^2 \otimes \mathbb{C}^2 = \mathbb{C}^4$, etc.

The singlet state is the vector $|\Psi\rangle = \{|12\rangle - |21\rangle\}/\sqrt{2}$. The corresponding density matrix is $\rho = |\Psi\rangle\langle\Psi| = \{|12\rangle - |21\rangle\}\{\langle 12| - \langle 21|\}/2$. It is a 4×4 complex matrix: non-negative, trace 1, self-adjoint (Hermitean). It moreover has rank 1.

The measurement directions are 3-vectors of length one, a and b. The Pauli spin matrices are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Define $\sigma = (\sigma_x, \sigma_y, \sigma_z)$. Define $\sigma_a = a \cdot \sigma$ and $\sigma_b = b \cdot \sigma$; these are 2×2 complex matrices.

Consider a two-particle system in the singlet state, on which we measure the spins in directions a and b. The two observables being measured on the composite system are $\sigma_a \otimes I$ and $I \otimes \sigma_b$. They commute, and their product is $\sigma_a \otimes \sigma_b$. Following the rules of conventional quantum mechanics, the statistics of measuring each spin separately and multiplying the two outcomes is the same as the statistics of measuring the product observable $\sigma_a \otimes \sigma_b$. The expectation value of this product is therefore $\mathrm{trace}(\rho\, \sigma_a \otimes \sigma_b) = \langle\Psi|\sigma_a \otimes \sigma_b|\Psi\rangle$. I call this function of a and b "the singlet correlation". Quantum mechanics does tell us more. It is easy to check that $\sigma_a^2 = I$, $\sigma_b^2 = I$. The eigenvalues of $\sigma_a \otimes I$ and of $I \otimes \sigma_b$ must therefore be ± 1 since the squares of the (real) eigenvalues all equal $+1$. One can compute the two mean values $\mathrm{trace}(\rho\, \sigma_a \otimes I) = 0$ and $\mathrm{trace}(\rho\, I \otimes \sigma_b) = 0$. Knowing the fact that the joint measurement of the two observables takes values in the set of four possible joint outcomes $\{(\pm 1, \pm 1)\}$, that the mean values are zero, and knowing the mean of the product, we can easily compute the complete probability distribution. The probability of outcome (x, y) is $(1 - xy\, a \cdot b)/4$ for $x, y = \pm 1$.

We assumed the trace rule for computation of mean values of observables. Notice that a collection of commuting observables can be expressed as a function of one single observable. The probability distribution of the measured values of a function of an observable is equal to the probability distribution of the same function of the measured values of the observable. Measurement of an observable results in observation of an eigenvalue of the observable.

We could alternatively have started by assuming a generalization of the Born rule to the situation of the joint measurement of a collection of commuting observables. We could then have easily derived the other just mentioned properties. The important thing to note is that these rules are all that are needed to derive the usual results on quantum teleportation, the GHZ experiment, and so on. The notion that there is a wave function which collapses on measurement was never used.

One also has rules for the state of a system after an ideal measurement of an observable, and hence for the joint probability distribution of the outcomes of a sequence of measurements. One might consider that these rules do involve a "collapse" assumption. However, this is an illusion. Certainly, after a measurement has been made and its outcomes are available to some agent, that agent's predictions about results of future measurements will be different from what they would be, without the knowledge of the intermediate outcomes. The rules together just allow one to compute the joint probability distribution of the results of a sequence of ideal measurements, by decomposing it in Markov fashion. There is no implication that the physical system under study has changed in a non-local way, though the rules certainly do bring that suggestion uncomfortably to mind.

The usual colourful language involving non-local collapse of the wave function can be thought just to be a description of a useful computational tool, not a description of physical changes to something existing in physical reality. The only thing assumed to exist are measurement outcomes, and the theory allows us to compute probability distributions of their outcomes, also in complex, composite, sequential, experimental set-ups. One can compute what one needs to know by *pretending* that the wave function collapses as suggested by the von Neumann-Lüders extension of the Born law are somehow real. One gets the right answer, as directly as possible. There is however no need to think of wave function collapse as being something physical (and necessarily non-local). Such thinking is an optional extra. Some people find it distasteful. Tastes differ.

An original "hidden variable" of quantum mechanics was quite simply the wave function. The original rules of quantum mechanics, including von Neumann-Lüders collapse, are a hidden variable theory. However, it is a non-local hidden variable theory, and it is not complete, since dispersion free states still give random measurement outcomes.

Helicity

Now we can expand

$$\rho = \{|12\rangle\langle 12| + |21\rangle\langle 21|\}/2 - \{|12\rangle\langle 21| + |21\rangle\langle 12|\}/2 = \rho_{\text{collapsed}} - \tau_{\text{remainder}}.$$

Notice the minus sign and the fact that the second term is not a density matrix. But the first term is.

This allows Bryan Sanctuary to write the singlet correlations as the difference of two terms

$$\langle\Psi|\sigma_a \otimes \sigma_b|\Psi\rangle \;=\; \text{trace}(\rho_{\text{collapsed}}\, \sigma_a \otimes \sigma_b) - \text{trace}(\tau_{\text{remainder}}\, \sigma_a \otimes \sigma_b),$$

where the first term is the correlation observed if the two particles' joint state had collapsed on separation. He attempts to give the remainder term a physical interpretation by introducing "anti-Hermitean observables". As operators, such objects have purely imaginary eigenvalues. Sanctuary considers them as quantum hidden variables. As far as I can see, his approach is to write the minus sign as the square of the square root of minus one, and to multiply both of the two occurrences of the vector of observables σ in the "trace rule formula" trace($\tau_{\text{remainder}}\, \sigma_a \otimes \sigma_b$) by i, taking as it were the real 3-vectors a and b to the "outside" of the whole expression, two occurrences of $i\sigma$ to the inside. This can be neatly expressed in higher-order tensor notation, and appears to be related to calculations in quantum field theory. He calls $i\sigma$ the helicity. He calls all observables and anti-observables "elements of reality" and he asserts that this model is "local" and "realistic". Having enlarged the meanings of the words in the dictionary of quantum mechanics, he can now assert that helicity accounts for the singlet correlations and that his model is local and realistic.

He also says he has disproved Bell's theorem through a counter-example but so far he has not provided any example. Bell's theorem states that a classical local hidden variables theory cannot reproduce certain quantum correlations without violating locality (or worse – superdeterminism). It does not say anything about what Sanctuary asks us to call a quantum local hidden variables theory. At present, we have an introduction of new terminology which allows him to state that the previously unrecognised quantum hidden variable helicity accounts for the violation of Bell's inequalities in the EPR-B situation. I don't think it can be thought of as a completion of existing basic quantum mechanics. Moreover, it does not reproduce existing basic quantum mechanical prediction, nor can it explain existing experimental results.

I plan to work out those computations in suitable notation; I have no doubt that Sanctuary's computations are correct.