Can Bell's cosine correlation be reduced to the Stern Gerlach result?

Jay R. Yablon
yablon@alum.mit.edu

Here is something I have in mind since late 2019 regarding Stern-Gerlach (SG) and the singlet correlations. The only problem is that to me it is so simple, I would be surprised if nobody has seen this before:

Start with the expectation value of the measurement functions product (Bell's (2) and (3) restated), where θ is the angle between the two (Alice and Bob) detectors:

$$\begin{aligned}&\int d\lambda \rho(\lambda) \mathcal{A}(\mathbf{a}, \lambda, \neg \mathbf{b}, \neg \mathcal{B}) \mathcal{B}(\mathbf{b}, \lambda, \neg \mathbf{a}, \neg \mathcal{A}) \\ &= \left\langle \mathcal{A}_k(\mathbf{a}, \lambda_k, \neg \mathbf{b}, \neg \mathcal{B}_k) \mathcal{B}_k(\mathbf{b}, \lambda_k, \neg \mathbf{a}, \neg \mathcal{A}_k) \right\rangle = -\mathbf{a} \cdot \mathbf{b} = -\cos\theta\end{aligned} \tag{1}$$

in which the measurement functions for any proposed LRHV theory would have to be (Bell's (1)):

$$\mathcal{A}_k(\mathbf{a}, \lambda, \neg \mathbf{b}, \neg \mathcal{B}) = \pm_k 1; \quad \mathcal{B}_k(\mathbf{b}, \lambda, \neg \mathbf{a}, \neg \mathcal{A}) = \pm_k 1 \tag{2}$$

The negation $\neg$ inside the measurement function means "is NOT a function of…," and $k = 1, 2, 3 \ldots n$ labels each individual singlet split / detection event. The view is held by many that this cosine correlation, rather than a sawtooth correlation, can only be explained by nonlocal communication between the two detection events.

Now, (1) is an expectation value which runs over the range:

$$-1 \le \left\{ \left\langle \mathcal{A}_k(\mathbf{a}, \lambda_k, \neg \mathbf{b}, \neg \mathcal{B}_k) \mathcal{B}_k(\mathbf{b}, \lambda_k, \neg \mathbf{a}, \neg \mathcal{A}_k) \right\rangle = \left\langle (\pm_k 1)(\pm_k 1) \right\rangle = \left\langle \pm_k 1 \right\rangle = -\cos\theta \right\} \le +1, \tag{3}$$

with $\pm_k 1 = (\pm_k 1)(\pm_k 1)$ computed over all four binary sign combinations, that is, $+_k 1 = (+_k 1)(+_k 1)$ or $+_k 1 = (-_k 1)(-_k 1)$ for correlated outcomes, and $-_k 1 = (+_k 1)(-_k 1)$ or $-_k 1 = (-_k 1)(+_k 1)$ for anticorrelated. So, while not suggesting that one can *predict* the outcome for any individual event, we certainly can use this to compute the *probability* of the outcome for any individual event.

In the regard, let us use ρ_+ to denote the probability of the result being $+_k 1 = (\pm_k 1)(\pm_k 1)$ and ρ_- to denote the probability of the result being $-_k 1 = (\pm_k 1)(\mp_k 1)$ for any given outcome *k*. Of course, $\rho_+ + \rho_- = 1$, because there are only two possible outcomes for each event. Then, we know that when $\theta = 0$ we will have $\left\langle \mathcal{A}_k \mathcal{B}_k \right\rangle = -1$ and therefore that $\mathcal{A}_k \mathcal{B}_k = -1$ for *each and every event*, thus probabilities $\rho_- = 1$ and $\rho_+ = 0$. Likewise, when $\theta = \pi$ we will have $\left\langle \mathcal{A}_k \mathcal{B}_k \right\rangle = +1$ and therefore $\mathcal{A}_k \mathcal{B}_k = +1$ for *each and every event*, thus $\rho_- = 0$ and $\rho_+ = 1$.

So, given that $-1 \le -\cos\theta \le +1$, we can fit these probability extrema with the correlations by calculating more generally that:

$$\begin{aligned} &\rho_+ = \tfrac{1}{2}(1-\cos\theta) = \sin^2\left(\tfrac{1}{2}\theta\right) \\ &\rho_- = 1-\rho_+ = 1-\tfrac{1}{2}(-\cos\theta+1) = \tfrac{1}{2}(1+\cos\theta) = \cos^2\left(\tfrac{1}{2}\theta\right) \end{aligned}. \tag{4}$$

Obviously, $\rho_+ + \rho_- = 1$. We may then confirm that this is a proper fit for all angles by calculating the expectation value:

$$\langle \mathcal{A}_k \mathcal{B}_k \rangle = \rho_+ \times (+1) + \rho_- \times (-1) = \tfrac{1}{2}(1-\cos\theta) - \tfrac{1}{2}(1+\cos\theta) = -\cos\theta\,. \tag{5}$$

These (4) however, are precisely the SG probabilities for two detectors in series which have an angle θ between them, but for one detail: If we add the subscript "singlet" to the probabilities defined in (4) for the singlet correlation, then a direct comparison of the two shows that:

$$\begin{aligned} &\rho_{+\mathrm{SG}} = \cos^2\left(\tfrac{1}{2}\theta\right) = \tfrac{1}{2}(1+\cos\theta) = \rho_{-\mathrm{singlet}} \\ &\rho_{-\mathrm{SG}} = \sin^2\left(\tfrac{1}{2}\theta\right) = \tfrac{1}{2}(1-\cos\theta) = \rho_{+\mathrm{singlet}} \end{aligned}. \tag{6}$$

And this in turn stems from the fact that in singlet splits, the spins toward Alice and Bob are equal and opposite, which is a feature absent from SG. Indeed, if we wish to calculate an expectation value for an SG experiment with + and – spin results in the way that we do for singlet correlations, we simply use (6) above to obtain:

$$\mathrm{E}_{\mathrm{SG}} = \rho_{+\mathrm{SG}} \times (+1) + \rho_{-\mathrm{SG}} \times (-1) = \tfrac{1}{2}(1+\cos\theta) - \tfrac{1}{2}(1-\cos\theta) = +\cos\theta\,. \tag{7}$$

So, if Bob is asked to report his "tails" as "heads" and vice versa thus correlating Alice's heads with Bob's tails and vice vera, the minus sign in $\cos\theta$ in (1) can be flipped to a plus sign, fully matching (7).

In this way, Bell's negative cosine correlations are no more and no less troublesome than the SG results. They are both decidedly non-classical. And Feynman stated that all the essential features of quantum mechanics versus classical physics can be distilled down to the Stern-Gerlach experiment. But because of (7) when compared to Bell's results stated in (1), to the extent we need to resort to instantaneous action at a distance to explain singlet correlations, we would also need to do so for SG. Conversely, if we do not have to resort to instantaneous action at a distance to explain SG, the neither do we need to do so for the singlet correlations.

Stated differently, it seems that the Bell's nonclassical cosine correlation widely thought to require action at a distance, can be reduced to just a variant of the Stern-Gerlach result; no more and no less "spooky."